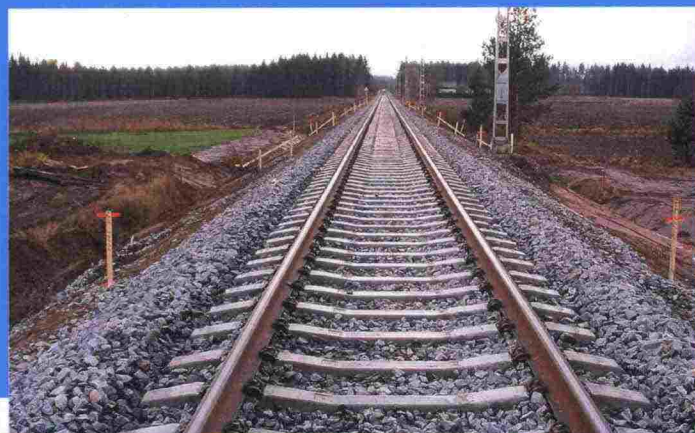


# Deformation behaviour of railway embankment materials under repeated loading

Literature review



Fabrizio Brecciaroli - Pauli Kolisoja



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repeated loading: Literature review

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**Key words:** unbound granular material, resilient deformation, permanent deformation, modelling, repeated loading triaxial test, stress, strain, stress level, moisture content, density, grading, fines content

## ABSTRACT

This is a literature survey of the topic "Granular materials under repeated loading" for a research project that deals with railway embankments width and slope. The survey was done in the Laboratory of Foundation and Earth Structures at Tampere University of Technology by MSc Fabrizio Brecciaroli under the supervision of prof. Pauli Kolisoja. The survey is the continuation of Brecciaroli's earlier research, the results of which were reported in the Publications of Finnish Rail Administration A 8/2004 ("Railway embankments with critical stability, preliminary study") and A 9/2004 ("Railway embankments width and slope, preliminary study"). The research project is going to continue with laboratory experiments, field measurements, and modelling of railway structures that supplement the study. When the project ends the results will be reported in the form of a doctoral thesis.

Railway embankments are made of unbound granular materials. An unbound granular material consists of a conglomeration, often inhomogeneous and non-isotropic, of a large number of individual, solid, and macroscopic particles in contact. One consequence of having a granular nature is that these materials, when untreated, have no inherent strength as a continuum and are unable to withstand any tension. On the other hand, they can support (small) shear stresses indefinitely. Placed in a layer and well compacted such materials possess the ability to carry traffic loads and distribute them onto the underlying layers or the sub grade. The deformation resistance of an unbound granular material depends on the applied stresses. The behaviour of unbound granular materials under compressive stresses is highly complex because of the existence of both resilient and permanent strains even at small levels of stress. The resilient strain response is important for the load-carrying ability of the embankment, while the permanent strain response characterizes the long-term deformation behaviour of the embankment.

The state-of-the-art modelling of unbound granular materials requires the use of constitutive models for all the materials in the embankment and in the sub grade. The models are divided into continuum mechanics models and particulate mechanics models. The continuum mechanics approach does not consider the granular nature of the material. Instead, a fictitious element of granular material isolated from the surrounding soil mass is assumed to have a homogenous composition. The stresses in the fictitious element are assumed to be continuously distributed, whereas the forces acting at the contact points between the particles are not taken into account. In the particulate mechanics modelling, on the other end, the interactions between the distinct particles are explicitly investigated. The basic idea of the approach is that, when the number of particles is sufficiently large, the behaviour of the group of particles can be extended to describe the macroscopic behaviour of the actual material.



Past investigations have shown clearly that the resilient response is influenced most by the level of applied stresses and the moisture content in the material. The influence of the other factors on the resilient response of granular materials is somewhat unclear. The literature reveals that researchers do not always agree on the nature or the extent of the impact of these factors and different, or even completely opposite, conclusions are often found. The discrepancies found in the literature emphasise the need for more intensive research into this area in the future.

Most of the research carried out to study the mechanical properties of granular materials deals with the resilient behaviour of these materials. In comparison, the amount of work on plastic behaviour is relatively limited. This is probably due to the fact that monitoring long term behaviour is a very time-consuming and cumbersome process when very large numbers of load applications need to be employed ( $10^5$  to  $10^6$  cycles). Furthermore, each specimen can only be subjected to a single stress path since permanent deformation behaviour is strongly influenced by stress history.

A railway embankment is exposed to a large number of load applications during its service life. Although the permanent deformation is normally a fraction of the total deformation produced by each load repetition, the gradual accumulation of a large number of these small plastic deformation increments could lead to eventual failure. Failure and also excessive permanent deformations of the embankment must of course be prevented. To do so the knowledge of the plastic behaviour of unbound granular materials is very important.

The permanent strain development in granular materials is affected by several factors: stress level, stress history, number of load applications, principal stress rotation, moisture content, density, grading and aggregate type, and physical properties of aggregate particles.

Modelling is an important necessity for an analytical approach in describing material performance. Many researchers have outlined several different procedures for predicting the resilient and permanent strain responses of unbound granular materials. However, the great number of models available is per se further evidence of the complexities that overshadow this research area. While many researchers present mathematical formulations that fit their particular data, greater effort is clearly needed in developing more general models and procedures that have a sound theoretical basis and wide applicability.

In general, two approaches are employed for mathematical modelling of the resilient behaviour of granular materials. In the first approach, the stress-strain relationship is given by a stress-dependent resilient modulus and a constant or stress-dependent Poisson's ratio (e.g. K- $\theta$  model and Uzan model). In the second approach, the stress-strain relationship is characterised by decomposing both stresses and strains into volumetric and shear components. The resilient response of the material is then defined using bulk and shear moduli instead of resilient modulus and Poisson's ratio (e.g. Boyce model and contour model). Models of this kind are usually more complex in nature and the parametric values are more difficult to determine from collected test data.

In modelling the long-term behaviour of granular materials, it is essential for the analysis to take into account the gradual accumulation of permanent strain with number of load applications and the important role played by stress conditions. Hence, one of the main objectives of research into long-term behaviour of granular materials is establishing constitutive relationships, which allow accurate predictions of permanent strain at any number of cycles at a given stress level (Lekarp 1997 and Lekarp 1999). Over the years, several researchers have attempted to outline procedures for predicting permanent strain in unbound granular materials and the permanent deformation of such materials has been modelled in a variety of ways. Some of these are logarithmic with respect to number of loading cycles, whilst others are hyperbolic, tending towards an asymptotic value of deformation with increasing numbers of load cycles.

In spite of the significant progress made over the years in understanding the behaviour, especially the resilient behaviour, of granular materials, there is still a great need for further research into developing more general and theoretically valid models and procedures for prediction of both the resilient and the permanent strain response of granular materials.

In addition, it is worth noting that the structural response of granular materials is normally studied in the laboratory, often using repeated load triaxial testing. Although the triaxial testing devices presently available are all based on the same principles, the quality and constraints of the facilities and the test procedures vary, sometimes greatly, between laboratories. It remains to be investigated into more detail how such differences influence the test results. The common repeated load triaxial apparatus applies repetitive loading on cylindrical materials for a range of specified stress conditions. The output is deformation (shortening of the cylindrical sample) versus number of load cycles (usually 50,000) for a particular set of stress conditions. Multi-stage repeated load triaxial tests are used to predict deformation behaviour for a range of stress conditions. In order to provide a reasonable simulation of traffic-type loading using triaxial equipment, the loading system should be able to cycle both the vertical (deviator) stress and the confining pressure in phase, and at levels and frequencies corresponding to the actual field conditions.



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**Avainsanat:** karkearakeinen materiaali, palautuva muodonmuutoskäyttäytyminen, pysyvä muodonmuutoskäyttäytyminen, mallintaminen, kolmiakselinen toistokuormituskoe, jännitys, muodonmuutos, jännitystaso, vesipitoisuus, tiheys, rakeisuus, hienoaainespitoisuus

## TIIVISTELMÄ

Tämä kirjallisuusselvitys aiheesta ”toistokuormitetut ratapengermaamateriaalit” on osa tutkimusprojektia, joka käsittelee ratapenkereitten leveyttä ja luiskakaltevuutta. Selvityksen on tehnyt diplomi-insinööri Fabrizio Brecciaroli Maa- ja pohjarakenteiden laitoksella professori Pauli Kolisojan ohjauksessa. Selvitys on jatkoa Brecciarolin aiempaan tutkimukseen, jonka tulokset on raportoitu Ratahallintokeskuksen julkaisuissa A 8/2004 ”Stabilitetiltaan kriittiset ratapenkereet, esitutkimus” ja A 9/2004 ”Ratapenkereitten leveys ja luiskakaltevuus, esitutkimus”. Tutkimusprojekti jatkuu laboratoriokokeilla, maastomittauksilla ja ratarakenteiden mallintamisella, jotka täydentävät tutkimusta. Kun projekti päättyy, tulokset raportoidaan väitöskirjan muodossa.

Ratapenkereet muodostuvat karkearakeisista maamateriaaleista. Karkearakeinen materiaali on usein epähomogeeninen ja anisotrooppinen konglomeraatio, joka koostuu suuresta määrästä toistensa kanssa kosketuksessa olevia yksittäisiä makroskooppisia rakeita. Yksi seuraus rakeisesta luonteesta on, että käsiteltäessä karkearakeisilla materiaaleilla ei ole luontaista lujuutta kontinuumina eivätkä ne kestä käytännössä lainkaan vetojännitystä. Toisaalta ne voivat kestää suurehkoja puristusjännityksiä ja kohtuullisia leikkausjännityksiä loputtomasti. Kun tällaiset materiaalit rakennetaan kerroksittain ja tiivistetään hyvin, ne kykenevät kantamaan liikennekuormia ja jakamaan kuormat alla oleviin kerroksiin tai pohjamaahan. Karkearakeisten materiaalien muodonmuutosvastus riippuu kuitenkin aina vaikuttavista jännityksistä.

Karkearakeisten materiaalien käyttäytyminen puristusjännityksen alaisuudessa on hyvin monimutkaista, koska pienilläkin jännitystasoilla esiintyy sekä palautuvia että pysyviä muodonmuutoksia. Palautuvat muodonmuutokset vaikuttavat ratapenkereen kykyyn kantaa ja jakaa kuormia, kun taas pysyvät muodonmuutokset vaikuttavat ratapenkereen pitkäaikaiseen muodonmuutoskäyttäytymiseen. Toisin sanoen ratapenkereissä esiintyy palautuvia muodonmuutoksia, jotka palautuvat kunkin kuormituskerran jälkeen, ja pysyviä muodonmuutoksia, jotka kertyvät kunkin kuormituskerran jälkeen.

Karkearakeisten materiaalien tämänhetkistä osaamisen huipputasoa edustava mallintaminen vaatii konstutiivisten mallien käyttöä kaikille materiaaleille ratapenkereessä ja pohjamaassa. Mallit jaetaan kontinuumimekaniikan malleihin ja partikkelimekaniikan malleihin. Kontinuumimekaniikan menettelytapa ei ota huomioon materiaalin rakeista luonnetta. Sen sijaan ympäröivästä maamassasta eristetyn karkearakeisen materiaalin kuvitteellisen elementin oletetaan olevan homogeeninen koostumukseltaan. Jännityksien kuvitteellisessa elementissä oletetaan olevan jatkuvasti jakautuneita, kun taas rakeiden välissä oleviin kosketuspisteisiin vaikuttavia voimia ei



oteta huomioon. Toisaalta partikkelimekaniikan mallinnuksessa vuorovaikutuksia partikkelien välillä tutkitaan eksplisiittisesti. Menettelytavan perusidea on, että partikkelien määrä ollessa riittävän suuri partikkeliryhmän käyttäytyminen voidaan laajentaa kuvaamaan kyseisen materiaalien makroskooppista käyttäytymistä.

Aiemmat tutkimukset ovat selvästi osoittaneet, että palautuviin muodonmuutoksiin vaikuttavat erityisesti jännitystaso ja materiaalin vesipitoisuus. Joidenkin muiden tekijöiden osalta johtopäätökset niiden vaikutuksesta karkearakeisen materiaalin palautuviin muodonmuutoksiin ovat epäyhtenäisempiä ja jossakin määrin jopa keskenään ristiriitaisia. Tällöin on kuitenkin syytä muistaa se, että tuloksia on eri yhteyksissä saatu varsin paljonkin toisistaan poikkeavilla materiaaleilla ja myös erilaisilla koejärjestelyillä.

Suurin osa tutkimuksista, jotka on tehty karkearakeisten materiaalien mekaanisista ominaisuuksista, käsittelee näiden materiaalien palautuvaa muodonmuutoskäyttäytymistä. Toisaalta pysyvistä muodonmuutoskäyttäytymisestä tehty tutkimus on suhteellisen vähäistä. Tämä johtuu todennäköisesti siitä, että pitkäaikaisen käyttäytymisen monitorointi on hyvin aikaa vievä ja hankala prosessi, johon tarvitaan erittäin suuri määrä kuormituskertoja ( $10^5 - 10^6$  sykliä). Jokainen näyte voidaan lisäksi altistaa periaatteessa vain yhdellä jännitystasolla tehtävälle kuormitukselle, koska pysyvä muodonmuutoskäyttäytyminen riippuu huomattavasti näytteen jännityshistoriasta.

Ratapenger altistuu suurelle määrälle kuormituskertoja käyttöikänsä aikana. Vaikka pysyvä muodonmuutos on tavallisesti hyvin pieni osa yksittäisen kuormituskerran aiheuttamasta kokonaismuodonmuutoksesta, näiden pienten pysyvien muodonmuutosten asteittainen kerääntyminen voi johtaa lopulliseen murtumiseen. Ratapenkereen murtuminen sekä liian suuret pysyvät muodonmuutokset tulee tietysti estää. Tämän tavoitteen saavuttamiseksi karkearakeisten materiaalien pysyvän muodonmuutoskäyttäytymisen tunteminen on hyvin tärkeää.

Palautuvien muodonmuutosten tavoin myös pysyvien muodonmuutosten kehittymiseen karkearakeisissa materiaaleissa vaikuttavat monet tekijät kuten jännitystaso, jännityshistoria, kuormituskertojen määrä, pääjännitysten kiertyminen, vesipitoisuus, tiheys, rakeisuus, materiaalin tyyppi sekä materiaalin partikkeleiden fysikaaliset ominaisuudet.

Mallintaminen on välttämätöntä karkearakeisten materiaalien mekaanisen toiminnan kuvaamisessa. Monet tutkijat ovat esittäneet erilaisia menetelmiä ennustamaan karkearakeisten materiaalien palautuvia ja pysyviä muodonmuutoksia. Suuri määrä olemassa olevia malleja on toisaalta sinällään todiste tutkimusalueen monimutkaisuudesta ja hankaluudesta. Monet tutkijat ovat esittäneet mallinnusmenetelmiä, jotka sopivat heidän omiin tutkimustuloksiinsa. Tarvitaan kuitenkin enemmän työtä kehittämään yleisempiä malleja, joilla olisi tukeva teoreettinen perusta ja laaja käytettävyys.

Karkearakeisten materiaalien palautuvan muodonmuutoskäyttäytymisen matemaattisessa mallintamisessa käytetään yleensä kahta menettelytapaa. Ensimmäisessä menettelytavassa jännitys-muodonmuutos-suhde annetaan jännityksestä riippuvana resilient-moduulina ja vakiona tai jännityksestä riippuvana Poissonin lukuna (esim. K- $\theta$  malli ja Uzanin malli). Toisessa menettelytavassa jännitys-muodonmuutos-suhde

kuvataan jakamalla jännitykset ja muodonmuutokset tilavuudenmuutos- ja leikkausmuodonmuutoskomponentteihin. Materiaalin palautuvat muodonmuutokset määritellään tällöin käyttäen tilavuusmoduulia ja leikkausmoduulia resilient-moduulin ja Poissonin luvun sijaan. Tällaiset mallit ovat yleensä monimutkaisempia luonteeltaan, ja parametriarvojen fysikaalinen merkitys on usein vaikeampaa päätellä kerätyistä mittaustuloksista.

Karkearakeisten materiaalien pitkäaikaisen käyttäytymisen mallintamisessa on olennaista ottaa huomioon asteittainen pysyvien muodonmuutosten kerääntyminen, kuormituskertojen määrä ja jännitysolosuhteiden tärkeä rooli. Siksi yksi tärkeimmistä tavoitteista karkearakeisten materiaalien pitkäaikaiskäyttäytymiseen liittyen on kehittää konstitutiivinen malli, joka mahdollistaisi pysyvien muodonmuutosten ennustamisen millä tahansa kuormituskertojen määrällä annetulla jännitystasolla (Lekarp 1997 ja Lekarp 1999). Vuosien varrella useat tutkijat ovat yrittäneet kehittää menetelmiä karkearakeisten materiaalien pysyvien muodonmuutosten ennustamiseen, ja materiaalien pysyvää muodonmuutuskäyttäytymistä on mallinnettu usein eri tavoin. Jotkut malleista ovat logaritmisia kuormituskertojen määrän suhteen, toiset taas ovat hyperbolisia lähestyen asympotoottisesti muodonmuutoksen raja-arvoa kuormituskertojen määrän kasvaessa.

Huolimatta vuosien varrella tapahtuneesta edistyksestä karkearakeisten materiaalien muodonmuutuskäyttäytymisen ja erityisesti palautuvan muodonmuutuskäyttäytymisen ymmärtämisessä, kirjallisuustutkimuksen perusteella on todettavissa, että edelleen on olemassa tarvetta tutkimukselle, jonka avulla kehitettäisiin yleisempiä ja teoreettisesti parempia malleja ja menetelmiä karkearakeisten materiaalien sekä palautuvan että pysyvän muodonmuutuskäyttäytymisen ennustamiseen.

On huomattavaa myös, että karkearakeisten materiaalien käyttäytymistä tutkitaan yleensä laboratoriossa käyttämällä kolmiaksaalisia toistokuormituskokeita. Vaikkakin nykyisin tarjolla olevat kolmiaksaalikolaitteet perustuvat samaan perusperiaatteeseen, koejärjestelyjen laatu ja rajoitteet vaihtelevat laboratorioden välillä. Pitäisi myös selvittää tarkemmin, kuinka tällaiset erot vaikuttavat koetuloksiin. Tavallinen kolmiaksaalikolaitte käyttää toistuvaa kuormaa sylinterinmuotoiselle näytteelle erilaisissa jännitysolosuhteissa. Keskeisin mitattava suure on näytteen aksiaalinen muodonmuutos (sylinterinmuotoisen näytteen lyhenemä) kuormituskertojen määrän funktiona (tavallisesti suuruusluokkaa 50000) tietyissä jännitysolosuhteissa. Moniportaisia kolmiaksaalikokeita käytetään muodonmuutuskäyttäytymisen mittaamiseen useissa erilaisissa jännitysolosuhteissa. Jotta mahdollisimman realistinen simulaatio liikenteen tyypiselle kuormitukselle käyttäen kolmiaksaalikolaitetta voitaisiin toteuttaa, kuormitussysteemin tulisi pystyä tuottamaan sekä pystysuuntainen pääjännitysero että sellipaine sellaisilla tasoilla ja taajuuksilla, jotka vastaavat todellisia kenttäolosuhteita.



## FOREWORD

This is a literature survey of the topic "granular materials under repeated loading" for a research project that deals with railway embankments width and slope. The survey was done in the Laboratory of Foundation and Earth Structures at Tampere University of Technology under the supervision of prof. Pauli Kolisoja. The survey is written by MSc Fabrizio Brecciaroli.

The main subscriber and sponsor of the research was the Finnish Rail Administration, whose representative Matti Levomäki guided the work. Additional financial support was provided by the Graduate School of Tampere University of Technology.

The research project continues with laboratory experiments, field measurements, and modelling of railway structures that supplement the study. When the project ends the results will be reported in the form of a doctoral thesis.

Helsinki, June 2006

Finnish Rail Administration  
Rail Network Department

## TABLE OF CONTENTS

ABSTRACT .....	3
TIIVISTELMÄ.....	6
FOREWORD.....	9
1 INTRODUCTION.....	13
1.1 Background.....	13
1.2 Objectives.....	15
1.3 Content of research and structure of the literature review.....	15
2 INTRODUCTION TO THE MECHANICAL BEHAVIOUR OF GRANULAR MATERIALS .....	17
2.1 Stresses in engineering materials.....	17
2.2 Deformation characteristics of unbound granular materials.....	21
2.3 Interaction between water and granular materials.....	27
3 BASIC APPROACHES IN MODELLING OF GRANULAR MATERIALS .....	29
3.1 Introduction .....	29
3.2 Models of continuum mechanics.....	29
3.2.1 Principle of continuum mechanics modelling approach.....	29
3.2.2 Linear elastic material models.....	30
3.2.3 Non-linear elastic material models.....	35
3.3 Models of particulate mechanics.....	36
4 FACTORS AFFECTING RESILIENT DEFORMATION.....	39
4.1 Introduction .....	39
4.2 Stress level.....	39
4.3 Loading characteristics.....	43
4.3.1 Stress history .....	43
4.3.2 Number of load cycles.....	44
4.3.3 Load duration and load frequency .....	44
4.3.4 Load sequence .....	44
4.4 Strain level.....	44
4.5 Density.....	46
4.6 Grading characteristics .....	49
4.6.1 Introduction .....	49
4.6.2 Grain size distribution .....	49
4.6.3 Fines content.....	54
4.6.4 Maximum grain size .....	56
4.7 Moisture content.....	58
4.8 Particle characteristics .....	66
5 MODELLING OF RESILIENT DEFORMATION BEHAVIOUR .....	72
5.1 Introduction .....	72
5.2 Models based on resilient modulus and Poisson's ratio.....	73
5.2.1 Definition of resilient modulus and Poisson's ratio .....	73
5.2.2 Resilient modulus as a function of hydrostatic stress.....	74

5.2.3 Resilient modulus as a function of deviatoric stress.....	75
5.2.4 Influence of different factors on the equations parameters .....	79
5.2.5 Poisson's ratio as a function of stress.....	82
5.3 Resilient modulus using shear-volumetric approach.....	86
<b>6 FACTORS AFFECTING PERMANENT DEFORMATION.....</b>	<b>94</b>
6.1 Introduction .....	94
6.2 Stress level.....	95
6.3 Stress history .....	99
6.4 Number of load applications.....	102
6.5 Principal stress rotation .....	105
6.6 Moisture content.....	109
6.7 Density.....	117
6.8 Fines content and grading.....	121
6.8.1 Fines content.....	121
6.8.2 Grading .....	122
6.9 Physical properties of aggregate particles .....	125
6.9.1 Introduction .....	125
6.9.2 Grain type and shape .....	125
6.9.3 Particle Surface Roughness .....	126
6.9.4 Electro-chemical properties.....	126
6.10 Temperature.....	126
<b>7 MODELLING OF PERMANENT DEFORMATION BEHAVIOUR .....</b>	<b>128</b>
7.1 Introduction .....	128
7.2 Correlation between static and dynamic loading tests .....	129
7.3 Correlation between resilient and plastic behaviour.....	130
7.4 Permanent deformation moduli .....	131
7.5 Permanent strain and number of cycles.....	132
7.6 Permanent strain and its relationship to stresses .....	137
7.7 The Shakedown Theory .....	141
<b>8 REPEATED LOAD TRIAXIAL APPARATUS .....</b>	<b>151</b>
8.1 Introduction .....	151
8.2 Repeated load triaxial test: advantages and drawbacks.....	151
8.3 Constant (CCP) and variable (VCP) confining pressure tests.....	152
8.4 General guidelines on specimen size.....	153
8.5 General guidelines on instrumentation.....	154
8.6 University of Nottingham's repeated load triaxial apparatus.....	154
8.6.1 Introduction .....	154
8.6.2 Repeated load triaxial tests by Lekarp (1997).....	156
8.6.3 Repeated load triaxial tests by Werkmeister (2003).....	158
8.7 Stockholm Royal Institute of Technology's repeated load triaxial apparatus.....	160
8.7.1 Introduction .....	160
8.7.2 Description of the repeated load triaxial apparatus used by Lekarp (1999).....	161
8.7.3 Description of the test series performed by Lekarp (1999).....	169
8.8 Laboratoire des Ponts et Chaussées' repeated load triaxial apparatus .....	169
8.8.1 Introduction .....	169
8.8.2 Description of the repeated load triaxial test apparatus.....	170
8.8.3 Description of the test procedure.....	172



8.9 Trondheim NTNU/SINTEF's repeated load triaxial apparatus.....	173
8.10 Triaxial tests with measurement of negative pore pressure.....	174
9 CONCLUSIONS .....	180
9.1 Resilient deformation behaviour .....	180
9.2 Permanent deformation behaviour .....	181
REFERENCES .....	183

## APPENDIX 1

## Résumé in Finnish

## 1 INTRODUCTION

### 1.1 Background

Due to the severity of the climatic conditions in Finland, the thickness of the structural layers in all the high-grade railway lines is considerably bigger than in many warmer countries. On the other hand, in order to reduce the expenses, the railway embankments have had to be designed and constructed with relatively narrow widths and steep slopes. As a result, under repeated train loading, gradual, permanent deformation occurs within the railway embankment. Eventually, the deformations lead to the railway embankment being flattened.

The optimum width and slope of a railway embankment has great importance, because the widening of the embankment and/or the flattening of its slopes would require considerably high investments. In contrast, the flattening of the railway embankment increases the need for maintenance. As an indirect consequence, the repeated tamping of the track increases the degradation of the ballast. The choices related to the shape and width of the embankment have an especially big importance in those rail sections where the allowed axle loads and train speeds are being increased, because an increase of the trainload always influences the embankment deformation.

The values of the static critical loads are generally very high on the railway network compared to the allowed trainloads (Brecciaroli and Kolisoja 2004). However, also the displacements are then so big that, although the real displacements would be only a small fraction of these values, in practice they would not be elastic anymore. This means that stage-by-stage the embankment would collapse.

Railway embankments are made of unbound granular materials. These materials are different from most natural soils in their physical characteristics and also in their response to applied cyclic load. One consequence of having a granular nature is that these materials, when untreated, have no inherent strength as a continuum and are unable to withstand any tension. On the other hand, they can support (small) shear stresses indefinitely. Only gravity and applied outside forces create intergranular contact pressures and frictional forces that are able to resist relative movement of particles. Granular interlocking contributes to this type of strength. Placed in a layer and well compacted such materials possess the ability to carry traffic loads and distribute them onto the underlying layers or the sub grade. The deformation resistance of an unbound granular material depends on the applied stresses.

The behaviour of unbound granular materials under compressive stresses is highly complex because of the existence of both resilient and permanent strains even at small levels of stress. Unbound granular materials in a railway embankment are subjected to a large number of load cycles during the service life of the embankment. The deformational response of these materials under repeated, traffic-type loading is defined by a resilient deformation response and a permanent deformation response. The resilient strain response is important for the load-carrying ability of the embankment, while the permanent strain response characterizes the long-term deformation behaviour of the embankment. In other words, these layers exhibit a



combination of resilient strains that are recovered after each load cycle, and permanent strains that accumulate with every load cycle.

Since 1960, numerous research efforts have been devoted to characterising the resilient behaviour of unbound granular materials. It is well known that these materials exhibit a complex non-linear and time-dependent elasto-plastic response under repeated, traffic-type loading. To deal with this non-linearity and to differentiate from the traditional elasticity theories, the resilient response of unbound granular materials is usually defined by resilient modulus  $M_r$  and Poisson's ratio  $\nu$ . Alternatively, the use of shear and bulk moduli has been suggested. For design purposes, it is important to consider how the materials involved in the design react to and the resilient behaviour varies with changes in various influencing factors. From the studies found in the literature, it appears that the resilient behaviour of unbound granular materials under repeated, traffic-type loading depends, with varying degrees of importance, on many influential factors such as stress level, moisture content, density, grading, fines content, maximum particle size, aggregate type, particle shape, load duration, load frequency, load sequence, and compaction.

Past investigations have shown clearly that the resilient response is influenced most by the level of applied stresses and the moisture content in the material. The influence of the other factors on the resilient response of granular materials is somewhat unclear. The literature reveals that researchers do not always agree on the nature or the extent of the impact of these factors and different, or even completely opposite, conclusions are often found. The discrepancies found in the literature emphasise the need for more intensive research into this area in the future.

In addition, it is worth noting that the structural response of granular materials is normally studied in the laboratory, often using repeated load triaxial testing. Although the triaxial testing devices presently available are all based on the same principles, the quality and constraints of the facilities and the test procedures vary, sometimes greatly, between laboratories (Hoff 2004 and CEN 2004). It remains to be investigated into more detail how such differences influence the test results. Even when the same material is tested using the same equipment, some variations between laboratories and also some variations for parallel samples tested at the same laboratory can be found (Hoff et al. 2005).

A railway embankment is exposed to a large number of load applications during its service life. Although the permanent deformation is normally a fraction of the total deformation produced by each load repetition, the gradual accumulation of a large number of these small plastic deformation increments could lead to eventual failure. Failure and also excessive permanent deformations of the embankment must of course be prevented. To do so the knowledge of the plastic behaviour of unbound granular materials is very important.

Most of the research carried out over the years has concentrated on the resilient behaviour of granular materials. In comparison to resilient behaviour, less research has been devoted to plastic response and permanent deformation development in granular materials. This is perhaps due to the practical difficulties in studying permanent deformation behaviour. While resilient tests are fairly quick and each laboratory specimen

can be studied for a great number of stresses, the monitoring of the build-up of permanent deformation in unbound granular materials and permanent deformation tests are very time-consuming and usually separate specimens are required for each set of stresses. As a result, greater advances have been made in understanding the resilient response than the long-term performance of granular materials.

The permanent strain development in granular materials is affected by several factors: stress level, stress history, number of load applications, principal stress rotation, moisture content, density, grading and aggregate type, and physical properties of aggregate particles.

## **1.2 Objectives**

The main objectives of the research have been:

1. to review the up-to-date knowledge on the deformation behaviour of unbound granular materials in a railway embankment under repeated loading,
2. to determine how and to what extent factors such stress level, moisture content, density, grading, fines content, maximum particle size, aggregate type, particle shape, load duration, load frequency, load sequence, and compaction influence both resilient and permanent deformation behaviour of unbound granular material in a railway embankment under repeated loading,
3. to review the up-to-date knowledge on the available models used to model both resilient and permanent deformation behaviour of unbound granular material in a railway embankment under repeated loading,
4. to review the different repeated load triaxial test apparatus most commonly used around the world to study the response of unbound granular material under cyclic, traffic-type loads.

## **1.3 Content of research and structure of the literature review**

An overview of the mechanical behaviour of granular materials is given in Chapter 2. Stresses in engineering materials are reviewed and general guidelines on the deformation characteristics of unbound granular materials are given. Finally, the interaction between water and granular materials is analysed.

Chapter 3 deals with modelling of granular materials from a general point of view. The models are divided into models of continuum mechanics and models of particulate mechanics.

Chapter 4 and Chapter 6 analyse the effect of different factors (stress level, moisture content, density, grading, fines content, maximum particle size, aggregate type, particle shape, load duration, load frequency, load sequence, compaction, etc) on the resilient and permanent deformation behaviour of unbound granular materials respectively.



Modelling is an important necessity for an analytical approach in describing material performance. Many researchers have outlined different procedures for predicting the resilient and permanent strain responses of granular materials. Chapter 5 and Chapter 7 give an overview of the models that have been used so far to model the resilient and permanent deformation behaviour of unbound granular materials respectively.

Chapter 8 deals with the different repeated load triaxial test apparatus used around the world to study the response of unbound granular material under cyclic, traffic-type loads.



## 2 INTRODUCTION TO THE MECHANICAL BEHAVIOUR OF GRANULAR MATERIALS

### 2.1 Stresses in engineering materials

The stress state acting on a given infinitesimal cubic soil element can be defined by its normal and shear stress components oriented according to a cartesian reference system, as illustrated in Figure 2.1:1.

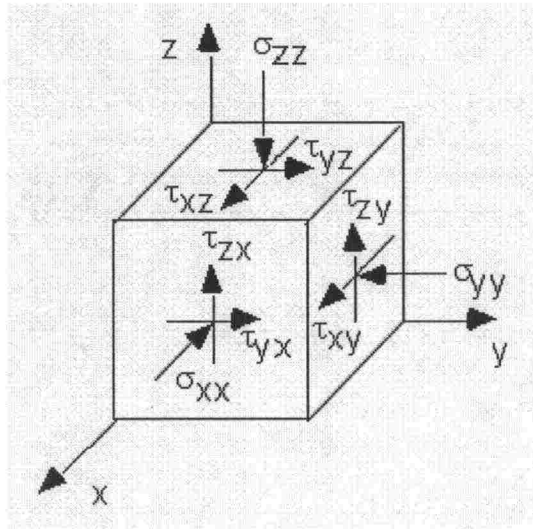


Figure 2.1:1 Stress components on a cubic soil element.

The stress components are often given in a matrix form. Stress is a tensor which can be represented by a matrix in Cartesian coordinates:

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}. \quad (\text{Eq. 2.1:1})$$

The indices refer to the coordinate system x, y, z. The first index specifies the direction in which the stress component acts, and the second identifies the orientation of the surface upon which it is acting.

We can use equilibrium of moments to study the relationship between  $\tau_{xy}$  and  $\tau_{yx}$ . Assuming a unit thickness, the horizontal and vertical faces are of area  $dx$  and  $dy$  respectively (Figure 2.1:2).

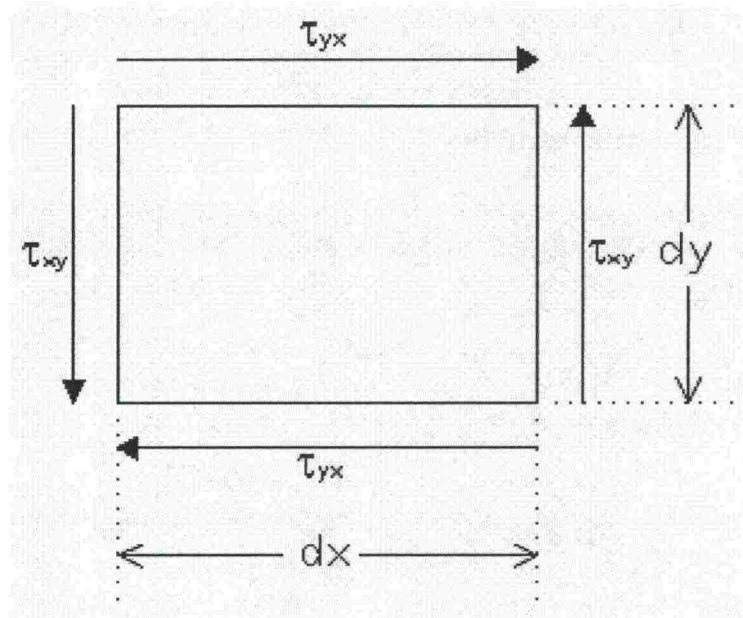


Figure 2.1:2 Two-dimensional state of shear stress.

Let us now take moments about the centre:

$$2 \cdot \tau_{xy} \cdot dy \cdot \frac{dx}{2} - 2 \cdot \tau_{yx} \cdot dx \cdot \frac{dy}{2} = 0, \quad (\text{Eq. 2.1:2})$$

i.e.

$$\tau_{xy} = \tau_{yx}. \quad (\text{Eq. 2.1:3})$$

We can look at other planes, the y-z plane and the z-x plane, and do similar calculation to show that

$$\tau_{yz} = \tau_{zy} \quad (\text{Eq. 2.1:4})$$

and

$$\tau_{xz} = \tau_{zx}. \quad (\text{Eq. 2.1:5})$$

The existence of these relations is the reason for there being only six independent stress components. The stress state is uniquely defined by means of these six stress components referring to a specific coordinate system. The matrix representing the state of stress is symmetric. The actual values of the 3 normal and 3 shear components vary according to the orientation of the reference system.

It is often useful to use principal stresses rather than Cartesian stress components especially when formulating material models. The principal stresses  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  (Figure 2.1:3) are stresses acting on a plane where no shear stresses occur. The principal directions are the directions of the normal vectors of these planes. The stresses  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  are the major, intermediate, and minor principal stresses respectively. Note that compression is positive in geotechnical applications, which makes  $\sigma_1$  the largest

compressive stress. Since the principal stresses represent the greatest magnitude of stress that can exist in a particular loading situation, they are of great importance in engineering design.

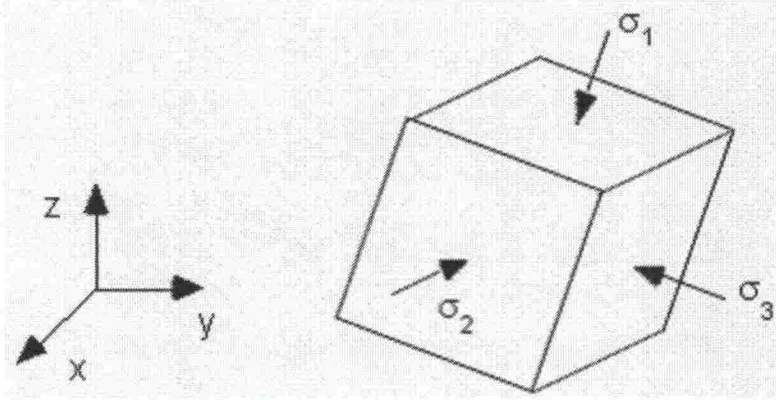


Figure 2.1:3 Principal stresses.

It can be proved that for any general state of stress through any point in a body, three mutually perpendicular planes exist on which no shear stresses act. Given an arbitrary stress state, we may use Cauchy's law to show that three principal stresses and correspondingly three principal directions exist. According to Cauchy's law the stress traction acting on a plane with normal vector  $\underline{n} = [n_1, n_2, n_3]$  is

$$\underline{t} = \underline{\underline{\sigma}} \bullet \underline{n}. \quad (\text{Eq. 2.1:6})$$

Let us assume that the stress traction  $\underline{t}$  is directed along the plane normal  $\underline{n}$  and that the scalar value of the normal stress is  $\sigma$ . Equation 2.1:6 becomes

$$\underline{t} = \sigma \bullet \underline{n}. \quad (\text{Eq. 2.1:7})$$

Combining the two expressions for the stress traction we obtain an eigenvalue problem

$$(\underline{\underline{\sigma}} - \sigma \bullet \underline{\underline{I}}) \bullet \underline{n} = 0. \quad (\text{Eq. 2.1:8})$$

The identity matrix  $\underline{\underline{I}}$  has diagonal elements equal to 1 and every other element is zero. A non-trivial solution for the eigenvalue problem requires that

$$\det(\underline{\underline{\sigma}} - \sigma \bullet \underline{\underline{I}}) = \det \begin{vmatrix} \sigma_{xx} - \sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} - \sigma \end{vmatrix} = 0 \quad (\text{Eq. 2.1:9})$$

and



$$\det \begin{vmatrix} \sigma_{xx} - \sigma & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} - \sigma \end{vmatrix} = \sigma^3 - I_1 \cdot \sigma^2 - I_2 \cdot \sigma - I_3 = 0, \quad (\text{Eq. 2.1:10})$$

where

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}, \quad (\text{Eq. 2.1:11})$$

$$I_2 = -\sigma_{xx} \cdot \sigma_{yy} - \sigma_{xx} \cdot \sigma_{zz} - \sigma_{yy} \cdot \sigma_{zz} + \sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2, \quad (\text{Eq. 2.1:12})$$

$$I_3 = \det(\underline{\underline{\sigma}}). \quad (\text{Eq. 2.1:13})$$

The equation above is guaranteed to provide three real roots (three values of  $\sigma$ ). This is due to mathematical properties of the real symmetric matrix  $\underline{\underline{\sigma}}$ . The three values of the scalar  $\sigma$  are the principal stresses  $\sigma_1, \sigma_2$ , and  $\sigma_3$ . The three principal stresses act on the three planes with plane normals  $\underline{n}_1, \underline{n}_2, \underline{n}_3$  solved from

$$(\underline{\underline{\sigma}} - \sigma_i \cdot \underline{\underline{I}}) \cdot \underline{n}_i = \underline{0}. \quad (\text{Eq. 2.1:14})$$

Since the normal vectors  $\underline{n}_1, \underline{n}_2, \underline{n}_3$  are orthogonal, the stresses  $\sigma_1, \sigma_2, \sigma_3$  are found as normal stresses on a cubical soil element oriented along the principal directions  $\underline{n}_1, \underline{n}_2, \underline{n}_3$ .

The principal stresses  $\sigma_1, \sigma_2$ , and  $\sigma_3$  are invariant quantities, i.e. quantities that do not depend on the orientation of the chosen reference system. Since the principal stresses are the solution of a third order equation, the coefficients  $I_1, I_2$ , and  $I_3$  of the equation are invariant quantities as well. Note that since the stress matrix has six independent components, there should exist six invariant quantities to describe the stress state uniquely. These invariant quantities are the principal stresses  $\sigma_1, \sigma_2$ , and  $\sigma_3$  and the three principal stress directions, or, equivalently, the stress invariants  $I_1, I_2, I_3$  and the three principal stress directions.

In order to simplify the stress-strain analysis, the applied stresses can be divided into volumetric (non-deviatoric) and shear (deviatoric) components. As a result, the general stress state in a three-dimensional system can be given by the following functions:

$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \sigma_{mean} = \frac{I_1}{3}, \quad (\text{Eq. 2.1:15})$$

$$\tau_{oct} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \frac{\sqrt{2}}{3} \cdot \sqrt{I_1^2 + 3I_2}. \quad (\text{Eq. 2.1:16})$$

Here,  $\sigma_{oct}$  is the non-deviatoric (volumetric) stress and  $\tau_{oct}$  is the deviatoric (shear) stress. Both  $\sigma_{oct}$  and  $\tau_{oct}$  are called stress invariants, as they are functions of the principal

stresses but independent of the orientation of the axes. In the case of linearly elastic materials the non-deviatoric stress does not cause distortion or damage. It causes volumetric change and it can change the material's state and behaviour. Instead, the deviatoric stress causes shear distortion and material damage.

The stress state can always be broken down into deviatoric (shear) and non-deviatoric (volumetric) components:

$$\begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix} = \begin{vmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{vmatrix} + \begin{vmatrix} \sigma_1 - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_2 - \sigma_m & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_3 - \sigma_m \end{vmatrix}. \quad (\text{Eq. 2.1:17})$$

Total Stress  
State

Hydrostatic  
Stress State  
( $\Delta$  Volume)

Stress Deviation  
(Distortion)

When the behaviour of granular materials is analysed at the macroscopic level, the observed deformation may be volumetric, shear, or a combination of the two. Volume changes occur due to changes in particle arrangements and orientation and/or contraction or expansion of the whole soil structure without modifications of the single soil particles. Shear strains are governed by sliding friction between contracting particles, interlocking friction between adjacent particles, and disruption on interlocking when particles are forced to slide over the adjacent particles with a large distortion of the grain arrangement.

## 2.2 Deformation characteristics of unbound granular materials

A granular material consists of a conglomeration, often inhomogeneous and non-isotropic, of a large number of individual solid, macroscopic particles in contact with each other. The particles are of different shapes and sizes and are characterised by a loss of energy whenever the particles interact (the most common example is friction when two or more grains collide). The pore space between the particles is filled with air, water, or a mixture of the two. The constituents that compose granular materials must be large enough so that they are not subject to thermal motion fluctuations. Thus, the lower size limit for grains in granular material is about 1  $\mu\text{m}$ . Granular materials are ubiquitous in nature and are the second most manipulated material in the industry (the first is water).

Unbound granular materials are different from soils not only in their physical characteristics but also in their response to applied cyclic load. One consequence of having a granular nature is that these materials, when untreated, have no inherent strength as a continuum and are unable to withstand any tension. On the other hand, they can support (small) shear stresses indefinitely. Only gravity and applied outside forces create intergranular contact pressures and frictional forces that are able to resist relative movement of particles. Granular interlocking contributes to this type of strength. Placed in a layer and well compacted, such materials possess the ability to carry train traffic loads and distribute them onto the underlying layers or the sub grade. The deformation resistance of an unbound granular material depends on the applied



stresses. As it can be seen from Figure 2.2:1, an increase in stress causes an increase in strain, i.e. a decrease in material's resistance to further deformation.

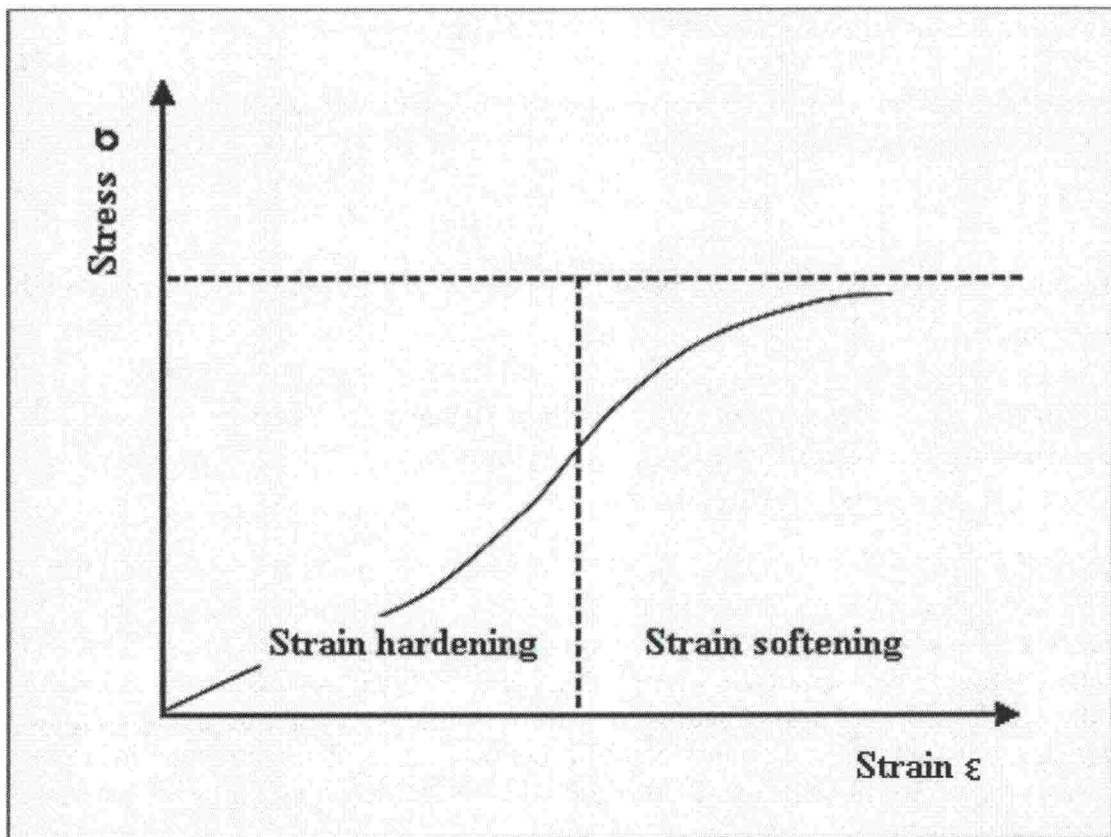


Figure 2.2:1 Stress-strain behaviour of unbound granular materials (Werkmeister 2003).

At low levels of stress, the stiffness of the material increases as the stress increases (strain hardening). The particles are forced into new interlocked positions and the compacted material becomes more closely packed and harder to move. As the stress approaches failure, the stiffness of the material decreases (strain softening) until the material eventually reaches failure. The non-linearity of the stress-strain relationship is affected by the structure of the grain assembly.

The behaviour of unbound granular materials under compressive stresses is highly complex because of the existence of both resilient and permanent strains even at small stresses. Unbound granular materials in a railway embankment are subjected to a large number of load cycles during the service life of the embankment. The deformational response of these materials under repeated, traffic-type loading is defined by a resilient response and a permanent strain response. The resilient response is important for the load-carrying ability of the embankment, while the permanent strain response characterizes the long-term performance of the embankment. In other words, these layers exhibit a combination of resilient strains that are recovered after each load cycle and permanent strains that accumulate with every load cycle. As a result of this complex behaviour, the stress-strain relationship for unbound granular materials is given by a non-linear curve. The curve is not retraced on the removal of stresses but forms a hysteresis loop indicating the permanent strain that occurs during the application of the

stress. Evaluation of a particular hysteresis loop produces the values for the permanent and resilient strains per load cycle. Figure 2.2:2 gives a simplified presentation of the stress-strain curve for both linear elastic and granular materials. It also gives a general idea of a single hysteresis loop.

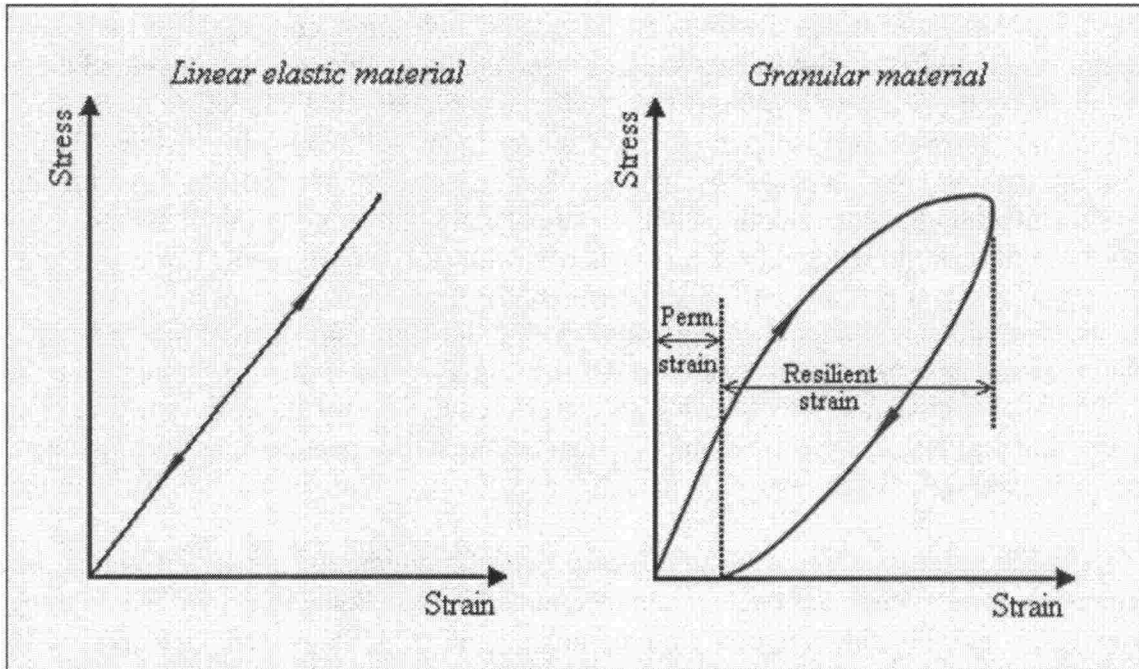


Figure 2.2:2 Schematic comparison of stress-strain curve for linear elastic and granular materials (Lekarp 1997).

The area of the hysteresis loop corresponds to the deformation work per volume element. Most of this work is transformed into heat energy, while only a small part is accumulated (Werkmeister 2003).

After the initial phase, the resilient response may be independent of the development of permanent deformations. It is therefore possible to look at the resilient behaviour and the resistance to permanent deformations separately.

Despite years of research, the deformation mechanisms of unbound granular materials are not yet fully understood. However, it has been postulated (Chan 1990) that the deformation of unbound granular materials under dynamic loading results from three main mechanisms: consolidation, distortion, and attrition.

- Consolidation → The consolidation mechanism is a change in shape and compressibility of particle assemblies. As a result of consolidation, the volume decreases because of changes in grain arrangements, particle orientation, and generalised contraction of the assembly (i.e. the whole soil structure) without modification of the single soil particles.
- Distortion → The distortion mechanism is characterised by bending, sliding, and rolling of individual particles. Distortion is governed mainly by microscopic interlocking of contacting particles. Particle bending is important in the case of flat particles, whereas sliding and rolling are usually associated with rounded



grains. The resistance to particle sliding and rolling depends on the interparticle friction in the grain assembly. Permanent deformation is mainly caused by this mechanism of distortion and reorientation.

- Attrition → The attrition mechanism is a change in a material's fabric and packing because of crushing, breakage, and abrasion occurring when the applied load and in turn the contact stresses between the grains exceed the strength of the particles. Particle crushing is a progressive process that can begin at relatively low stresses and results in gradual changes in the soil fabric and packing. Particle crushing is governed by grain size, magnitude of applied stresses, and mineralogy and strength of individual particles. Crushing is important especially for weak stones. Grain abrasion is a spalling of small particles from the grain surfaces at the contact points between the grains and may occur even at low stress levels. Loading of the grains during the laying and compaction process is much more severe than under traffic loading. For this reason, grain fragmentation is of minor importance during the service conditions of the material. It is likely that accumulation of permanent deformation continues as long as attrition occurs in the grain assembly.

The resilient deformation is mainly caused by deformations of individual grains. In a stressless state, contacts between grains are punctual (number 0 in Figure 2.2:3). When the force  $F$  transmitted by the interparticle contacts is increased, the size of the interparticle contact areas must increase due to the compression of those contacts. The resistance of the centres of individual aggregate particles approaching each other increases, too. As illustrated in Figure 2.2:3 the displacement  $\Delta\delta$  between particles (resilient deformation of the particles) decreases with the increase of the contact force  $\Delta F$ .

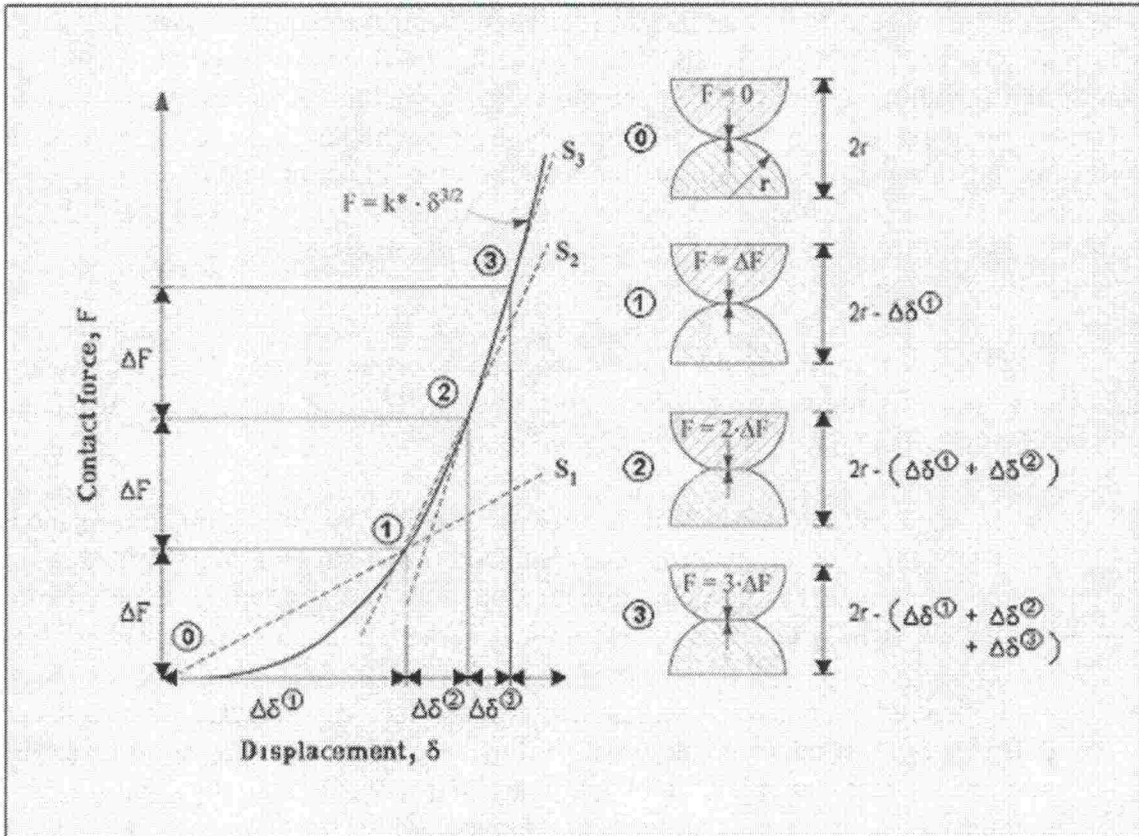


Figure 2.2:3 Dependence between contact force  $F$  and displacement  $\delta$  between two particles (Kolisoja 1997).

When the number of contacts between particles increases, the force exerted on a single particle contact by a certain external load most evidently decreases. When the load level is low, the deformations are elastic and occur mainly in the neighbourhood of the particle contacts. As a result of the smaller contact force, the deformations are smaller. Correspondingly, when the load level is high, the tangential force component in some of the contacts between particles will have already reached the maximum value of the interparticle frictional force. At high stress levels additional effects probably affect non-linear resilient deformation behaviour (Van Niekerk 2002). If the mineral skeleton is densely packed, the particulate system cannot be significantly rearranged, because there is not enough pore space between the particles, and each particle contact limits the directions of free movements of the particles. Shear strain forces particles to climb on each other, and the volume increases. The increase in the total volume of the material is called dilation. If the expansion is restricted, the dilatation results in increased stiffness because it requires work to be done against the external load (Van Niekerk 2002 and Kolisoja 1997).

Thom and Brown (1989) used repeated load triaxial tests to assess elastic stiffness, shear strength, and susceptibility to permanent deformation. They recognised that elastic stiffness correlates, among other things, with the frictional resistance at particle contact points, which depends on the microscopic properties. The shear strength and the resistance against permanent deformation were found to be a function of visible roughness. The ranking of resistance against permanent deformation showed some similarity to that for the shear strength; however, notable differences were recognised.



The reason was argued to be that the shear strength is influenced by the overall particle shape as well as roughness. It was also argued that the loading (whether dynamic or static) and the stress levels between the individual grains must be different. This was reflected by the fact that a direct dependency between the shear strength and the resistance against permanent deformation was not observed. For this reason, different micromechanical processes must be in play. According to Werkmeister (2003), however, to confirm these results further investigations are necessary.

Kolisoja (1997) has presented a concept influenced by, among others, Lambe and Whitman (1979) for outlining the particle-level phenomena related to the deformation behaviour of coarse-grained aggregates subject to cyclic loading. The concept also helps understand the event related to the degradation of material:

- When the load on a particulate system increases, only elastic deformations develop at first in the material particles. Due to the stresses concentrated at the contact points of the particles, the deformations are apparently largest in the immediate vicinity of the contact points.
- At high load levels, the stresses at contact points can increase so much that particles or their edges break down. In this case the loaded particles move with respect to each other and both particles and stresses rearrange until a new equilibrium with the external loads is achieved.
- The resultant forces acting on the contact points between particles are usually not perpendicular to the tangential planes of the contact points due to the direction of the load applied to the particulate system, the non-symmetric shape of the particles, and to the irregular packing arrangement of the particle skeleton. Therefore, the particles start to slide at the contact points as the tangential force at the contact point reaches the maximum value of the frictional force. This value depends on the normal force between the particles, the coefficient of friction between the particle surfaces, and the strength of the possible bonds between the particles. The mineral skeleton of the particulate system is again rearranged until the equilibrium with the external loads is achieved.
- When the load applied to the mineral skeleton of the material is removed, the elastic deformations of the particles recover, but the deformations caused by sliding and breakage of the particles are mainly permanent. The ratio between the permanent and recoverable deformations greatly depends on the magnitude of the applied stresses (especially shear stresses).
- If the load is repeated several times, the deformation mechanism is in principle exactly the same for all load applications. However, in the later load cycles permanent deformations are reduced by the fact that the mineral skeleton of the particulate system has already been rearranged during the previous cycles. This is normally observed as stabilising behaviour of the material when the load pulse series proceeds, for example in connection with the cyclic loading triaxial tests.
- Whether the loading direction changes in between or during the loading cycles or not presumably influences the accumulated amount of permanent deformations.

If the directions of the principal stresses do not change during the repeated loading cycles, the behaviour of the material is probably stabilised. However, in the loading conditions where the principal stress directions change the particulate system must rearrange itself many times in order to achieve a stable arrangement under all loads. Obvious consequences of this are larger permanent deformations compared to the loading conditions where the principal stress directions remain constant. This has also been a common result of the laboratory experiments where moving wheel loading is applied (e.g. Chan 1990).

### 2.3 Interaction between water and granular materials

A granular material consists of a group of individual particles in contact. The pore space between the particles is filled with air, water, or a mixture of the two. When a stress  $\sigma$  is applied to a soil or granular material, it is carried partly through the skeleton of the particles, called effective stress  $\sigma'$ , and partly by the pore water, called pore water pressure  $u$ .

The relationship between stresses can be expressed by

$$\sigma = \sigma' + u. \quad (\text{Eq. 2.3:1})$$

Since the total stress  $\sigma$  and the pore water pressure  $u$  can be measured, the effective stress  $\sigma'$  can be easily calculated. However, this equation is valid only in the case of positive pore water pressure. When the soil is unsaturated, the pore pressure may be negative or positive. The negative pressure or suction is caused by capillary forces due to curved air-water interfaces in the voids or by surface forces bonding water molecules together.

In partially saturated soils the total stress depends not only on the effective stress and the pore water pressure but also on the pore air pressure. According to Bishop and Blight (1963), in the case of partially saturated soils the relationship between stresses changes, and it takes into account both pore air pressure and suction. The relationship now assumes the form

$$\sigma = \sigma' + u_a - \chi \cdot (u_a - u_w), \quad (\text{Eq. 2.3:2})$$

where

$\sigma$	=	total stress,
$\sigma'$	=	effective stress,
$u_a$	=	pore air pressure,
$u_w$	=	pore water pressure,
$(u_a - u_w)$	=	suction,
$\chi$	=	parameter between 0 and 1 depending on the degree of saturation ( $\chi = 0$ if the material is dry and $\chi = 1$ if the material is saturated).

The aforementioned matric or soil suction ( $u_a - u_w$ ) is the suction exerted by the soil material (matrix) that induces water to flow in unsaturated soils. By convention, the



term matric suction implies the positive or absolute value of the negative pore pressure calculated in the unsaturated zone. Water flows from a soil with low matric suction (wet soil) to soil with a high matric suction (dry soil). The attraction that the soil exerts on the water manifests itself as a tensile hydraulic stress in a saturated piezometer with a porous filter placed in intimate contact with the water in the soil. The magnitude of this attraction is governed by the size of the voids in a manner similar to the way the diameter of a small bore glass tube governs the height to which water will rise inside the tube when the tube is immersed in water. The smaller the void is, the harder it is to remove the water from it. The meniscus formed between adjacent particles of soil by the soil suction creates a normal force between the particles. This force bonds the particles in a temporary way. Thus the soil suction, if it can be relied upon, can enhance the stability of earth structures. However, the soil suction also provides an attractive force for free water, which can result in a loss of stability in loosely compacted soils or swelling in densely compacted soils. Soil suctions can be found in all grounds that lie above the water table. This may be natural level ground or slopes, fill materials, and other earth structures that are constructed above the water table. Soil suctions will also be present in samples that have been recovered from a ground investigation. Laboratory measurements of suction can be very useful for assessing the quality of the samples, estimating the in situ effective stress, and detecting the presence of desiccation.

Another important quantity is the shear strength which, in the case of a saturated soil, depends on the effective (not total) shear strength parameters and is given by the Mohr-Coulomb equation

$$\tau = c' + \sigma' \cdot \tan \varphi', \quad (\text{Eq. 2.3:3})$$

where

$\tau$	=	shear strength,
$c'$	=	cohesion,
$\sigma'$	=	effective stress,
$\varphi'$	=	effective friction angle.

According to Fredlund et al (1978), in partially saturated soils the expression for the shear strength becomes

$$\tau = c' + (\sigma - u_a) \cdot \tan \varphi' + (u_a - u_w) \cdot \tan \varphi^b, \quad (\text{Eq. 2.3:4})$$

where

$\varphi^b$	=	friction angle with respect to suction ( $u_a - u_w$ ).
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The two expressions for  $\sigma$  show that any reduction of the pore pressure is compensated by an increase in the effective stress since the total applied stress should remain unchanged. This reduces particle movements, resulting in improved material performance. On the other hand, an increased pore pressure has a negative effect on the strength and stiffness of the material due to the consequent reduction of the effective stress.

### **3 BASIC APPROACHES IN MODELLING OF GRANULAR MATERIALS**

#### **3.1 Introduction**

The state-of-the-art modelling of granular materials requires the use of constitutive models for all the materials in the structure and in the sub grade. Models that describe the behaviour of the materials accurately are needed to obtain correct analyses of stress-strain responses under railway traffic loading. These analyses form the basis for the estimation of the structure's performance and thus aid in the selection of the best materials and optimal cross-section of the railway embankment.

Due to improvements in stress and strain analyses by computerized calculation methods, mathematical modeling of the behaviour of soils and granular materials has become a very active field of soil mechanics.

#### **3.2 Models of continuum mechanics**

##### ***3.2.1 Principle of continuum mechanics modelling approach***

Material models based on continuum mechanics have traditionally been used to model mechanical behaviour of unbound granular materials. The continuum mechanics approach does not consider the granular nature of the material, i.e. distinct particles and their interactions. Instead, a fictitious element of granular material isolated from the surrounding soil mass is assumed to have a homogenous composition (Figure 3.2.1:1). The stresses in the fictitious element are assumed to be continuously distributed, whereas the forces acting at the contact points between the particles are not taken into account.



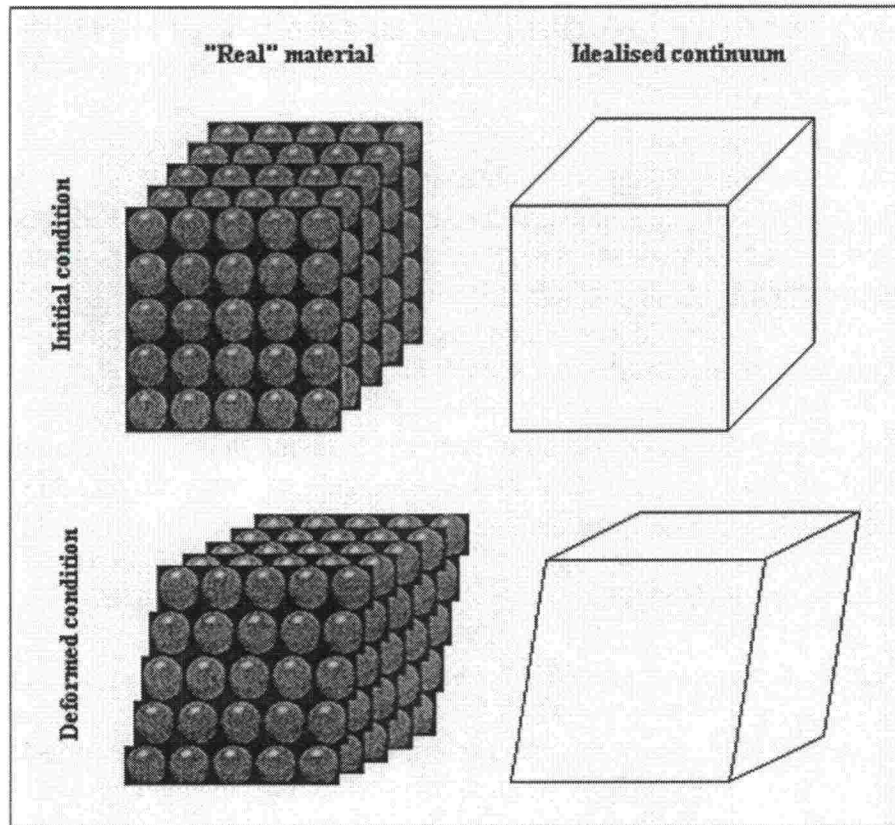


Figure 3.2.1:1 Idealisation of the granular material as a homogenous medium (Kolisoja 1997).

The mechanical model of the material aims to describe the volumetric and shear strains of the fictitious element resulting from a certain change in the stress state so that the deformations observed at the interfaces of the fictitious element are as large as they are at the interfaces of the corresponding group of real particles (Kolisoja 1997).

### 3.2.2 Linear elastic material models

The inclusion of only few constitutive parameters is characteristic of the simplest material models based on continuum mechanics. As a result, the mechanical behaviour of the material must be simplified considerably. The simplest continuum mechanical model used to describe the deformation behaviour of granular materials is Hooke's law, in which stresses and strains can be estimated using only two material parameters (Young's modulus and Poisson's ratio). In the Hooke's law deformations are assumed to be linearly elastic, and thus the deformation parameters are assumed to be independent of the stress level. Most materials for railway embankment structures do not behave linearly elastic (Figure 2.2:2). Furthermore, the embankment and the sub grade are assumed as a homogeneous, isotropic, and linear elastic half-space, which of course is not true. Nevertheless, this model is extensively used because it is relatively easy to use for stress and strain analyses.

The linear elastic deformation behaviour of a two-dimensional object is illustrated in Figure 3.2.2:1.

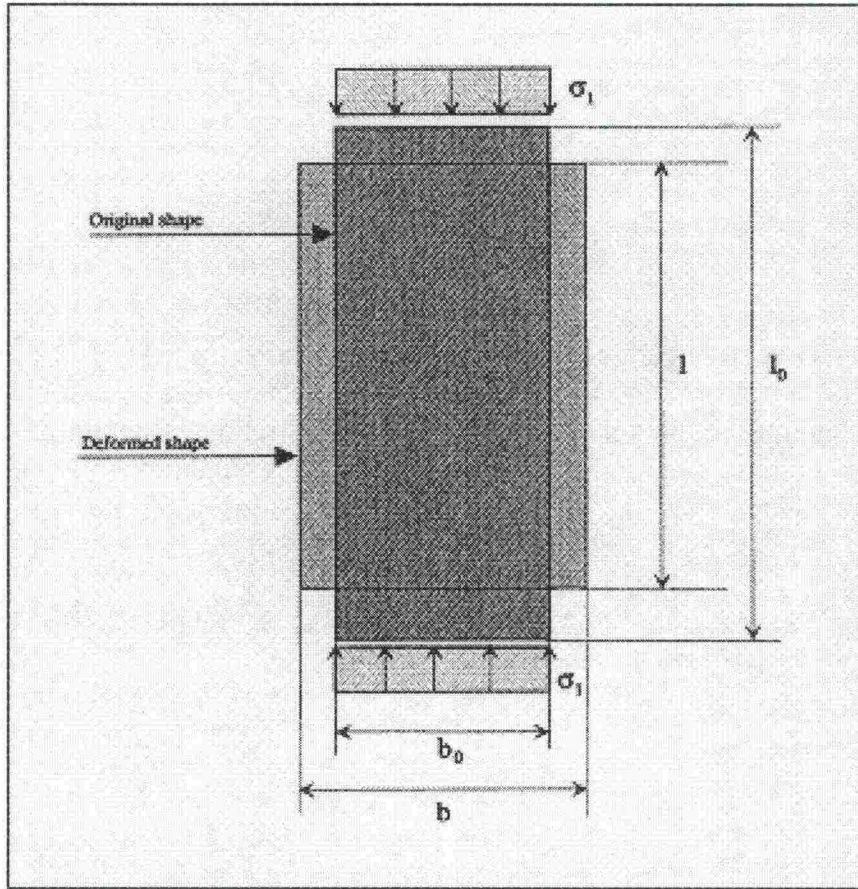


Figure 3.2.2:1 Linear elastic deformation behaviour of a two-dimensional object (Hoff 1999).

With reference to Figure 3.2.2:1, the strains  $\varepsilon_1$  and  $\varepsilon_3$  can be defined as

$$\varepsilon_1 = \frac{l - l_0}{l_0} \quad (\text{Eq. 3.2.2:1})$$

and

$$\varepsilon_3 = \frac{b - b_0}{b_0}. \quad (\text{Eq. 3.2.2:2})$$

These definitions for strain are called “engineering strains” and are based on the change in length relative to the original length. If the strains are relatively large, other strain definitions should be used. In railway embankments the practical strains are always small enough to be considered as “engineering strains”.

According to the theory of elasticity, the material model in 2-D (or asymmetric stress conditions  $\sigma_2 = \sigma_3$ ) can be written as

$$\sigma = E \cdot \varepsilon \quad (\text{Eq. 3.2.2:3})$$

and



$$\nu = \frac{\varepsilon_2}{\varepsilon_1}, \quad (\text{Eq. 3.2.2:4})$$

where

$$\begin{array}{ll} E & = \text{Young's modulus,} \\ \nu & = \text{Poisson's ratio.} \end{array}$$

Two independent elastic parameters are needed to describe the 3-D stress-strain state of a material that is isotropic and has a homogeneous structure. The two elastic parameters can be for example Young's modulus  $E$  and Poisson's ratio  $\nu$ . Alternatives to Young's modulus and Poisson's ratio are shear modulus  $G$ , bulk modulus  $K$ , Lamé constant  $\lambda$  and constrained modulus  $M$ . The relations between these parameters can be solved when two of them are known (Davis and Selvadurai 1996).

When Young's modulus and Poisson's ratio are used, the relationship between strains and stresses in the 3-D case has the form

$$\underline{\sigma} = \underline{D} \cdot \underline{\varepsilon}, \quad (\text{Eq. 3.2.2:5})$$

where

$$\underline{\sigma}^T = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}], \quad (\text{Eq. 3.2.2:6})$$

$$\underline{\varepsilon}^T = [\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{13}, \varepsilon_{23}], \quad (\text{Eq. 3.2.2:7})$$

$$D = \frac{E}{(1+\nu)(1-2\nu)} \cdot \begin{vmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) \end{vmatrix}.$$

Stiffness matrix  $D$  can be inverted to flexibility matrix  $D^{-1}$  that is more directly useful for manual calculations

$$D^{-1} = \frac{1}{E} \cdot \begin{vmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{vmatrix}.$$

The inverted relationship between strains and stresses is

$$\underline{\underline{\varepsilon}} = \underline{\underline{D}}^{-1} \cdot \underline{\underline{\sigma}}. \quad (\text{Eq. 3.2.2:8})$$

In this case the equations for the principal strains in the 3-D case have the form

$$\varepsilon_1 = \frac{1}{E} [\sigma_1 - \nu(\sigma_2 + \sigma_3)], \quad (\text{Eq. 3.2.2:9})$$

$$\varepsilon_2 = \frac{1}{E} [\sigma_2 - \nu(\sigma_3 + \sigma_1)], \quad (\text{Eq. 3.2.2:10})$$

$$\varepsilon_3 = \frac{1}{E} [\sigma_3 - \nu(\sigma_1 + \sigma_2)], \quad (\text{Eq. 3.2.2:11})$$

where

$$\begin{array}{ll} \varepsilon_{1,2,3} & = \text{strains in the principal stress directions,} \\ \sigma_{1,2,3} & = \text{principal stresses.} \end{array}$$

It is often convenient to divide the response into a shear and a volumetric response. The material can then be described with the aid of the bulk modulus  $K$  and the shear modulus  $G$ . The relationship between volumetric strain and mean stress is then described by

$$\sigma_m = K \varepsilon_v, \quad (\text{Eq. 3.2.2:12})$$

where

$$\sigma_m = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}), \quad (\text{Eq. 3.2.2:13})$$

$$\varepsilon_v = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}. \quad (\text{Eq. 3.2.2:14})$$

The 3-D relationship between shear stress and shear strain is given as

$$s_{ij} = 2G e_{ij}, \quad (\text{Eq. 3.2.2:15})$$



where

$$s_{ij} = \sigma_{ij} - \sigma_m, \quad (\text{Eq. 3.2.2:16})$$

$$e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_v. \quad (\text{Eq. 3.2.2:17})$$

The stiffness matrix can then be written as

$$D = \begin{vmatrix} K + \frac{4}{3}G & K - \frac{2}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ K - \frac{2}{3}G & K + \frac{4}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ K - \frac{2}{3}G & K - \frac{2}{3}G & K + \frac{4}{3}G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{vmatrix}.$$

Since K and G are just an alternative way to describe the elastic behaviour, these moduli are related to E and  $\nu$ . If one of the sets is known, the other set can be determined by the following equations:

$$K = \frac{E}{3(1-2\nu)}, \quad (\text{Eq. 3.2.2:18})$$

$$G = \frac{E}{2(1+\nu)}, \quad (\text{Eq. 3.2.2:19})$$

$$E = \frac{9KG}{1+3\frac{K}{G}}, \quad (\text{Eq. 3.2.2:20})$$

$$\nu = \frac{1}{2} \left( 1 - \frac{1}{K/G + 1/3} \right). \quad (\text{Eq. 3.2.2:21})$$

If the material in question cannot be assumed to be isotropic, the number of parameters needed to describe the general 3-D stress-strain state increases. Five parameters are needed for a material that has axisymmetric elastic properties and 21 parameters are needed for an elastic material that does not have any directions of symmetry (Kolisoja 1997).

If the structure to be analysed does not have a homogeneous composition, the above-mentioned elastic parameters must be known for each section of the structure that can be assumed to be homogeneous (Kolisoja 1997).

### 3.2.3 Non-linear elastic material models

Even though the deformations of an unbound granular material during repeated loading are almost entirely recoverable when the load remains clearly below the failure load, the linear elastic model is not very suitable for unbound granular materials, because the stress dependent behaviour of these materials is non-linear.

A non-linear elastic model means that the relationship between stresses and strains is not linear. This elastic modulus is not constant but varies with the stress and strain level. The same stress-strain path is followed for both loading and unloading. A typical non-linear stress - strain relationship is shown in principle in Figure 3.2.3:1.

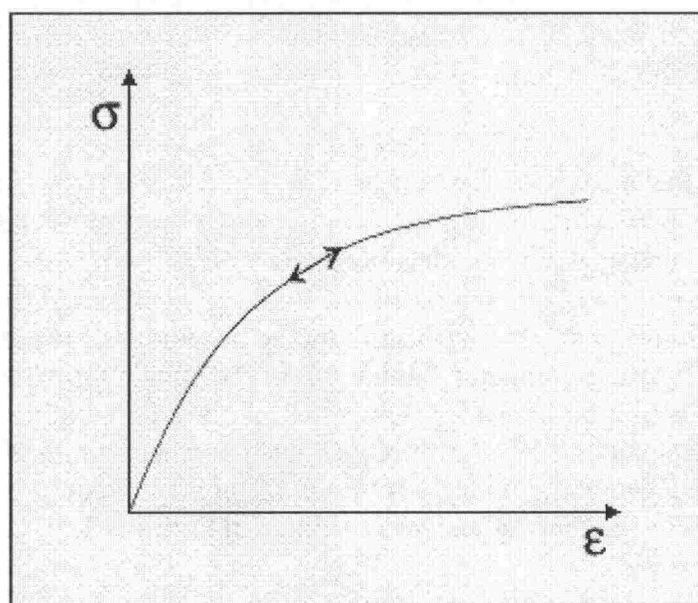


Figure 3.2.3:1 Typical non-linear stress-strain relationship (Hoff 1999).

Non-linear elastic models have been used to some extent for analyses of unbound granular materials' behaviour. They have a large number of constitutive parameters, and their determination obviously requires more extensive laboratory and site investigations than in the case of linear models. Nevertheless, the stress-strain state of the material can be modelled more realistically by using more advanced non-linear models. One disadvantage with non-linear elastic models is that permanent deformations cannot be modelled. Furthermore, these models cannot be handled with hand-calculations but must be solved using a computer software in which the stress distribution is normally computed using either the element method (Brown and Pappin 1981) or iterative processes (Huang 1993).

One of the strongest impediments to more extensive introduction of the most advanced non-linear material models is probably the restricted availability of the related material parameters due to their laborious, complex, and thus costly determination methods. In



addition, when both the complexity of the model and the number of parameters increase, the physical meaning of a single parameter is more difficult to conceive. It may also happen that the model describes very accurately the behaviour of the material in the loading condition of a certain laboratory test, but the applicability of the model to other loading conditions cannot always be guaranteed. Finally, it is not usually known how the changes in the quality and physical state of the material affect the model parameters. Instead, the changes must be investigated separately for all different conditions.

Non-linear elastic models for resilient deformation are described in detail in Chapter 5.

### **3.3 Models of particulate mechanics**

The counterpart of the continuum mechanics based modelling of the mechanical behaviour of granular materials is the so-called particulate mechanics modelling, where interactions between the distinct particles are explicitly investigated. The basic idea of the approach is that, when the number of particles is sufficiently large, the behaviour of the group of particles can be extended to describe the macroscopic behaviour of the actual material.

The approaches for modelling the mechanical behaviour of the granular materials starting from the particle level can be divided into two groups, namely, numerical simulation models and analytical models (Kolisoja 1997).

A natural way to implement a model describing the behaviour of materials composed of discrete particles is to use computer simulation. In the simulation model the material is modelled by means of a particulate system where the location and the surfaces of each particle are mathematically defined. Moreover, each contact point between the particles is individually modelled along with the forces at the contact point. A mechanical model of a contact point can be, for example, as illustrated in Figure 3.3:1.

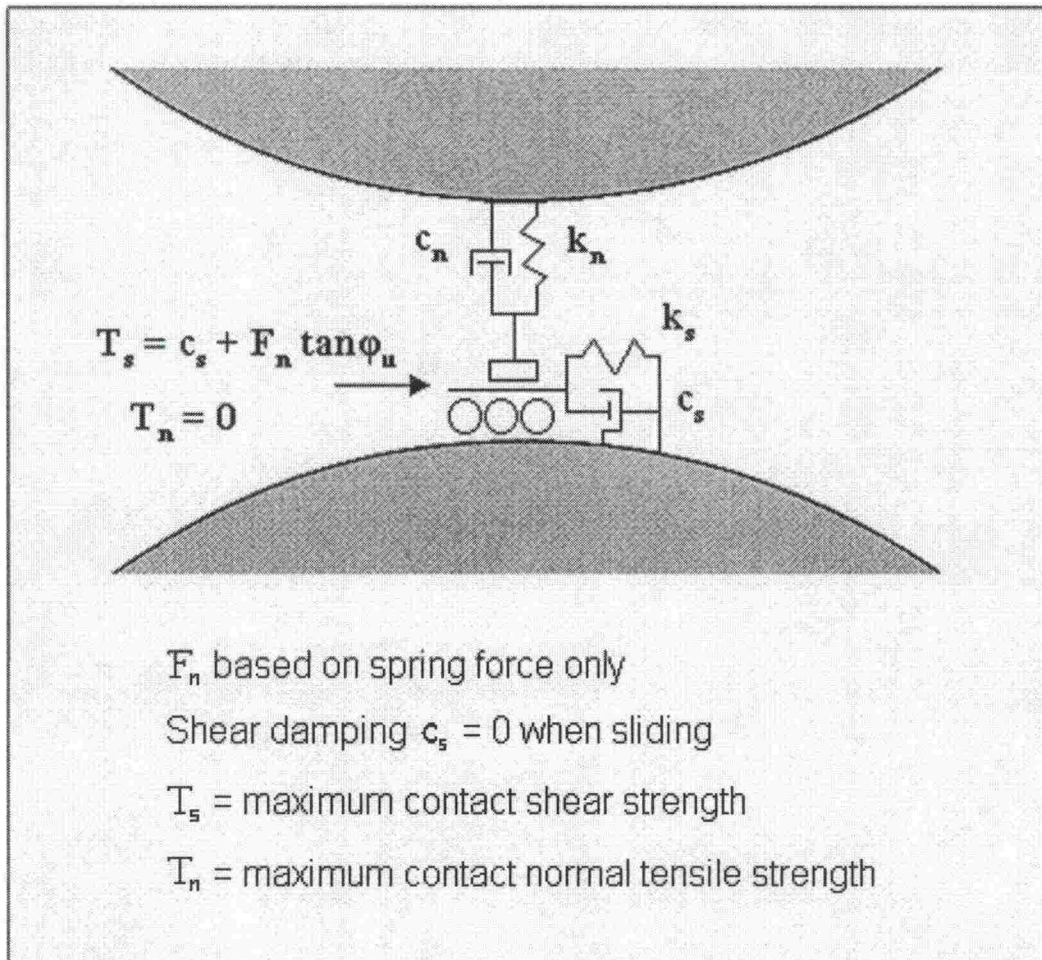


Figure 3.3:1 An example of a mechanical model of a contact point between two particles used in numerical simulations (Ting et al. 1989).

Another method, together with numerical simulation, for modelling the mechanical behaviour of the granular materials starting from the particle level is based on the analytical study of the particulate system. The constitutive equations that describe the macroscopic behaviour of the material can be derived by modelling the structure of the particulate system and the micro-level interactions between the particles.

Both the numerical and analytical modelling methods have still many restrictions when it comes to analysing real soil materials, real coarse grained aggregates, and general three dimensional stress states. In particular, continuous grain size distribution and variations in particle size and grain shape are very difficult to model correctly using particulate mechanics. One more impediment to the straightforward application of the particulate mechanics modelling is that initial modelling data, such as elastic and surface properties of the particles, are in most cases inadequately known. The application possibilities of the numerical simulation models in particular are, however, steadily improving due to the sharp increase in the capacity of computers.

Despite its shortcomings and limitations, the particulate mechanics modelling approach offers a physically well-justified point of view to the mechanical modelling of the aggregates and other soil materials. It can be used, for example, to support and



supplement continuum mechanics modelling when the influencing mechanisms of the variables affecting the deformation behaviour of granular materials are being studied.

## 4 FACTORS AFFECTING RESILIENT DEFORMATION

### 4.1 Introduction

Since 1960, numerous research efforts have been devoted to characterising the resilient behaviour of granular materials. It is well known that these materials exhibit a complex, non-linear, and time-dependent elasto-plastic response under repeated, traffic-type loading. To deal with this non-linearity and to differentiate from the traditional elasticity theories, the resilient response of granular materials is usually defined by resilient modulus and Poisson's ratio. Alternatively, the use of shear and bulk moduli has been suggested. For design purposes, it is important to consider how the materials involved react to and the resilient behaviour varies with changes in various influencing factors. From the studies found in the literature, it appears that the resilient behaviour of unbound granular materials under repeated loading depends, with varying degrees of importance, on many influential factors such as stress level, moisture content, density, grading, fines content, maximum particle size, aggregate type, particle shape, load duration, load frequency, load sequence, and compaction.

Past investigations have clearly shown that the resilient response is influenced mostly by the level of applied stresses and the moisture content of the material. The influence of other factors on the resilient response of granular materials is somewhat unclear. The literature reveals that researchers do not always agree on the nature or the extent of the impact of these factors and different, or even completely opposite, conclusions are often found. The discrepancies found in the literature emphasise the need for more intensive research into this area in the future.

In addition, it is worth noting that the structural response of granular materials is normally studied in the laboratory often using repeated load triaxial testing. Although the triaxial testing devices presently available are all based on the same principles, the quality and constraints of the facilities and the test procedures vary, sometimes greatly, between laboratories (Hoff 2004). It remains to be investigated into more detail how such differences influence the test results.

### 4.2 Stress level

Previous investigations have shown without exception that the level of applied stress is one of the factors that have the most significant impact on resilient properties of granular materials. Many studies (e.g. Mitry 1964, Monismith et al. 1967, Hicks 1970, Smith and Nair 1973, Uzan 1985, Sweere 1990) have shown a very high degree of dependence on confining pressure and sum of principal stresses for the resilient modulus of untreated granular materials. Hence, the resilient stiffness of granular materials is non-linear. Because of this, it is a very strong simplification to assume an unbound granular material to be linear elastic and to have a constant Young's modulus and a constant Poisson's ratio.

The typical deformation behaviour of coarse-grained aggregates during repeated load triaxial tests is presented in Figure 4.2:1 where the deformation behaviour is described using the concept of resilient modulus.



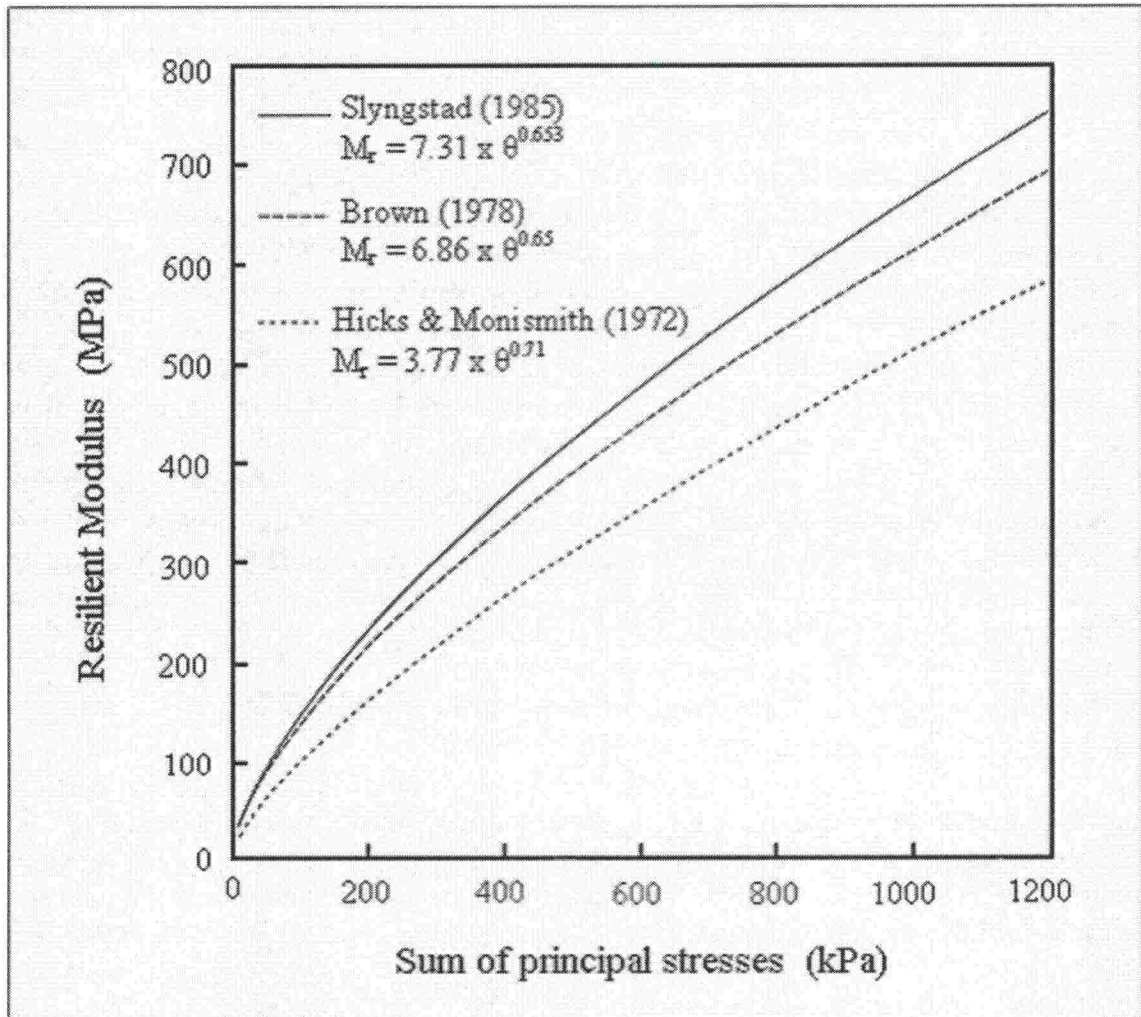


Figure 4.2:1 A typical variation in the resilient modulus of coarse grained materials as a function of the stress level (Ehrola 1996).

According to Figure 4.2:1, the resilient modulus increases when the stress level, expressed here as a sum of the principal stresses, increases. Monismith et al. (1967) reported an increase as great as 500 % in the resilient modulus for a change in confining pressure from 20 to 200 kPa. An increase of about 50 % in resilient modulus was observed by Smith and Nair (1973) when the sum of principal stresses increased from 70 to 140 kPa.

Although the resilient modulus increases greatly with increasing confining pressure and sum of principal stresses, it increases only slightly with increasing deviator stress (Hicks 1970, Uzan 1985, Sweere 1990). In a study conducted by Morgan (1966), the resilient modulus was shown to decrease slightly with increasing repeated deviator stress under constant confinement. Hicks (1970) suggested that the resilient modulus is practically unaffected by the magnitude of the applied deviator stress, provided that excessive plastic deformation is not generated. Hicks and Monismith (1971), on the other hand, reported slight softening of the material at low deviator stress levels and slight stiffening at higher stress levels.

Resilient Poisson's ratio is also believed to be influenced by the state of applied stresses. Hicks (1970), Brown and Hyde (1975), and Kolisoja (1997) reported that the Poisson's ratio of unbound granular materials increases with increasing deviator stress and decreasing confining pressure. However, the relationship is not as simple as that for the resilient modulus. Values of Poisson's ratio higher than 0.5 are often measured (Hoff 1999). This fact does not fit in with the linear elastic theory. However, this may be explained by resilient dilatancy (Hoff 1999).

Furthermore, increasing moisture content, especially at high degrees of saturation, results in a dramatic decrease in both resilient modulus and Poisson's ratio (Hicks and Monismith 1971, Barcksdale and Itani 1989, Dawson et al. 1996).

In laboratory triaxial testing, both constant confining pressure (CCP) and variable confining pressure (VCP) are used. Allen and Thompson (1974) compared the results obtained from these two types of test, and reported generally higher values of resilient modulus computed from CCP test data. The magnitude of the difference was itself non-constant and varied with the stress level. This study also showed that the CCP tests resulted in larger lateral deformations and higher values of Poisson's ratio. Figure 4.2:2 illustrates a typical result reported by Allen and Thompson. Brown and Hyde (1975) later suggested that VCP and CCP tests yield the same values of resilient modulus, provided that the confining pressure in the CCP test is equal to the mean value of the pressure used in the VCP test. As for the value of Poisson's ratio, Brown and Hyde confirmed considerable differences. In fact, their VCP tests yielded decreasing Poisson's ratio for increasing ratios of deviator stress to confining pressure, while their CCP tests showed the opposite.

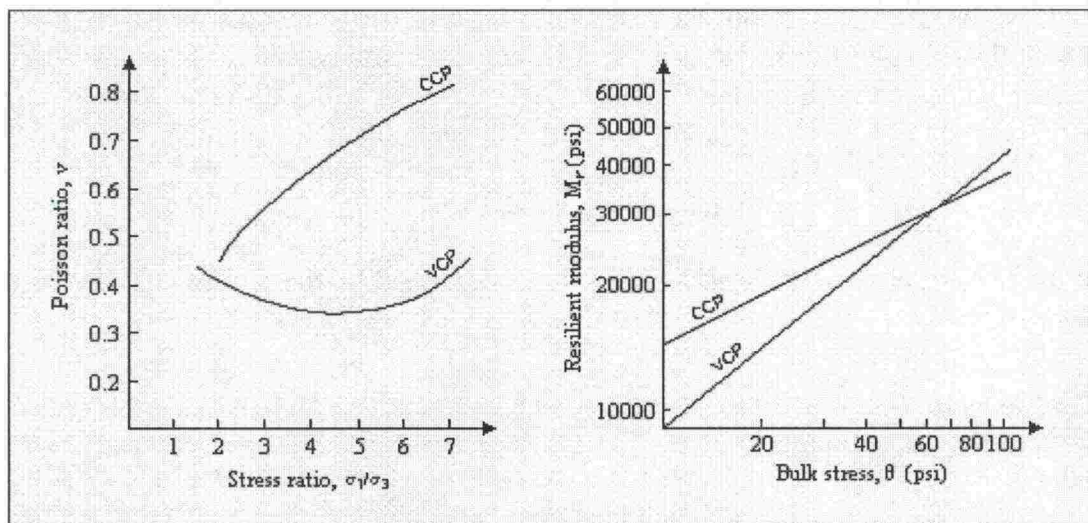


Figure 4.2:2 Examples of triaxial test results with CCP and VCP (Allen and Thompson 1974).

Hicks (1970) tested granular materials in a repeated load triaxial apparatus and showed that the material stiffness was strongly related to the confining pressure and somewhat to the axial load. This has been confirmed by several researchers during the last three decades. Figure 4.2:3 shows the modulus of resilience from Hicks' investigation as a function of the sum of principal stresses.



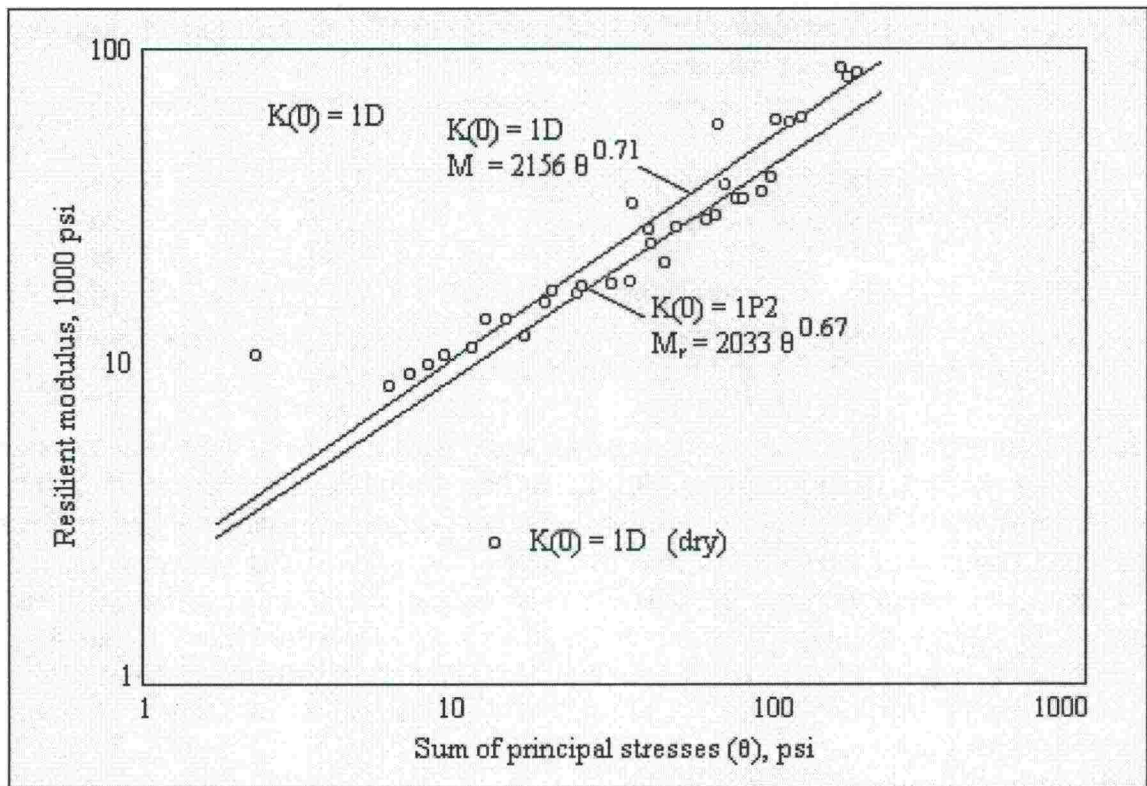


Figure 4.2:3 Modulus of resilience as a function of the sum of principal stresses (Hicks 1970).

Rada and Witczak (1981) published an interesting analysis of measured resilient moduli. They collected results from 10 different laboratories and universities and analyzed them together with their own results. The sample sizes used in the different tests were not specified but the sample diameters were in the range of 10 - 15 cm (4 to 6 in). The materials tested were mostly non-cohesive and ranged from silty sand to gravel and crushed rock. Some secondary materials were also tested.

All the tests gave an average resilient modulus described by the K- $\theta$  model (see section 5.2:2):

$$M_r = 23,34 \cdot \theta^{0,52}, \quad (\text{Eq. 4.2:1})$$

where

$$\begin{array}{lll} M_r & = & \text{resilient modulus (Mpa),} \\ \theta & = & \text{sum of principal stresses (kPa).} \end{array}$$

When only the crushed gravels were considered the parameters of the equation were changed to

$$M_r = 20.85 \cdot \theta^{0.45}. \quad (\text{Eq. 4.2:2})$$

Hoff (1999) measured the resilient modulus for various materials as a function of the stress state. Figure 4.2:4 shows the resilient modulus for all the different tests performed. The resilient modulus is plotted against the maximum mean stress. Although the levels are different, all the tests follow the same pattern with increasing stiffness for increasing maximum mean stress.

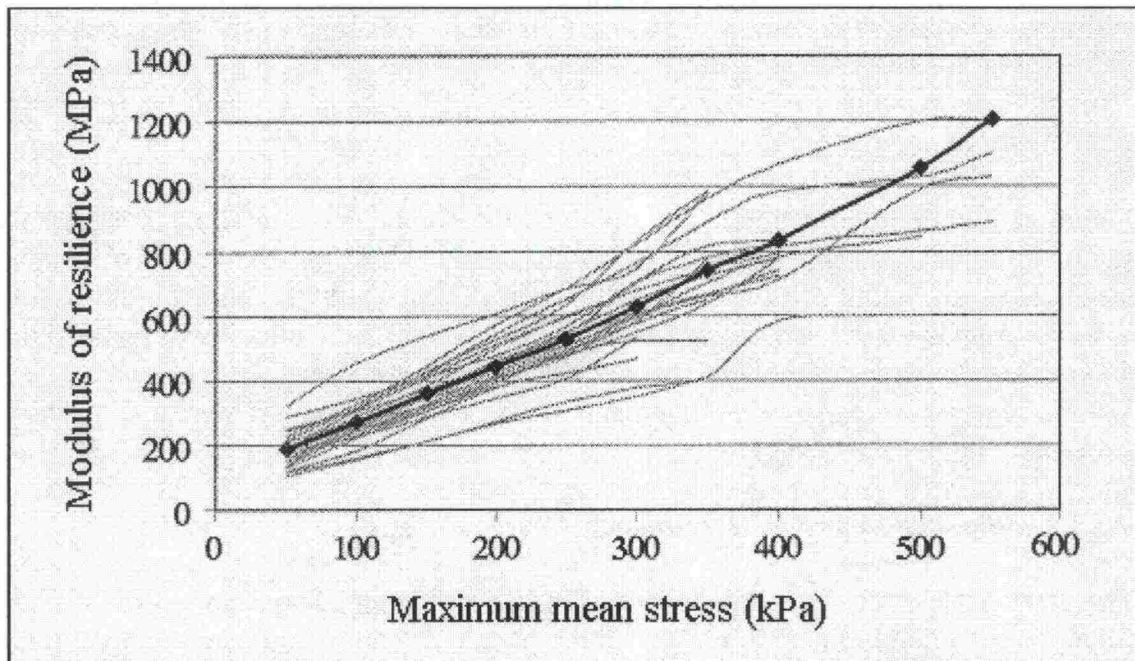


Figure 4.2:4 Resilient modulus plotted against mean stress (Hoff 1999).

### 4.3 Loading characteristics

#### 4.3.1 Stress history

Studies have indicated that stress history may have some impact on the resilient behaviour of granular materials. According to Dehlen (1969), the effects of the stress history appear as a consequence of progressive densification and particle rearrangement under repeated application of stress.

Boyce et al. (1976) carried out repeated load triaxial tests on samples of a well-graded crushed limestone all compacted to the same density in a dry state. The results showed that the material was subjected to the effects of the stress history. However, these effects could be reduced by pre-loading with a few cycles of the current loading regime and by avoiding high stress ratios in tests for resilient response.

Hicks (1970), on the other hand, reported that the effect of stress history is almost eliminated and a steady and stable resilient response is achieved after the application of approximately 100 cycles of the same stress amplitude. Similar observations were reported by Allen (1973), who suggested that a specimen should be conditioned for approximately 1,000 load repetitions prior to repeated load resilient tests.

Other researchers (Brown and Hyde 1975, Mayhew 1983) reported that the resilient characteristics of unbound granular materials are basically insensitive to stress history,



provided that the applied stresses are kept low enough to prevent substantial permanent deformation in the material. Therefore, large numbers of resilient tests can be carried out sequentially on the same specimen in order to determine the resilient parameters of the material.

According to the new European standard on triaxial testing (CEN 2004 EN 13286), when pre-stressing is done at a stress level higher than the one of the stress paths, the influence of the stress history can be neglected.

#### ***4.3.2 Number of load cycles***

Moore et al. (1970) investigated the effect of the number of load applications on the resilient response of granular materials. They concluded that the resilient modulus increases as the number of load repetitions increases, partly because of the loss of moisture from the specimen during testing. Hicks (1970), on the other hand, reported that the resilient properties of the granular materials tested were virtually the same after 50 to 100 load repetitions as after 25000 repetitions. Similar observations were also made by Allen and Thompson (1974).

#### ***4.3.3 Load duration and load frequency***

The general view regarding the impact of load duration and frequency on the resilient behaviour of granular materials is that these parameters are of little or no significance (e.g. Seed et al. 1965, Morgan 1966, Hicks 1970, Boyce et al. 1976, Thom and Brown 1987). For instance, Seed et al. (1965) reported a study in which the resilient modulus of sands increased only slightly (from 160 to 190 MPa) as the duration of load decreased from 20 minutes to 0.3 second. Hicks (1970) conducted tests at stress durations of 0.1, 0.15, and 0.25 seconds and found no change in the resilient modulus or Poisson's ratio. It is most likely that the resilient modulus will show a reduction with increasing loading frequency when the moisture content approaches saturation as transient pore pressures may then develop causing a reduction in effective stress. This will be most marked if there is little opportunity for drainage.

#### ***4.3.4 Load sequence***

Hicks (1970) and Allen (1973) studied the test sequence or the order in which the stresses are applied to a specimen. These studies clearly showed that the test sequence has almost no impact on the resilient properties of granular materials.

### **4.4 Strain level**

It is a well-known characteristic of soil materials that the deformation modulus decreases when the shear strain - that depends on the magnitude of the shear stresses - increases. Figure 4.4:1 illustrates this concept and shows the commonly observed principal relation between the value of the shear modulus and the shear strain. The value of the shear modulus has been normalised using the so-called maximum shear modulus  $G_{max}$  that corresponds to a very low level of strain.

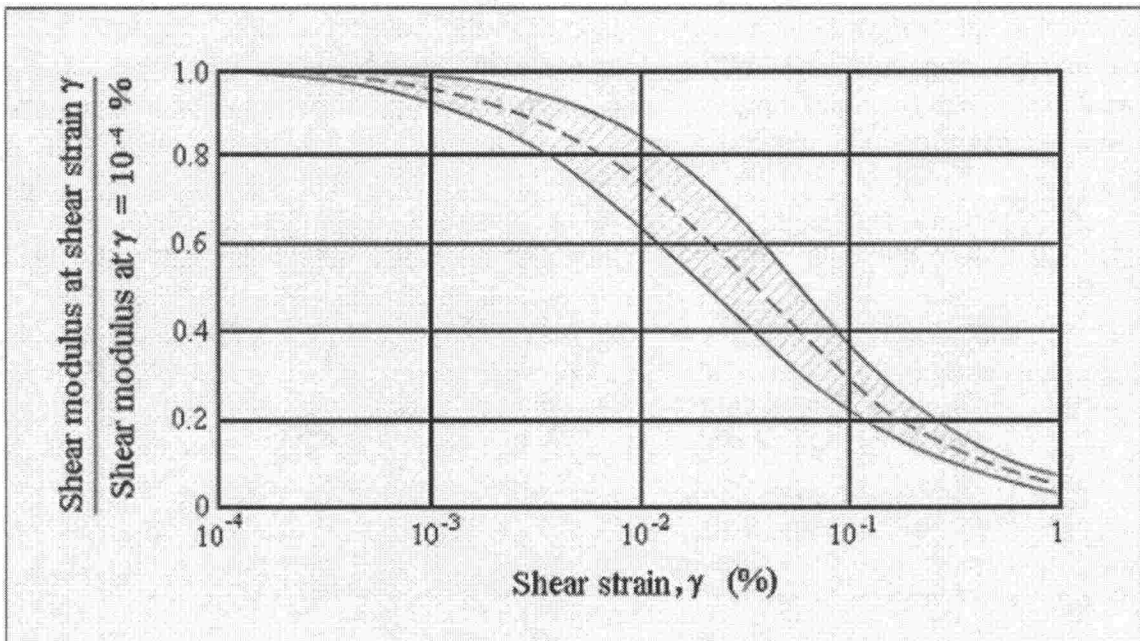


Figure 4.4:1 A typical variation in the shear modulus of soil materials as a function of the shear strain level (Seed and Idriss 1970).

In his work, Kolisoja (1997) observed that the deformation modulus of coarse grained aggregates increased as the stress level described using the sum of principal stresses increased. He also observed that the same deformation modulus decreased as the shear strain level, which depends on the level of deviator stress, increased. In repeated loading triaxial tests both the sum of the principal stresses and the deviator stress increase, when the confining pressure is constant and the axial load pulse increases. However, the deformation modulus cannot, of course, both steadily increase and decrease. Instead, the deformation behaviour in a certain laboratory test results from the interaction of these opposite actions, and the actually observed behaviour mainly depends on the stress-strain region that the used test method covers.

In general, the loading procedures typically applied to coarse grained aggregates used in railway embankments tend to locate in the stress-strain region where the increase of the modulus value due to an increase in stress level is more dominant than the decrease of the modulus value due to an increase in strain level. However, Uzan (1985) and Nataatmadja (1992) observed that for coarse grained materials the modulus values measured at very low loading levels using cyclic loading triaxial tests were higher than the values measured at a slightly higher loading level under the same confining pressure. Therefore, in these cases the decrease in the modulus values caused by an increase in shear strain level was evidently more dominant. Respectively, it is generally known that the deformation modulus decreases when the deviator stress is sufficiently high. This always happens in triaxial tests, for example, when the deviator stress approaches the value at failure conditions.

Figure 4.4:2 illustrates the principal dependency of the resilient modulus of a coarse grained aggregate on the level of strain during repeated load triaxial tests. The rise in the middle of the curve represents the increase in stiffness that happens when the strain level increases. A physical explanation for this phenomenon is the dilation of the



material (Uzan 1985). Dilation is the increase in the material volume produced by the particle rearrangement caused by shear stresses. In a particulate system where particles are densely packed, significant rearrangement is found only if particles are able to move around each other.

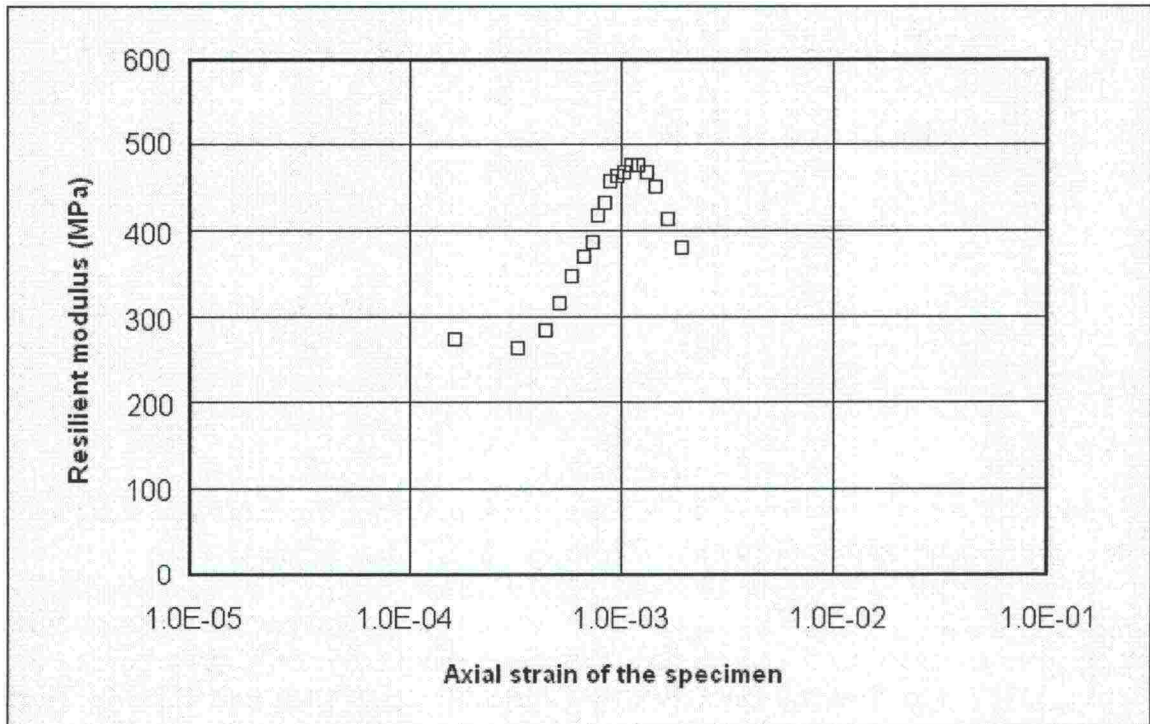


Figure 4.4:2 Principal dependence of resilient modulus of dense coarse grained aggregate on strain level in the loading condition of the triaxial test (Kolisoja 1997).

Limited resolution of measurement instruments makes it often difficult to observe the slight fall in the values of resilient modulus at very low levels of strain. In addition to the measurement resolution, also the loading procedure inevitably affects the observation of the phenomenon. For example, the fall can hardly be seen in Figure 4.4:2. Moreover, the slope of the rise and probably also its location in the strain axis depend on the density of the material, the level of the applied stresses, and many other factors related to the quality and the physical state of the tested material.

#### 4.5 Density

It has been known for a long time that increasing density of a granular material significantly alters its response to static loading, causing it to become both stiffer and stronger. However, the effect on resilient stiffness has been less thoroughly studied. The literature available is somewhat ambiguous in regard to the impact of density on resilient response of granular materials. Some investigations have considered the effect of density as relatively insignificant (Thom 1988, Thom and Brown 1988, Brown and Selig 1991), while some others have evaluated that the effect of density is dependent upon the material (Cheung 1994). On the other hand, many researchers (e.g. Allen and Thompson 1974, Coffman et al. 1964, Hicks and Monismith 1971, Trollope et al. 1962, Hicks 1970, Robinson 1974, Rada and Witczak 1981, Kolisoja 1997) have suggested that resilient modulus generally increases with increasing density. A similar conclusion has been

drawn for the moduli of low-level deformations from the particulate mechanical modelling of a theoretical material being composed of spherical particles (Chang et al. 1989 and Dorby et al. 1989) and from the resonant column tests of different sand materials (e.g. Hardin and Richart 1963).

Trollope et al. (1962) reported slow repeated load tests on a uniform sand and found that the resilient modulus increased by up to 50 % between loose and dense specimens. Similar observations were made by Robinson (1974) who also studied uniform sand. The number of particle contacts per particle increases greatly with increased density resulting from additional compaction of the particulate system. This, in turn, decreases the average contact stress corresponding to a certain external load. Hence, the deformation in particle contacts decreases and the resilient modulus increases (Kolisoja 1997).

Hicks and Monismith (1971) found the effect of density to be greater for partially crushed than for fully crushed aggregates. The resilient modulus was reported to increase with relative density for the partially crushed aggregate tested, whereas it remained almost unchanged when the aggregate was fully crushed. Hicks and Monismith further reported that the significance of changes in density decreased as the fines content of the granular material increased.

In the study conducted by Barksdale and Itani (1989), the resilient modulus increased markedly with increasing density only at low values of mean normal stress. At high stress levels, the effect of density was found to be less pronounced.

Vuong (1992) reported test results showing that at densities above the optimum value, the resilient modulus is not very density-sensitive.

The level of density also seems to have some influence on Poisson's ratio. This influence is believed by some researchers (Hicks 1970, Allen 1973, Allen and Thompson 1974) to be small and with no consistent variation. Others (Hicks and Monismith 1971, Kolisoja 1997) have reported a slight decrease in Poisson's ratio as the density of the material increases.

An example of the experimentally observed effects of density that is based on the results reported in literature is presented in Figure 4.5:1. The figure illustrates the values of resilient modulus at two different density levels - obtained using the compaction effort of standard (ASTM D 698-91) and modified (ASTM D 1557-91) Proctor compaction tests, respectively - of six different coarse grained granular materials. The presented values are computed using the values of parameters  $k_1$  and  $k_2$  in Equation 5.2.2:1 reported by Rada and Witczak (1981) and they correspond to the sum of principal stresses of 138 kPa (20 psi).



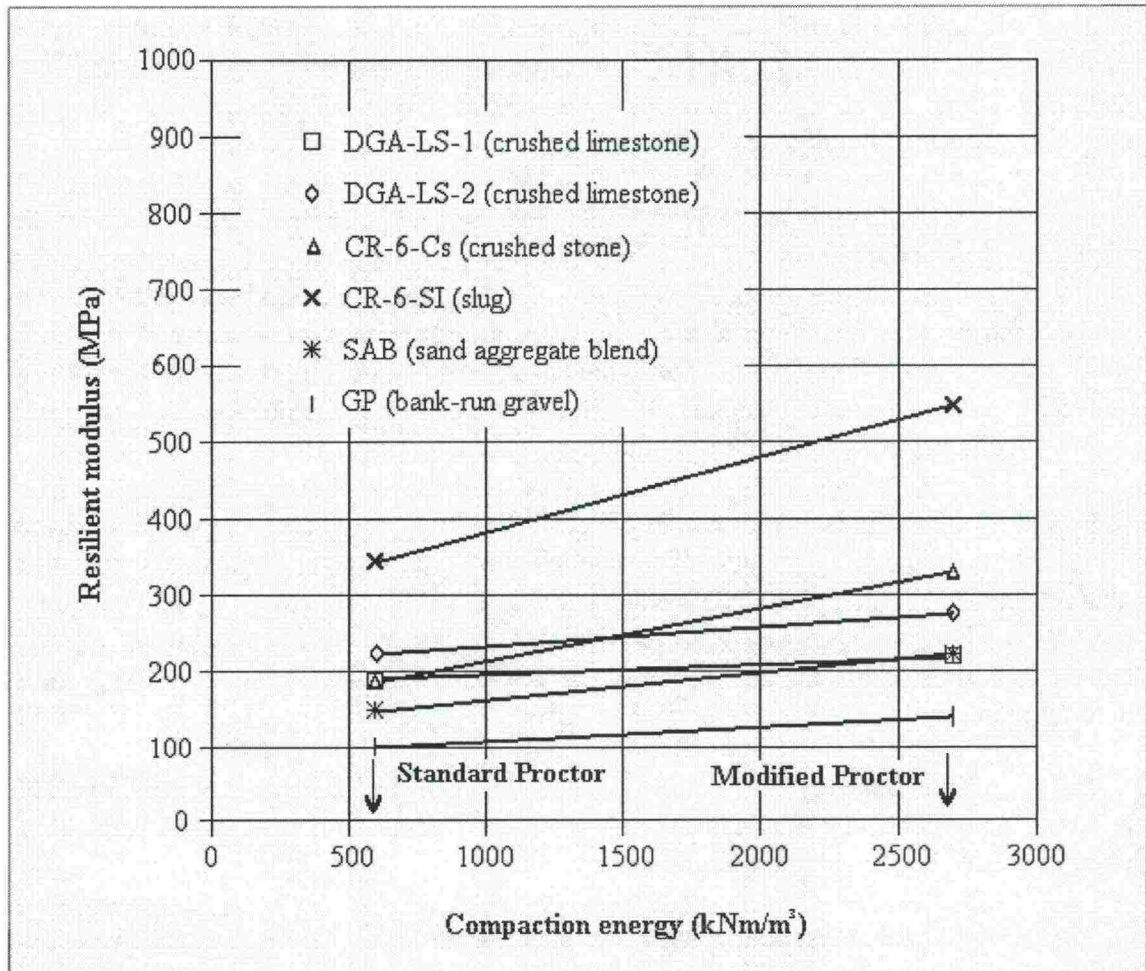


Figure 4.5:1 Resilient modulus of six different coarse grained granular materials as a function of compaction effort based on the results of Rada and Witczak (1981).

A heavily compacted material will have a dense grain structure where each grain has many contact points to the neighbouring grains for load distribution. This leads to a higher overall stiffness for the material. Several researchers (Hoff 1999, Kolisoja 1994) have shown that heavy compaction results in higher stiffness. Figure 4.5:2 shows the resilient stiffness for four different levels of compaction ( $R_c$ ) of the same material tested by Kolisoja (1994). As can be seen, there is a clear tendency to stiffer behaviour for the heavily compacted materials.

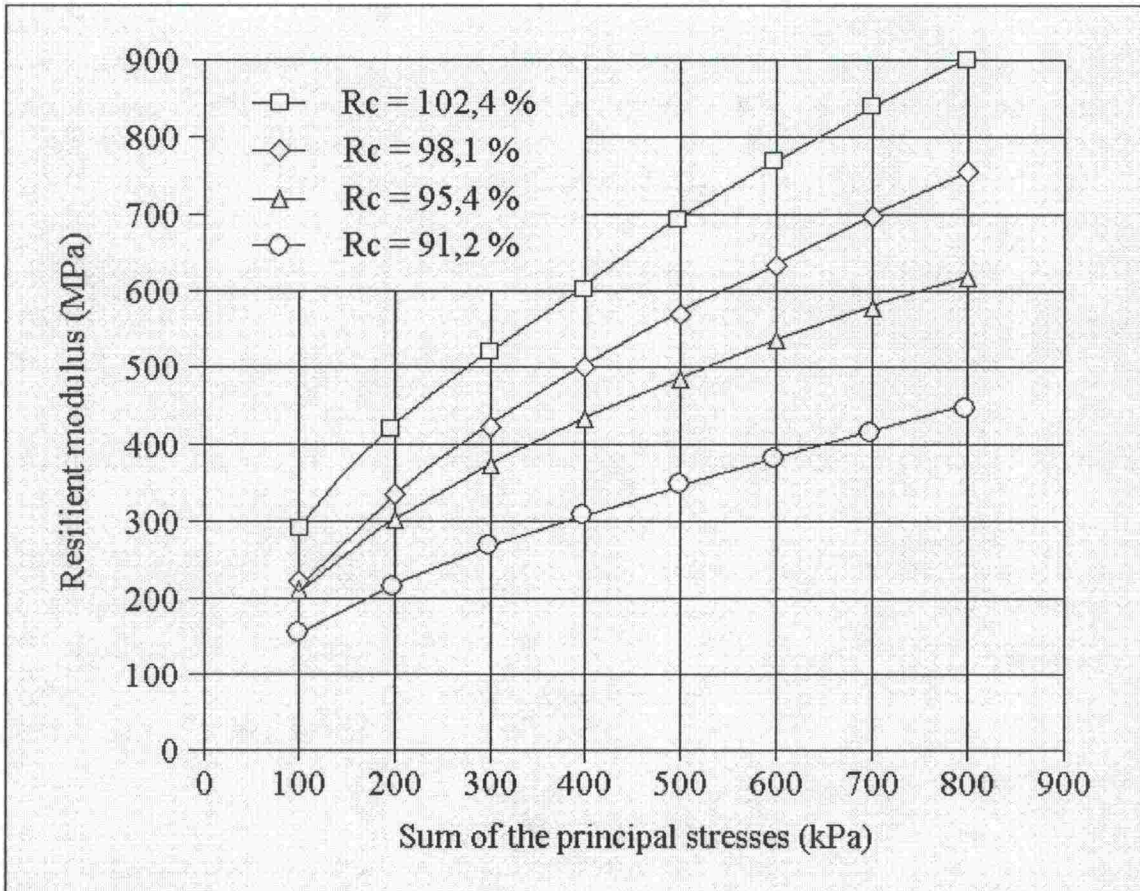


Figure 4.5:2 Modulus of resilience for four different levels of compaction,  $R_c$  (Kolisoja 1994).

## 4.6 Grading characteristics

### 4.6.1 Introduction

Granular materials consist of a large number of particles, normally of different sizes. Stresses in unbound granular materials are transferred from grain to grain through contact points. Because the overall stiffness of the grains themselves is normally much higher than the overall stiffness of the unbound granular materials, the local stresses and strains in these contact points are very important for the overall stiffness of the sample. The grain size distribution and the maximum grain size will also strongly influence the number of contact points that contributes to this overall stiffness of the sample. A well-graded material will have many contact points for each grain and should have lower stresses at each contact point. However, calculations done on idealized materials made of spheres show that the stresses are not equally distributed on all contact points, but seem to follow a few paths through the sample. A more open graded material will have fewer contact points but, a higher contact stress at each point.

### 4.6.2 Grain size distribution

The particle size distribution, or grading, of granular materials seems to have some influence on material stiffness, though it is generally considered to be of minor significance.



The grain size distribution of granular materials is usually presented graphically with the aid of a grading curve. The shape of the grading curve is also often described using single numerical values. Some of the most frequently used quantities are the coefficient of uniformity  $C_u$  (Equation 4.6.2:1), the coefficient of concavity  $C_c$  (Equation 4.6.2:2), and the exponent  $n$  of the Fuller (Equation 4.6.2:3):

$$C_u = \frac{d_{60}}{d_{10}}, \quad (\text{Eq. 4.6.2:1})$$

$$C_c = \frac{(d_{30})^2}{d_{10} \cdot d_{60}}, \quad (\text{Eq. 4.6.2:2})$$

$$p = 100 \cdot \left( \frac{d}{D_{\max}} \right)^n, \quad (\text{Eq. 4.6.2:3})$$

where

$C_u$	=	coefficient of uniformity,
$C_c$	=	coefficient of concavity,
$d_{10}$	=	grain size corresponding to per cent passing value of 10 %,
$d_{30}$	=	grain size corresponding to per cent passing value of 30 %,
$d_{60}$	=	grain size corresponding to per cent passing value of 60 %,
$p$	=	per cent passing value corresponding to the grain size $d$ ,
$D_{\max}$	=	maximum grain size of the material,
$n$	=	parameter describing the shape of the grading curve.

Thom (1988) and Thom and Brown (1988) studied the behaviour of crushed limestone at different gradings and the relation between the grading parameter “ $n$ ” from the Fuller curve and the shear stiffness. They concluded that uniformly graded aggregates were only slightly stiffer than well-graded aggregates, although they found a rather clear tendency to stiffer behaviour for open-graded materials. Brown and Selig (1991) and Raad et al. (1992) reported similar results, whereas tests performed by Kolisoja (1997) showed that the modulus values of the most efficiently compacted well-graded crushed rock material were higher than those of the open-graded material.

Thom (1988) and Thom and Brown (1988) systematically investigated the effects of the shape of the grain size distribution. According to their test series, the stiffness of dry crushed limestone having the maximum grain size of 10 mm increased 1.5 - 1.8 times, when the parameter  $n$  describing the shape of the grain size distribution in Equation 4.6.2:3 increased from the value  $n = 0.25$  to the value  $n = 5$  (Figure 4.6.2:1). The fact that the dry density of material for which  $n = 5$  was more than 25 % lower than that of the comparably packed material corresponding to the parameter value  $n = 0.25$  makes the results especially interesting.

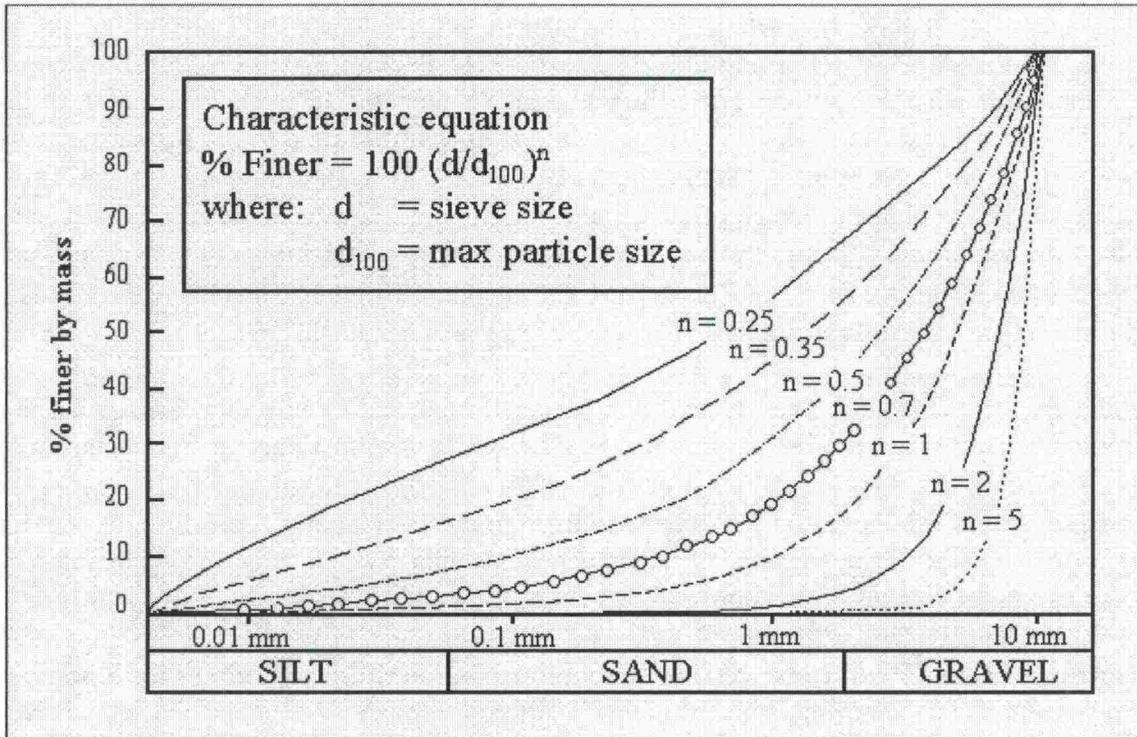


Figure 4.6.2:1 Grain size distributions of test materials used in Thom's test series (1988).

According to Kolisoja (1997), another example of the effects of the variations in the grain size distribution on the stiffness properties of granular materials is the so-called sand "bump" in the grading curve that is typical of many Finnish crushed gravel aggregates. The term sand "bump" means that medium size fractions typically located in the grain size region of sand in the grading curve are over-represented in the material, as it is compared to the grain size distribution corresponding to the parameter values of  $n = 0.35$  to  $n = 0.45$  of Equation 4.6.2:3 that has been found to result in optimal packing (Zheng et al. 1990). This type of grain size distribution has usually been observed to have an adverse effect on the stiffness properties of aggregate.

When explaining the variations in the shape of the grain size distribution, one of the major factors is probably the average size of the particles that transmit the load in the mineral skeleton of the material. The explanation is thus directly based on the stiffness properties of particle queues having different particle sizes. Therefore, the relatively high values of the resilient modulus of the open-graded crushed aggregates are not so surprising. This is due to the fact that the load is transmitted in the open-graded crushed aggregates via larger particles than in the aggregates which have the same maximum particle size but include plenty of fine fractions. Respectively, the low resilient modulus of materials containing plenty of sand fractions could be explained by the assumption that all sand fractions do not fit into the pore spaces between the coarse grained particles. Therefore, the coarse grained particles are not able to get in direct contact with each other and the loads are transmitted through larger number of contact points. The sand fraction can at worst be assumed to operate as some sort of bearing layer between the large particles.



The grain sizes that determine the deformation behaviour of the material could be recognised using theoretical studies of the particle packing. In the literature several investigations have been reported on the particle packing (e.g. Suzuki and Oshima 1985b and Yu and Standish 1988). Usually the studies are limited to the mixture of only a few different particle sizes (Standish and Borger 1979) or to only a few special cases where the grain shape is exactly determined (Ammi et al. 1987). Some researchers have also utilised computer simulations to study the particle packing (Powell 1980 and Suzuki and Oshima 1985a) and some have replaced the irregular particles with spherical particles having a suitable equivalent diameter (Yu and Standish 1993).

Plaistow (1994) argued that when moisture is introduced to well-graded materials, the effect of grading can be significantly increased, since these materials can hold water in the pores. They can also achieve higher densities than uniformly graded materials because the smaller grains fill the voids between the larger particles. Plaistow, therefore, concluded that grading has an indirect effect on the resilient behaviour of unbound aggregates by controlling the impact of moisture and density of the system.

Heydinger et al. (1996) compared the effect of grading on resilient modulus for limestone, gravel and slag. Limestone showed higher resilient modulus when open-graded, whereas no trend could be noted for the variation of modulus in gravel. For slag, however, the results were the opposite and the denser gradation tended to give higher stiffness.

In his study, Van Niekerk et al. (1998) investigated the behaviour of a limited number of specimens of sands, crushed masonry and crushed concrete. The results showed higher stiffness for well-graded than for uniformly-graded specimens. This conflicts with the conclusions drawn by other researchers, as mentioned above, and might not have held if even greater proportions of fines had been added, allowing their influence to dominate. Van Niekerk et al., however, argued that, as a result of a larger number of contact areas at equal confinement, a well-graded material can take up a larger deviatoric stress for equal deformation, thus resulting in higher stiffness.

Figure 4.6.2:2 shows the resilient stiffness as measured for the different grain size distributions for the material from Åndalen. The material in question is metamorphic gneiss whose main minerals are feldspar, quartz, and biotite (Hoff 1999). The figure shows the modulus of resilience for three levels of maximum mean stress. The grain size distributions used by Hoff for the material from Åndalen are illustrated in Figure 4.6.2:3.

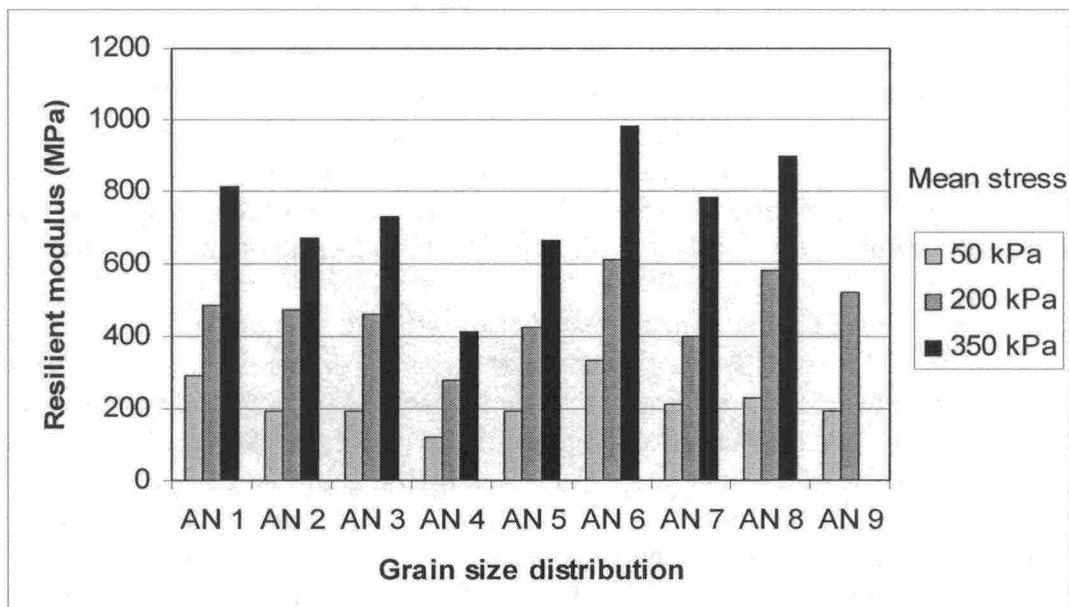


Figure 4.6.2:2 Modulus of resilience for different grain size distributions of Åndalen material (Hoff 1999).

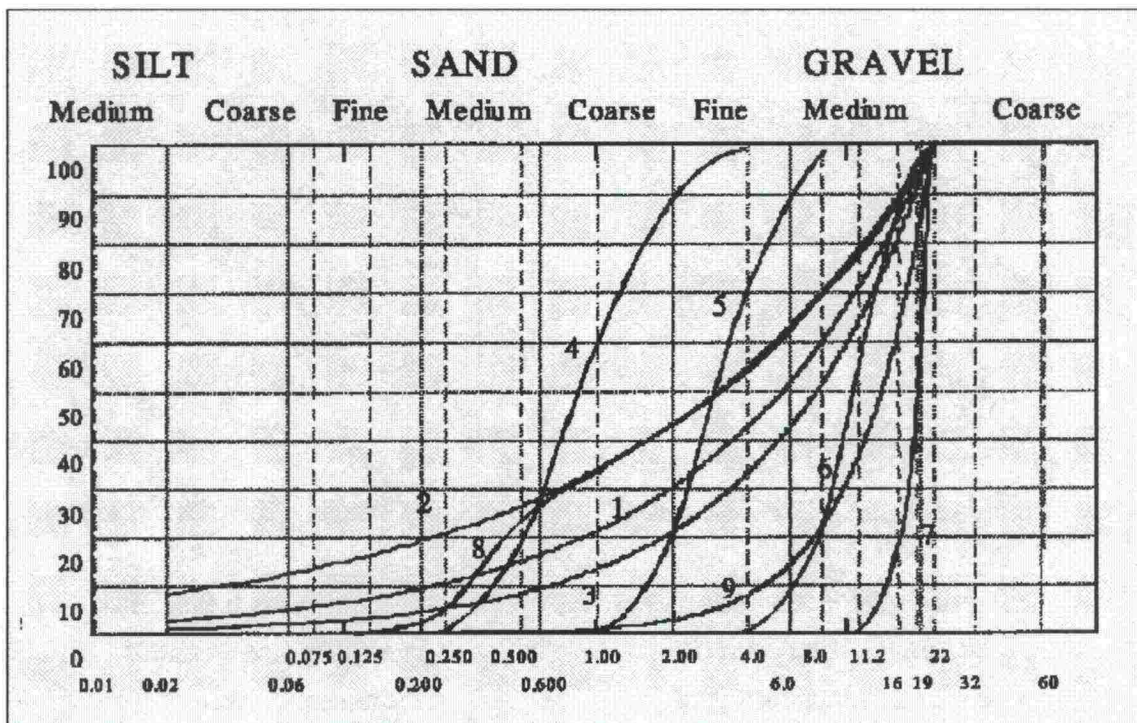


Figure 4.6.2:3 Tested grain size distributions (Hoff 1999).

The stiffness of grading 4, 5 and 6, which have similar shape of the grading curve but different maximal grain size, show that the stiffness increases for increasing grain size. Large grains give stiffer samples than smaller grains. If grading 6 is compared to grading 1, 2 and 3, it seems that the open-graded materials are stiffer than the well-graded materials. However, if the materials are very open-graded like grading 7, the stiffness is again reduced.



#### 4.6.3 Fines content

A simple characteristic number used to describe the grain size distributions of aggregates that are mainly composed of coarse-grained fractions is the amount of the fines fraction. The amount of fines is typically defined as the percentage by weight proportion of the dry material with a grain size smaller than 0.06 to 0.075 mm. The literature is not quite clear in regard to the impact of fines content on material stiffness.

At the particle level, the mechanism related to the adverse effect of the fines fraction is that if its amount as well as the amount of other small grains is relatively small, the larger particles are able to touch each other, and thus the loads are mainly transmitted by the mineral skeleton consisting of large particles (Kolisoja 1997). However, if the amount of fines and other small grain sizes is so high that all the fine-grained fractions cannot fit into the empty spaces located between the large particles, the large particles are not able to be in direct contact with each other. As a result, the stiffness of the mineral skeleton decreases and finally the coarse grained material can as if float in the fine-grained material. In that case, the fine-grained material dominates the macroscopic behaviour of the material (Figure 4.6.3:1).

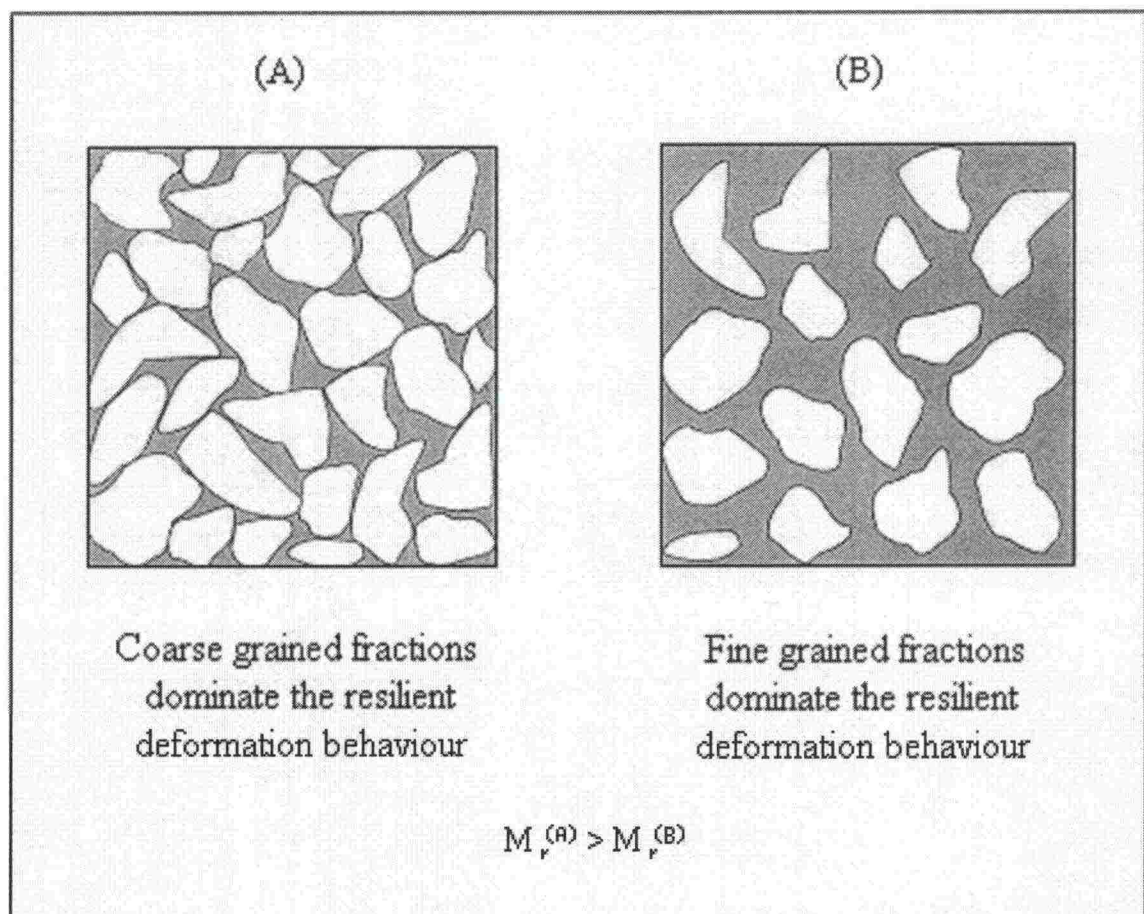


Figure 4.6.3:1 Quantitative effect of the fine-grained fractions on the mechanical behaviour of aggregates consisting mostly of coarse particles (Kolisoja 1997).

Not only the quantity of the fines fraction but also its quality presumably affects the deformation behaviour of the aggregates. In the case of fines fraction that is actively



absorbing water, the layers of absorbed water on the particle surfaces can become relatively thick. The fines fraction thus increases the water content of the material, decreases the permeability of the material, and, as a result, may efficiently advance the development of excess pore water pressure in the pore spaces of the loaded aggregate (Kolisoja 1997). As the effective stresses between the particles decrease, the deformation modulus of the aggregate also decreases.

Kim et al. (2004) reported that the dry density of optimum moisture content decreases as the fine content increases. According to them, aggregate gradation and amount of fines have an indirect effect on the resilient behaviour of unbound granular materials by affecting the impact of moisture and density of the system. A more direct impact of gradation on stiffness occurs due to the manner in which the fine particles fill the voids and impact the interaction among the coarser, angular particles. This can be visualised in the extreme when one compares a “floating matrix” where the coarse aggregate floats in the fines – preventing interaction – with a lack of fines where only coarse aggregate interaction provides a resistance to movement (Figure 4.6.3:1). The intermediate case where the coarse aggregate and fine aggregate blend is appropriately balanced to provide optimum density and maximum particle interaction.

Van Niekerk (2002) recognised that finer gradings will show less resilient dilatant behaviour (particles can resiliently rearrange under loading) than a coarse skeleton structure (particles will resiliently wedge sideways under loading).

Thom and Brown (1987) and Kamal et al. (1993) reported that resilient modulus generally decreases as the fine content increases. Hicks and Monismith (1971) found that resilient modulus decreases as fines content increases for partially crushed aggregates, but they found an opposite effect for fully crushed aggregates. A variation of fines content in the range of 2 – 10 % was reported by Hicks (1970) to have a minor influence on resilient modulus. Yet, a dramatic drop of about 60 % in resilient modulus was noted by Barksdale and Itani (1989), when the amount of fines was increased from 0 to 10 %.

Work performed by Jorenby and Hicks (1986) showed an initially increasing stiffness and then a considerable reduction as clayey fines were added to a crushed aggregate. The initial improvement in stiffness is attributed to increased contacts as pore space is filled. Gradually, excess fines displace the coarse particles so that the mechanical performance relies only on the fines, and stiffness decreases. For aggregates with the same amount of fines and similar shape of grain size distribution, the resilient modulus has been shown to increase with increasing maximum particle size (Gray 1962, Thom 1988, Kolisoja 1997). According to Kolisoja (1997), the particulate explanation of this response is that the major part of a load acting on a granular assembly is transmitted by particle queues. When the load is transmitted via coarser particles, the smaller number of particle contacts results in less total deformation and consequently higher stiffness.

The variation in resilient Poisson's ratio with fines content was investigated by Hicks (1970), who concluded that an increase in the amount of fines generally results in a reduction of Poisson's ratio. Kolisoja (1997) noted in his studies some influence of aggregate grading on Poisson's ratio, which was shown to be slightly smaller for coarse-



grained than for fine-grained aggregates. No study was found in the literature regarding the impact of maximum grain size on Poisson's ratio.

#### 4.6.4 Maximum grain size

Depending on the needs and application at hand, many characteristic numbers are used to describe the grain size distribution of granular materials. One of the simplest and at the same time one of the most approximate is the maximum grain size of the aggregate. The deformation modulus of aggregates having similar shape of grain size distribution has experimentally been shown to increase, when the maximum grain size increases (Thom 1988, Cheung 1994). Thom measured elastic shear strains of six different crushed limestone materials that had similar shape of grain size distribution but different maximum grain sizes (Figure 4.6.4:1) using two stress paths in a cyclic loading triaxial test. The results are presented in Figure 4.6.4:2.

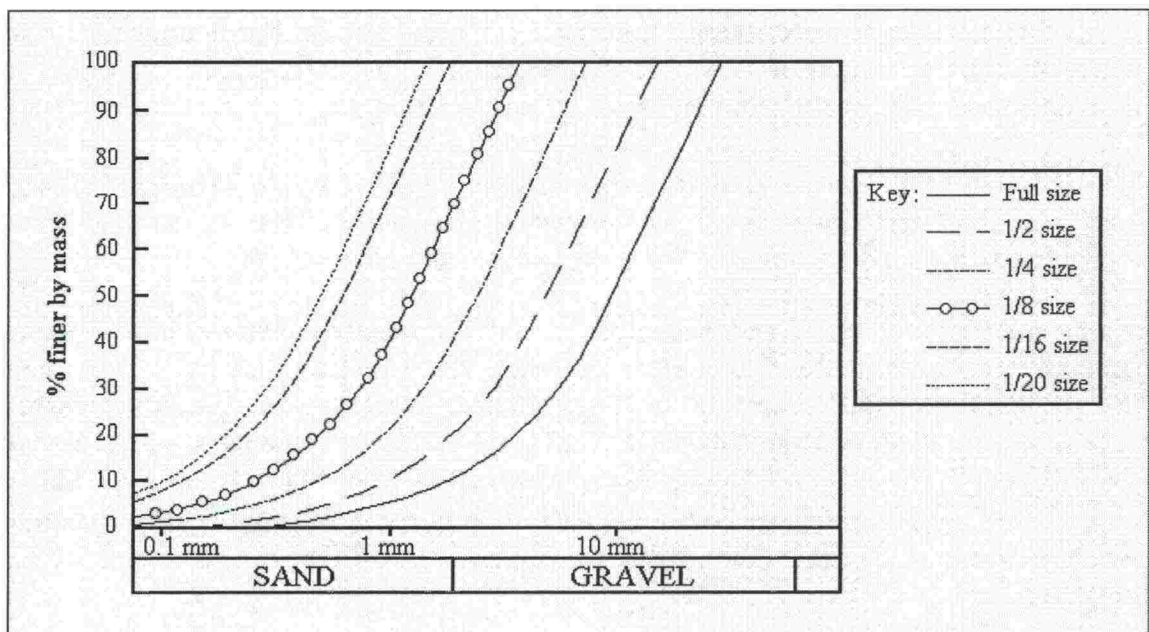


Figure 4.6.4:1 Grain size distributions of the test materials whose elastic shear strains are presented in Figure 4.6.4:2 (Thom 1988).

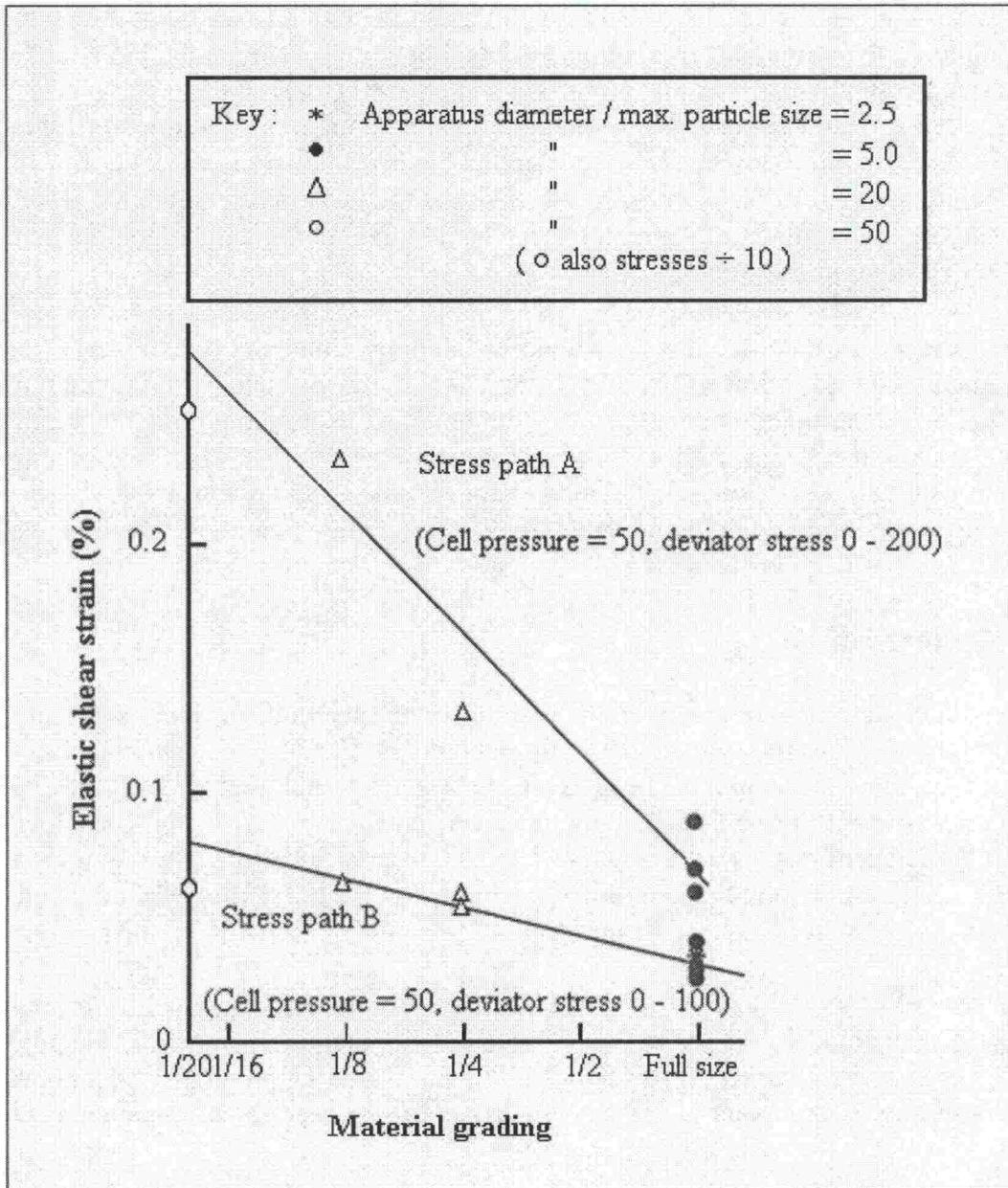


Figure 4.6.4:2 Elastic shear strains of six different crushed limestone materials having different maximum grain sizes for two stress paths (Thom 1988).

If the deformations produced in the mineral skeleton of a loaded coarse grained granular material are assumed to be concentrated mainly on the particle contacts and their immediate neighbourhoods, the explanation for the experimentally observed effects of the maximum grain size appears to be relatively straightforward: The coarser the material is, the smaller is the number of particle contacts needed to transmit the load exerted on the mineral skeleton (Kolisoja 1997).

The compression of a particulate system which is composed of spherical particles consisting of linear elastic material and being in simple cubic packing arrangement appears to be independent of the grain size if the load is applied in the direction of the axis of symmetry of the system and if the load is evenly distributed over the particle queues (Kolisoja 1997). However, as the packing arrangement of the particulate system



and the shape of the particles are very irregular in real coarse grained aggregates, according to the numerical simulation models (e.g. Ting and Khwaja 1993) and tests performed with photo elastic discs (e.g. Kuhn et al. 1991) most of the loads appear to be transmitted in granular materials by chain-like particle queues. Consequently, when the average size of the particle chains transmitting the load decreases, deflection of the particulate system can be assumed to increase and the deformation modulus describing the macroscopic stiffness of the material can be assumed to decrease in some relation to the particle sizes of the particle system.

One of the practical consequences of the conclusion presented above and found to be in agreement with experimental results is that when the mechanical behaviour of an aggregate is determined under laboratory circumstances, the grading of the specimens should be as close to the grading of the real materials as possible. Therefore, a so called scaled material, i.e. a material whose shape of the grading curve has been kept unchanged but whose maximum grain size is decreased, evidently does not behave similarly to the original material.

#### **4.7 Moisture content**

Some materials are more sensitive to moisture content than others. The most important factor that governs this sensitivity is the amount of fines in the material. For high content of fines the material becomes unstable when water is added. The degree of saturation or moisture content of most untreated granular materials has been found to affect the resilient response characteristics of the material in both laboratory and in situ conditions. However, the resilient properties do not seem to be affected as much as the resistance against permanent deformations.

It is generally agreed that the resilient response of dry and most partially saturated granular materials is similar, but as complete saturation is approached, the resilient behaviour may be affected significantly (Smith and Nair 1973, Vuong 1992). However, the consensus on this point is not unanimous, as can be seen for example from Figure 4.7:1.

Researchers (e.g. Haynes and Yoder 1963, Hicks and Monismith 1971, Barksdale and Itani 1989, Dawson et al. 1996, Heydinger et al. 1996), who studied the behaviour of granular materials at high degrees of saturation, have all reported a notable dependence of resilient modulus on moisture content, with the modulus decreasing with growing saturation level. Haynes and Yoder (1963), for instance, observed a 50 % decrease in resilient modulus in gravel as the degree of saturation increased from 70 % to 97 %. Hicks and Monismith (1971) showed that the resilient modulus decreases steadily as the moisture content increases above its optimum value.

Saturated granular materials develop excess pore water pressure under repeated loading. As pore water pressure develops the effective stress in the material decreases with a subsequent decrease in both strength and stiffness of the material. It can be argued that it is not the degree of saturation per se that influences the material behaviour, but rather that the pore pressure response controls deformational behaviour.

Mitry (1964), Seed et al. (1967), and Hicks (1970) stated that a decrease in resilient modulus due to saturation is obtained only if the analysis is based on total stresses. Similarly, Pappin (1979) observed that if the test results are analyzed on the basis of effective stresses, the resilient modulus remains approximately unchanged.

Thom and Brown (1987), however, argued that the presence of moisture in an aggregate assembly has some lubricating effect on particles. This would increase the deformation in the aggregate assembly with a consequent reduction of resilient modulus, even without generation of any pore water pressure. Thom and Brown confirmed this hypothesis with a series of repeated load triaxial tests on a crushed rock, where the moisture content was one of the parameters changed. Using drained tests and loading frequencies of 0.1 to 3 Hz, no noticeable pore pressures were developed for degrees of saturation of up to 85%. Despite the lack of pore pressure, the test results showed a reduction of resilient modulus with increasing moisture content and this was related to the lubricating effect of water. However, another way of interpreting these observations would be that localized pore suctions decrease with higher water content, leading to lower interparticle contact forces.

A study conducted by Raad et al. (1992) demonstrated that the effect of moisture on the resilient behaviour of unbound aggregates is perhaps most significant in well-graded materials with a high proportion of fines. This is because water is more readily held in the pores of such materials, whereas uniformly-graded materials allow water to drain freely.

Dawson et al. (1996) studied a range of well-graded unbound aggregates and found that below the optimum moisture content stiffness tends to increase with increasing moisture level, apparently due to development of suction. Beyond the optimum moisture content, as the material becomes more saturated and excess pore water pressure is developed, the effect changes to the opposite and stiffness starts to decline fairly rapidly.

Laboratory experiments have shown that aggregates having a low degree of saturation may have different stiffness properties depending on whether a certain moisture content has been achieved by moisturising or drying the material. In the latter case the stiffness of at least some materials has been observed to be much greater than the stiffness of the same material as it has been moisturised directly to the same moisture content (Thom 1988) or what have been the moduli values determined for the same material in a slightly higher moisture content (Sweere 1990).

The results of Thom's test series are presented in Figure 4.7:1 which illustrates how the wetting-drying cycles affect the modulus values of a crushed limestone having a maximum grain size of 10 mm. The effects were investigated using three different grain size distributions corresponding to different values of the grading parameter  $n$ . It should be noted that the modulus values shown in Figure 4.7:1 do not correspond to any certain stress path. Instead, the modulus values were computed as an average of the resilient modulus values that correspond to ten different stress paths. The measured elastic stiffnesses seem to decrease with increasing water content, but the decrease is strongly dependent on the grain size distribution. The largest differences are found in the dry end.



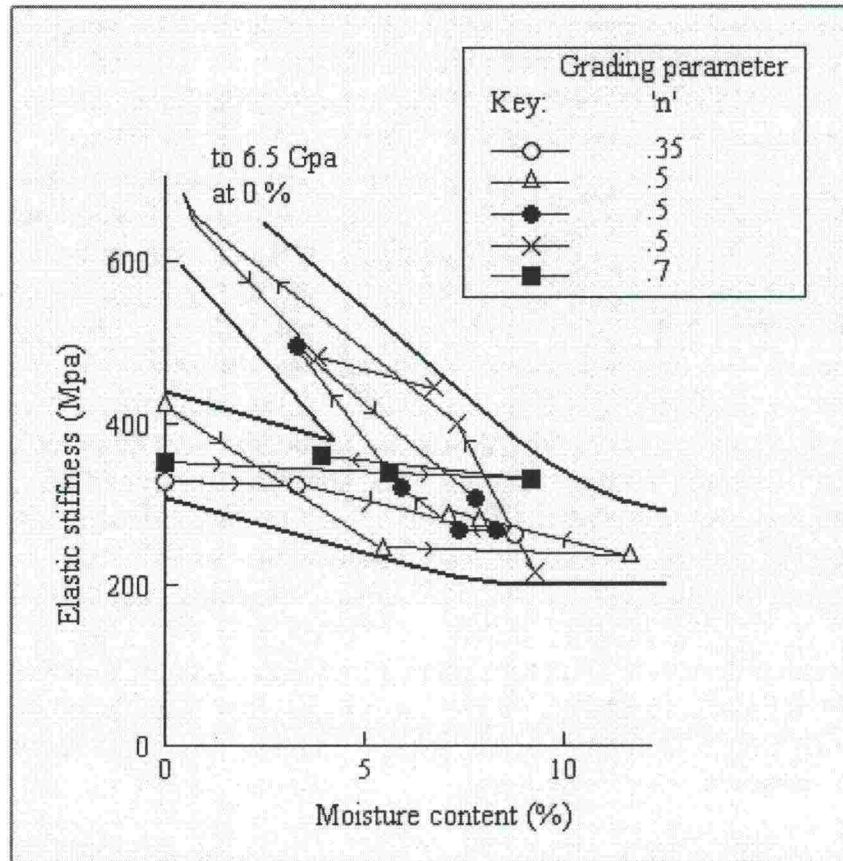


Figure 4.7:1 Effect of the wetting - drying cycles on the resilient behaviour of a crushed limestone (Thom 1988).

A physical explanation for the above-mentioned results is probably provided by the suction mechanism working very efficiently in the most fine-grained fractions of the aggregates. When the material dries, the available water concentrates at the particle contacts. The finest grained material gathered close to the contact points of the coarse grained particles can then be assumed to work as some sort of bond between the large particles. However, if dry material is directly wetted to a certain relatively low moisture content the suction mechanism does not necessarily work in the above described way, especially if the amount of water is insufficient to thoroughly moisturise all particle surfaces.

Wu et al. (1984) have presented an excellent example of a test series whose results illustrate clearly the effect of the degree of saturation on the stiffness of granular material under very low level of strains. Wu et al. performed resonant column tests for five different materials whose grading varied from silt to sand. The results show that in partly saturated conditions the values of the shear modulus can even double in comparison with dry and fully saturated conditions (Figure 4.7:2). They concluded that for the materials investigated in the research the optimum degree of saturation corresponding to the maximum value of the shear modulus varies from 5.0 to 17.0 % depending on the effective grain size  $d_{10}$ .

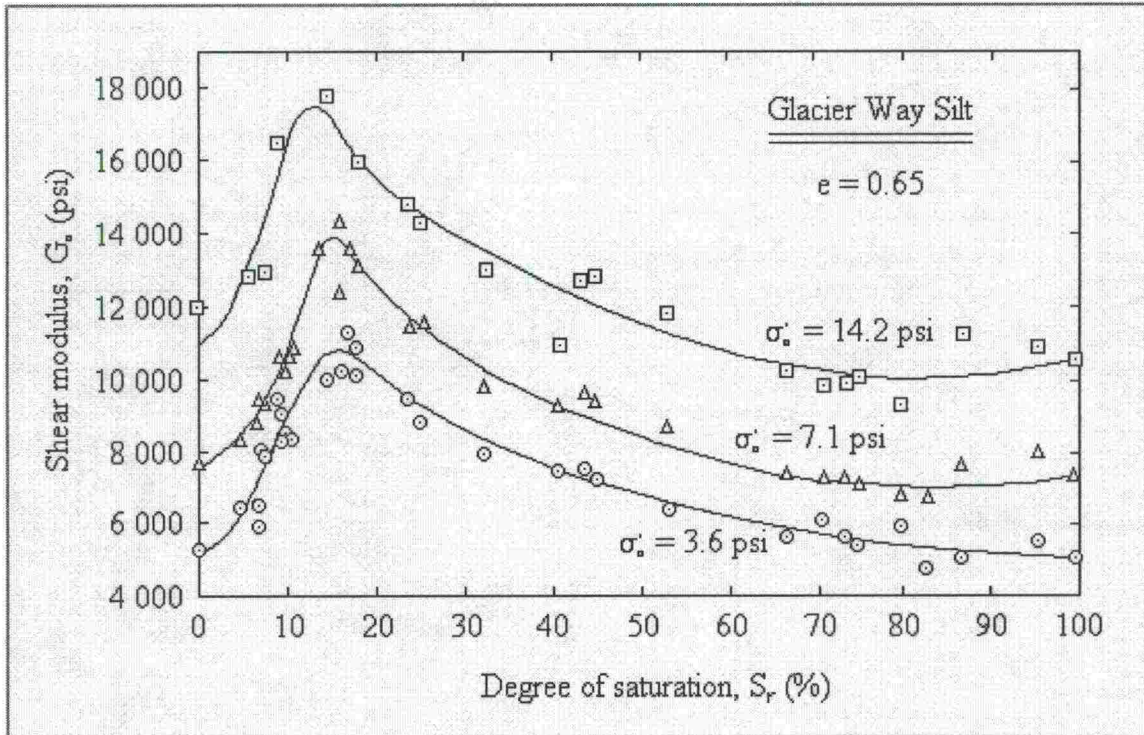


Figure 4.7:2 Variation in the shear modulus as a function of the degree of saturation and confining pressure  $\sigma'_0$  for a silt material (Glacier Way Silt) (Wu et al. 1984).

Figure 4.7:3 shows the resilient modulus as function of the sum of the principal stresses for five different levels of saturation tested by Kolisoja (1996b). The single curve that differs most from the other curves is the one for dry material that shows significant lower stiffness than curves for material with water. It should be noted that during the tests Kolisoja found a certain degree of internal instability, which resulted in the finer materials not settling to the bottom of the sample during the test.



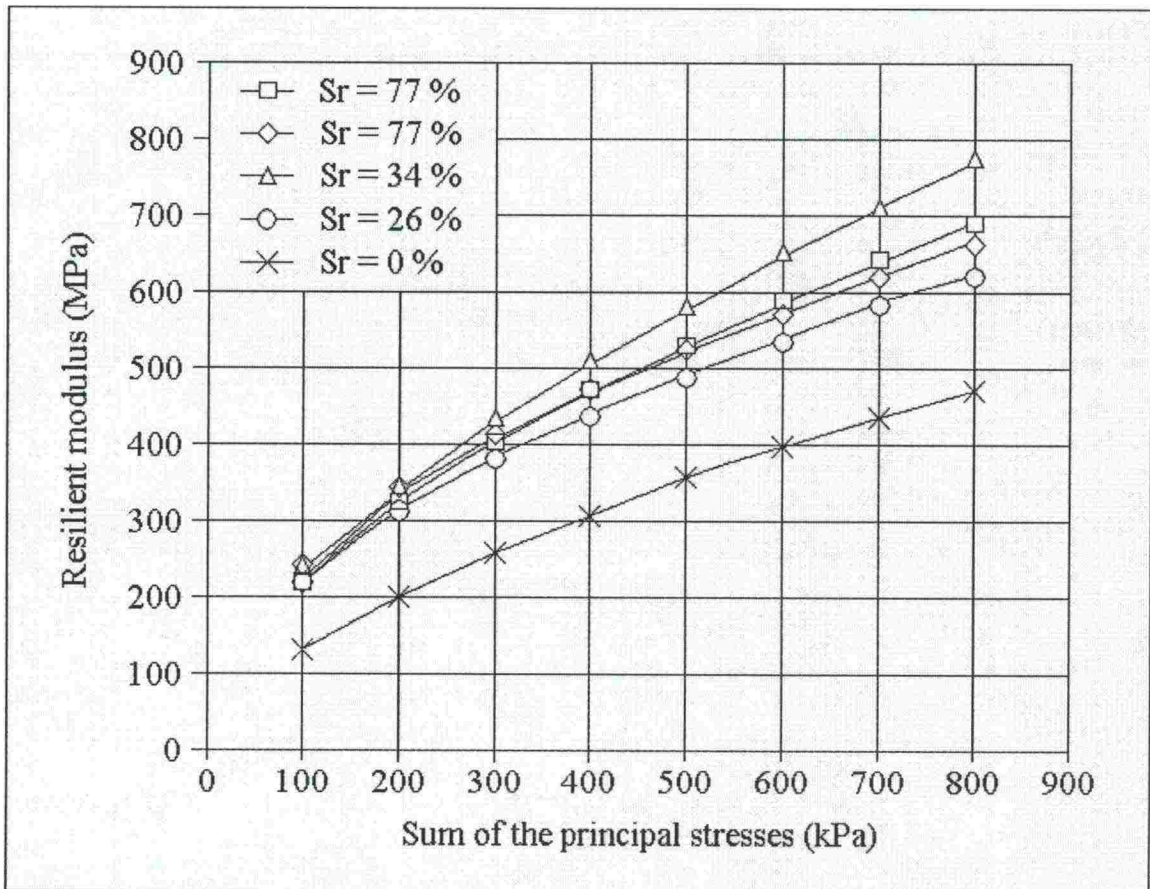


Figure 4.7:3 Modulus of resilience for different degree of saturation (Kolisoja 6/1996).

Hoff (1999) tested two different grain size distributions for different moisture contents. One distribution has a grading parameter  $n = 0.5$  (material from Åndalen AN1 – see Figure 4.6.2:3 for the grain size distribution), while for the other one  $n = 0.35$  (material from Åndalen AN2 – see Figure 4.6.2:3 for the grain size distribution). The resilient moduli are shown in Figure 4.7:4 for the grading curve with  $n = 0.5$  and in Figure 4.7:5 for the grading curve with  $n = 0.35$ .

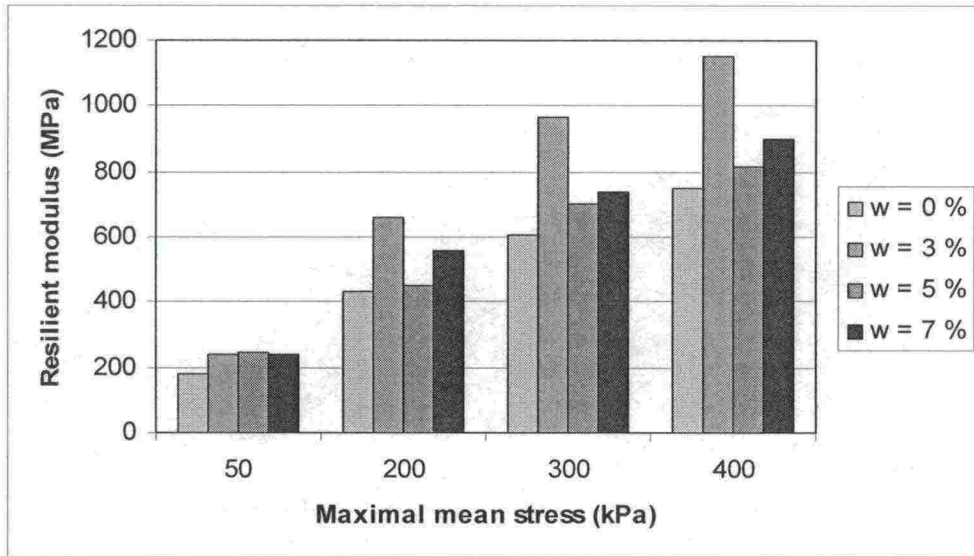


Figure 4.7:4 Resilient modulus for different moisture level for the distribution with  $n = 0.5$  (Hoff 1999).

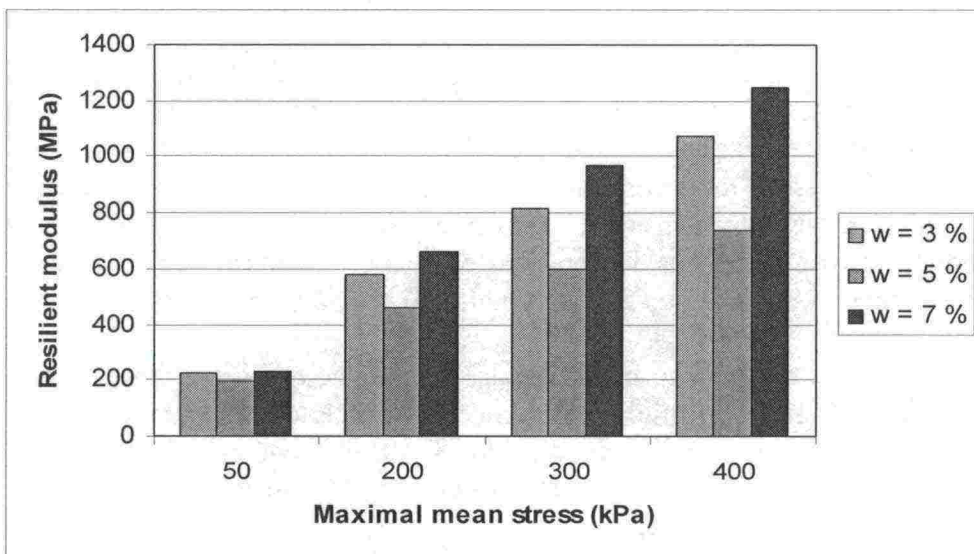


Figure 4.7:5 Resilient modulus for different moisture level for the distribution with  $n = 0.35$  (Hoff 1999).

These tests were performed on samples compacted with the same compaction energy. Because the effect of compaction is dependent on moisture content, this gives different void ratios for the samples. Figure 4.7:6 shows the maximum dry density as a function of moisture content for the samples with Fuller curve parameter  $n = 0.5$ . It seems like the relative high stiffnesses obtained for a water content of 3 % might be influenced by suction, whereas the high stiffnesses for water content of 7 % are related to the higher density.



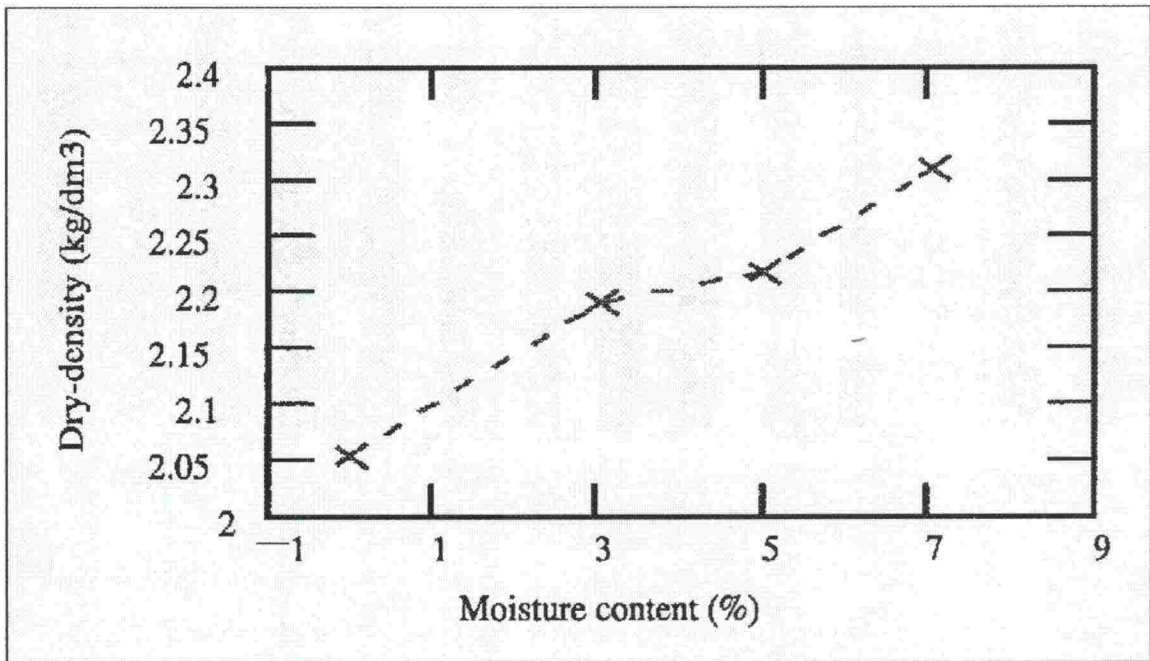


Figure 4.7:6 Dry-density as a function of moisture content (Hoff 1999).

As can be seen from the figure the dry-density increases with the moisture content. This may explain why the results seem to give highest stiffness for the highest moisture content. The testing of Thom and Brown (1987) shows an opposite effect with decreasing stiffness for increasing moisture content. However, other studies show an increasing stiffness for increasing dry-density. The water content of Hoff's tests was apparently not high enough to create any positive pore water pressure in these free draining materials that could cause a reduction in stiffness as indicated by Thom and Brown's results.

Saturation of unbound granular materials also affects the resilient Poisson's ratio. Hicks (1970) and Hicks and Monismith (1971) reported that Poisson's ratio is reduced as the degree of saturation increases. According to Hicks, the reduction in Poisson's ratio is noted whether the analysis is based on total or effective stresses. This suggests that a pore suction/pressure explanation for changing resilient Poisson's ratio is insufficient. However, the explanation involving the lubricating effect of water must also be questioned, as a higher Poisson's ratio should be expected with greater lubrication.

Ekblad (2004) tested materials with grading coefficients ranging from 0.3 to 0.8. He found that materials with high grading coefficients are less affected by a change in water content. This is mainly connected to the amount of fines in the materials, as the amount of fines in this case also increases with increasing grading coefficient. Ekblad found that an increase in water content causes a reduction in resilient modulus and an increase in Poisson's ratio as well. This behaviour was more pronounced as the grading parameter is decreased.

Similar results were found by Uthus et al. (2005), who studied the influence of water and fines on the behaviour of unbound granular materials. The basis is the Fuller curve, where two different grading coefficients ( $n = 0.5$  and  $n = 0.35$ ) are used to investigate the influence of grading, in particular the finer part of the curve. In this study, two materials were tested, one hard very fine grained gneiss (Gneiss 1) from a quarry in Askøy outside

Bergen in Norway ( $LA = 17.1$ ) and one weaker fine to medium grained gneiss (Gneiss 2) with about 30 % mica taken from a quarry near Göteborg in Sweden ( $LA = 24$ ). Both material gradings from Askøy (Gneiss 1) nearly showed the same resilient modulus function under low water contents. The material with the higher amount of fines, however, showed a significant reduction in modulus and resistance to permanent deformations under only slightly increased water contents, while the material with the lower amount of fines had to be influenced by higher water content before a reduction of the resilient modulus and the plastic limit occurred. The mica-rich gneiss from Göteborg (Gneiss 2) showed lower resilient moduli under the same mean normal stress compared to gneiss 1 for high stress levels. The materials with high amounts of fines showed lower resilient moduli under the same mean normal stress for high water contents than the materials with low fines contents. These materials were, however, less influenced by the water content in the ultimate stress stage. The dry density seemed to be more important for the shear strength than the water content in the ultimate stress stage, while the water content had a significant effect in the elastic stress stage / design stress stage. In particular, the following results were obtained:

- Gneiss 1 (Figure 4.7:7) → The resilient moduli for the two gradings  $n = 0.35$  and  $n = 0.50$  show about the same values for the highest water content of 6.7 - 6.8 %. The material with the higher amount of fines ( $n = 0.35$ ) shows, however, significantly lower values of  $M_r$  with increased water contents ( $w = 3.0 - 5.9$  %). The material with the lower amount of fines ( $n = 0.50$ ) has about the same module as a function of mean stress for water contents in the range 3.0 to 4.8 %, but lower resilient moduli for  $w = 6.8$  %. The material with the lower amount of fines seems to be less sensitive to small changes in water contents (3.0 - 4.8 %). Increased water content to  $w = 6.8$  % gives a significantly lower resilient moduli function. However, the modulus for the wet condition are very close for the two series  $n = 0.35$  and  $n = 0.50$  under the highest water conditions.
- Gneiss 2 (Figure 4.7:8) → This material shows significantly lower resilient moduli than Gneiss 1 under same mean normal stress and corresponding water contents. The material with the higher amount of fines ( $n = 0.35$ ) has approximately the same modulus function for low and medium water contents ( $w = 4.0 - 5.2$  %), while increased water content to 6.0 % gives a significantly lower resilient modulus function. In this case, the material with low fines contents ( $n = 0.50$ ) also gives near the same modulus function for the low water contents ( $w = 3.7 - 4.9$  %), while increased water contents to 6.1 % gives considerable lower resilient modulus under the same mean normal stress for high stress levels only. The unbound aggregate from Gneiss 2 with considerable amounts of mica gives significantly lower resilient moduli functions than Gneiss 1 from Askøy for high stress levels ( $\geq 150$  kPa).



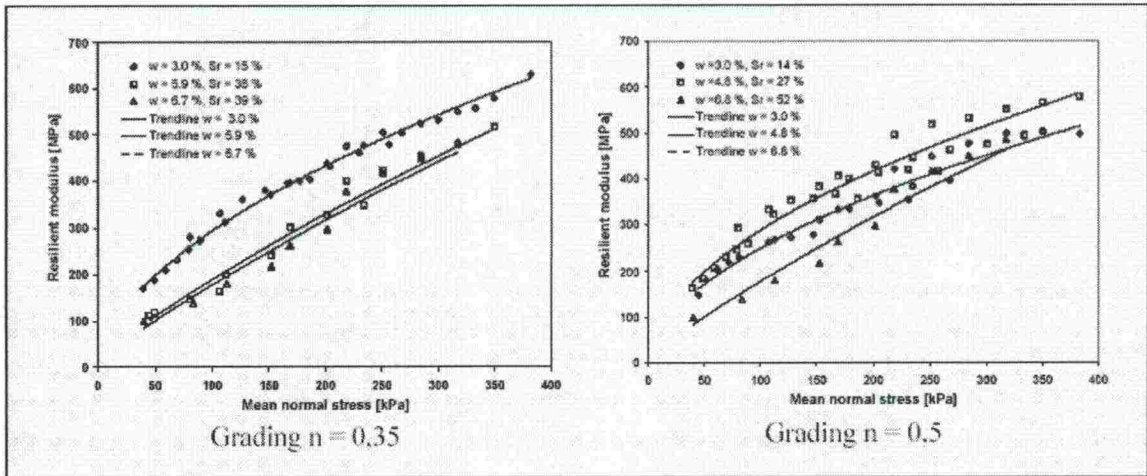


Figure 4.7:7 Resilient response for Gneiss 1 (Uthus et al. 2005)

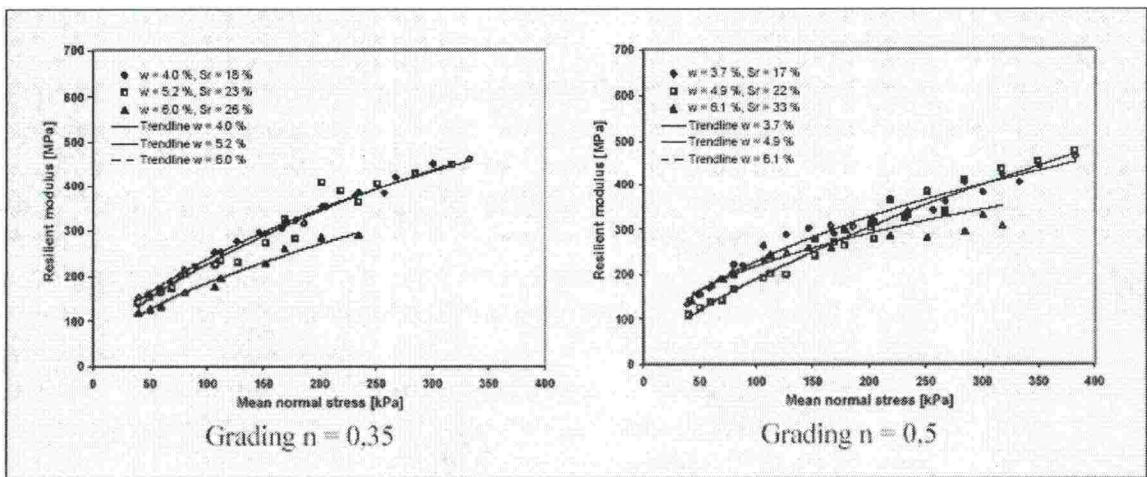


Figure 4.7:8 Resilient response for Gneiss 2 (Uthus et al. 2005)

#### 4.8 Particle characteristics

The most important factors influencing the grain shape of coarse-grained aggregates are the mineralogical composition and structure - especially the foliation of the structure - of the aggregate particles. Also the crushing technique used to produce the aggregates affects the grain shape of the crushed materials.

Research results on the effects of the grain shape on the deformation behaviour of coarse-grained materials have hardly been reported. One of the most important reasons is obviously that exact determination of the grain shape of a large number of particles is very laborious. Moreover, the grain shape of a certain material can even at best be described using statistical values that also vary with the grain size. On the other hand, the direct effect of the grain shape variation on the deformation behaviour of materials approved in practical use is likely to be so small that its separation from the effects of the other factors is probably rather difficult.

Aggregate particle shape can be expressed using three independent properties: form, angularity, and surface texture (Barrett 1980, Masad 2000). Form quantifies the

dimensional proportions of the aggregate. Angularity refers to the sharpness or roundness of the corners, while texture refers to the small-scale asperities at the surface of a particle. Figure 4.8:1 shows a two-dimensional schematic of these three properties.

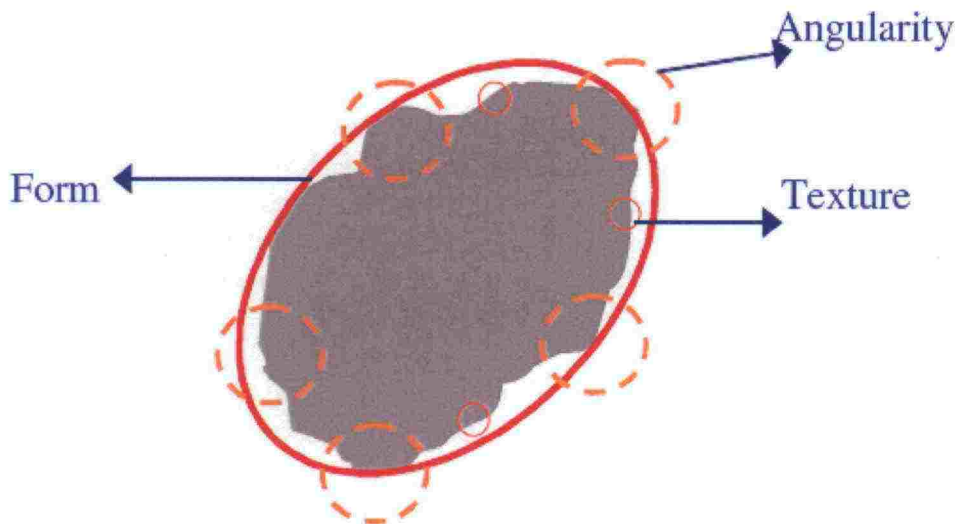


Figure 4.8:1 Schematic diagram of aggregate shape properties (Masad 2000).

Within a specific type of aggregate (mineralogy and classification), texture and aggregate shape significantly influence the resilient response of the granular material. Rough-textured and angular aggregates develop a stronger and stiffer mass by locking together, while smooth-textured and rounded aggregate particles tend to slide past one another (Kim et al. 2004). Studies have proven that the intuitive and obvious is true: crushed aggregates, high in angularity and roughly textured, provide better load carrying capacity and a higher resilient modulus than rounded, uncrushed particles (Hicks 1970, Hicks and Monismith 1971, Allen 1973, Allen and Thompson 1974, Barksdale and Itani 1989, Thom and Brown 1989).

Masad (2001) proposed the use of a parameter named the form index that utilises the incremental change in the particle radius in all directions. The radius is defined as the length of the line that connects the particles center to points on the boundary. The form index is expressed as

$$\sum_{\theta=0}^{\theta=360-\Delta\theta} \frac{|R_{\theta+\Delta\theta} - R_{\theta}|}{R_{\theta}}, \quad (\text{Eq. 4.8:1})$$

where

R            =            radius of the particle in different directions,  
 θ            =            directional angle.



The form index can be measured using the Aggregate Imaging System (AIMS) (Masad 2001). The system uses a video microscope to measure the depth of a particle. It analyses the other two dimensions from two-dimensional projections of particles. These three dimensions are obtained for every particle in the sample to generate a cumulative distribution of form index in an aggregate sample. Aggregate angularity is measured through AIMS using the gradient method (Kim et al. 2004). In this method, the change in the gradients on the boundary of a two-dimensional projection of a particle is calculated. The differences between these gradients along the boundary are taken as a measure of angularity. Smooth particles have small gradients, while rough particles have higher gradients. Texture is analysed using the wavelet transform, which captures the changes of texture on grey scale images (Kim et al. 2004). The wavelet transform gives a higher texture index for particles with rougher surfaces. Similar to the sphericity index, form and texture are represented by cumulative distribution functions as it is common practice to represent aggregate gradation using a cumulative distribution function.

In a study conducted by Heydinger et al. (1996), gravel was shown to have a higher resilient modulus than crushed limestone. However, many researchers (Hicks 1970, Hicks and Monismith 1971, Allen 1973, Allen and Thompson 1974, Thom 1988, Barksdale and Itani 1989, Thom and Brown 1989) have reported that crushed aggregate, having angular to sub angular shaped particles, provides better load spreading properties and a higher resilient modulus than uncrushed gravel with sub rounded or rounded particles. A rough particle surface is also said to result in a higher resilient modulus.

Barksdale and Itani (1989) investigated several types of aggregate, and observed that the resilient modulus of the rough, angular crushed materials was higher than that of the rounded gravel by a factor of about 50 % at low mean normal stress and about 25 % at high mean normal stress.

While increasing particle angularity and surface roughness could result in higher resilient modulus, studies show that Poisson's ratio decreases for the same conditions (Hicks 1970, Hicks and Monismith 1971, Allen 1973). This observation tends to reinforce the questioning of a lubrication explanation for the relationship between Poisson's ratio and moisture reported a few paragraphs earlier, as lateral resilient movements are indicated here as being controlled by interparticle contact condition.

In his investigation, Hoff (1999) tested both open graded and well-graded samples. Figure 4.8:2 shows large differences between crushed rock and gravel materials. The four different types of rock materials (Åndalen, Steinskogen, Hedrum, and Visnes) have stiffnesses that are significantly higher than the corresponding ones of the gravel materials (Hovinmoen natural gravel and Hovinmoen crushed gravel).

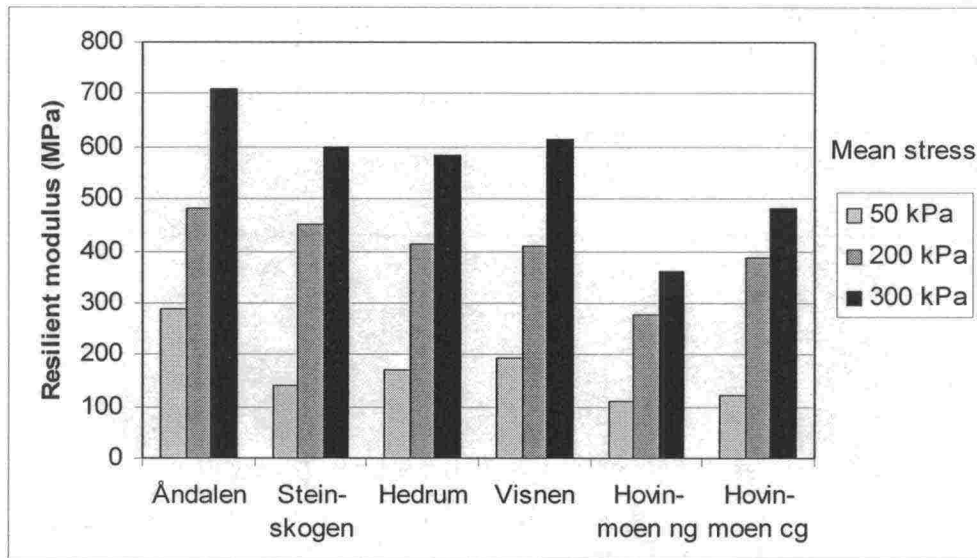


Figure 4.8:2 Modulus of resilience for the six different materials (well-graded samples) (Hoff 1999).

Samples of these six types of materials with open gradings (4 - 22 mm) were tested to study the stiffness of a more open-grained skeleton. Figure 4.8:3 shows the resilient modules for the 4 - 22 mm grain size distribution. For these coarse grained materials the differences between the stiffnesses of the rock materials (Åndalen, Steinskogen, Hedrum, and Visnes) and the gravel materials (Hovinmoen natural gravel and Hovinmoen crushed gravel), though still significant, are somewhat smaller than they were in the case of well-graded samples (Hoff 1999).

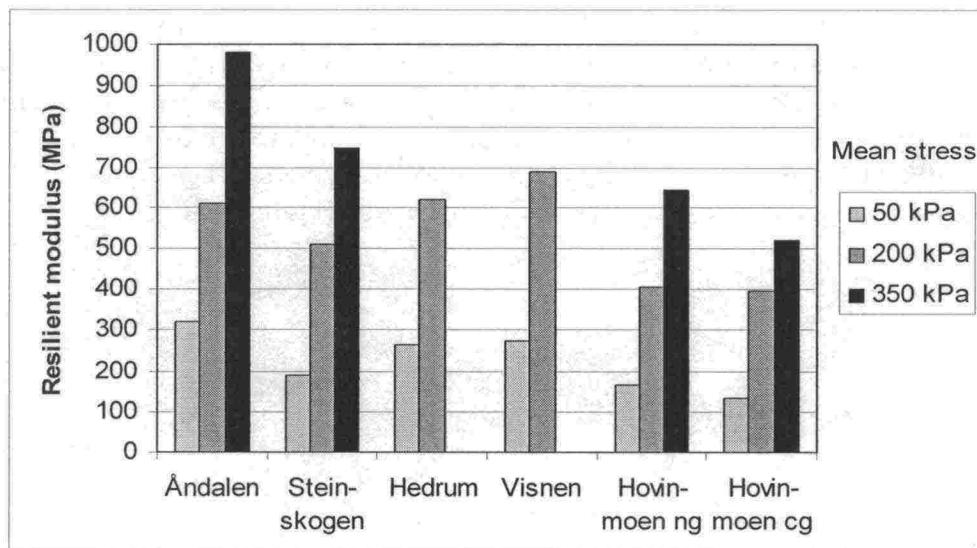


Figure 4.8:3 Modulus of resilience for the six different materials (open graded samples) (Hoff 1999).

The same materials tested in Hoff's study have also been tested in Tampere, Finland, and are reported by Kolisoja in (1996a and 1996b). Also in this case the materials from Åndalen, Steinskogen, Hedrum, and Visnes are rock materials, while the materials from



Hovinmoen are gravel materials. Figure 4.8:4 shows resilient moduli for some of the tests performed in Tampere. The moduli cannot be directly compared because compaction, test procedures, and grain size distribution are different, but the results seem to have the same tendencies as the results from this investigation.

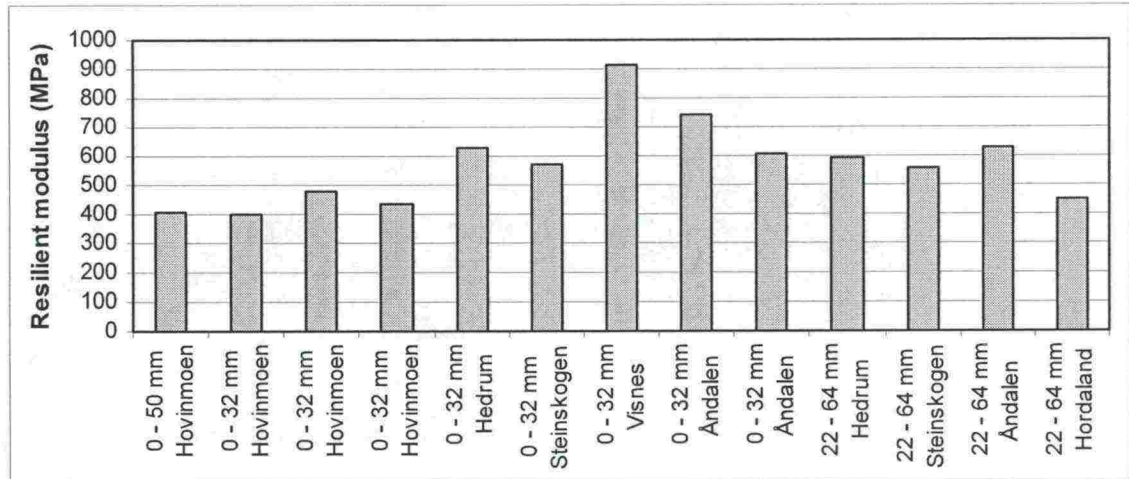
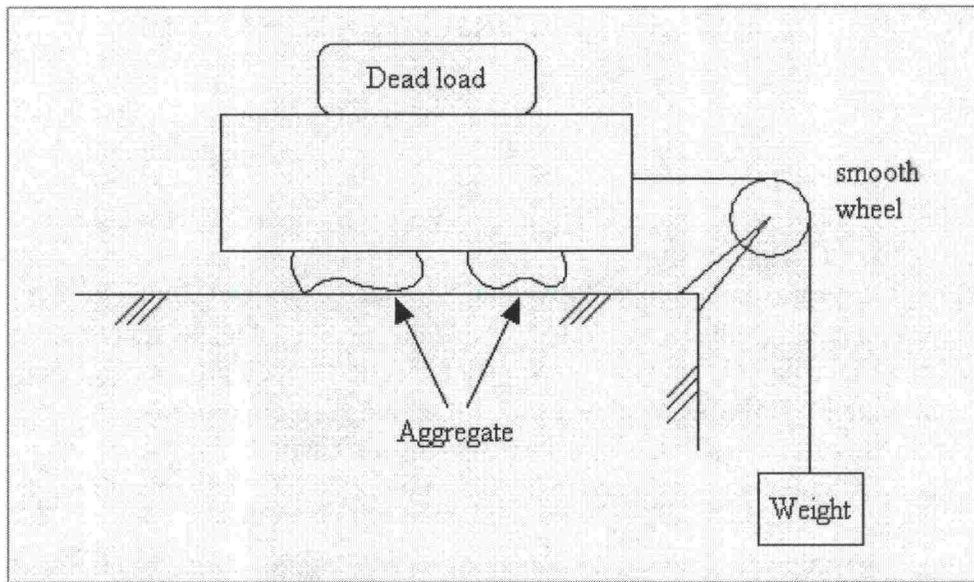


Figure 4.8:4 Modulus of resilience different materials (Kolisoja 1/1996 and Kolisoja 6/1996).

Six different materials were tested in the very large (625 x 1250 mm) repeated load triaxial apparatus at the Norwegian Geotechnical Institute (NGI) and are reported by Vik (1996). These tests also show the differences between gravel and crushed rock materials. For high deviatoric stress levels, the crushed rock is significantly stiffer.

Many investigations have addressed the effect of macro and micro roughness of the particles on the deformation behaviour (Cheung 1994, Kolisoja 1997, Thom and Brown 1989). The surface friction angle can be measured using a sliding type test performed by pulling a representative specimen of aggregate loaded on a rough test surface (Cheung 1994). In the test, the horizontal force required to move the aggregate particles that are compressed against the base by a dead load is measured (Figure 4.8:5).



*Figure 4.8:5 Functional overview of the test arrangement used to measure the surface friction angle of aggregates (Cheung 1994).*

The surface friction at the contact points of particles can be assumed to affect the resilient deformation behaviour, especially when the external load reaches the value that makes the particles slide. The reason is that the maximum frictional force value resisting sliding is defined as a mathematical product of the coefficient of friction and the normal force acting at the contact point. It should be mentioned that the value of the surface friction angle determined using the friction test as proposed by Cheung (1994) does not necessarily correlate with the visual assessment of the surface roughness. Therefore, the recoverable deformation and the macro level surface roughness do not necessarily interrelate. However, the macro level surface roughness (visible roughness of the individual particles based on the number of protrusions in the surface) probably correlates better with the ability of the material to resist permanent deformations (Thom and Brown 1989, Brown and Selig 1991). The surface structure of mineral crystals probably has more significant influence on the friction, because the surface structure basically determines the micro level influencing mechanisms of the sliding resistance.



## 5 MODELLING OF RESILIENT DEFORMATION BEHAVIOUR

### 5.1 Introduction

Despite the fact that the resilient behaviour of granular materials is affected by several factors, the effect of stress parameters is the most significant. It is therefore essential that the stress-strain relationship be modelled as accurately as possible with constitutive laws. Over the years, many constitutive models have been developed to express the resilient response of granular materials. The complexity of the behaviour of unbound granular materials has made it a very difficult task for researchers to combine the theoretical principles of soil mechanics with the simplicity required in procedures for routine analysis of the material characteristics. In this chapter, the models found in the literature are presented and some are reviewed in more detail.

Modelling is an important necessity for an analytical approach in describing material performances. Many researchers have outlined different procedures for predicting the resilient and permanent strain responses of granular materials. However, the great number of models available is per se further evidence of the complexities that overshadow this research area. In spite of the significant progress made over the years in understanding the resilient behaviour of granular materials, there is still a great need for further research into developing more general models and procedures which have a sound theoretical basis and wide applicability.

In general, two approaches are employed for mathematical modelling of the resilient behaviour of granular materials. In the first approach, the stress-strain relationship is given by a stress-dependent resilient modulus and a constant or stress-dependent Poisson's ratio. This type of modelling is widely used and several numerical formulations, using different stress components, are found in the literature. The K- $\theta$  model (Hicks 1970, Seed et al. 1967) and the Uzan model (Uzan 1985) are examples of this type of modelling.

In the second approach, the stress-strain relationship is characterised by decomposing both stresses and strains into volumetric and shear components. The resilient response of the material is then defined using bulk and shear moduli instead of resilient modulus and Poisson's ratio. Models of this kind are usually more complex in nature, and the parametric values are more difficult to determine from collected test data. Examples of the models using the shear/volumetric approach are the Boyce model (Boyce 1980), the contour model (Brown and Pappin 1985), and the modified inelastic and anisotropic versions of the Boyce model suggested by Sweere (1990) and Hornych et al. (1998), respectively.

The models analysed in this chapter are based on the results obtained by repeated loading triaxial tests. In order to provide a reasonable simulation of traffic-type loading using triaxial equipment, the loading system should be able to cycle both the vertical (deviator) stress and the confining pressure in phase, and at levels and frequencies corresponding to the actual field conditions.

There are two test methodologies for conducting repeated load triaxial tests: the constant confining pressure test (CCP) and the variable confining pressure test (VCP). Both the CCP test and the VCP test are described into more detail in Chapter 8.

## 5.2 Models based on resilient modulus and Poisson's ratio

### 5.2.1 Definition of resilient modulus and Poisson's ratio

In the traditional theories of elasticity, the elastic properties of a material are defined by the modulus of elasticity ( $E$ ) and Poisson's ratio ( $\nu$ ), which are material constants. A similar approach has been widely used in dealing with granular materials, but the modulus of elasticity is replaced with the resilient modulus to indicate the non-linearity, i.e. the dependence on the stress level, of the behaviour. For repeated load triaxial tests with constant confining pressure, the resilient modulus and Poisson's ratio are defined by

$$M_r = \frac{\Delta(\sigma_1 - \sigma_3)}{\varepsilon_{1,r}}, \quad (\text{Eq. 5.2.1:1})$$

$$\nu = -\frac{\varepsilon_{3,r}}{\varepsilon_{1,r}}, \quad (\text{Eq. 5.2.1:2})$$

where

$M_r$	=	resilient modulus,
$\nu$	=	resilient Poisson's ratio,
$\Delta$	=	indicates "change in",
$\sigma_1, \sigma_3$	=	major and minor principal stress,
$\varepsilon_{1,r}, \varepsilon_{3,r}$	=	recoverable axial and horizontal strain.

This method of calculating the resilient parameters is the same as would apply to an isotropic, linear-elastic material under uniaxial stress conditions. When cyclic confining pressure is applied, the generalized Hooke's law is employed for 3-dimensional stress-strain relationships of an isotropic, linear-elastic material. The resilient modulus and Poisson's ratio are then derived from

$$M_r = \frac{\Delta(\sigma_1 - \sigma_3) \cdot \Delta(\sigma_1 + 2\sigma_3)}{\varepsilon_{1,r} \cdot \Delta(\sigma_1 + \sigma_3) \cdot 2\varepsilon_{3,r} \cdot \Delta\sigma_3}, \quad (\text{Eq. 5.2.1:3})$$

$$\nu = \frac{\Delta\sigma_1 \varepsilon_{3,r} - \Delta\sigma_3 \varepsilon_{1,r}}{2\Delta\sigma_3 \varepsilon_{3,r} - \varepsilon_{1,r} \Delta(\sigma_1 + \sigma_3)}. \quad (\text{Eq. 5.2.1:4})$$

Many researchers have tried to outline mathematical procedures for describing the stress dependence of the resilient modulus using various stress variables. The great majority of the models found in the literature are based on simple curve fitting procedures using the data from laboratory triaxial testing.



### 5.2.2 Resilient modulus as a function of hydrostatic stress

It is well known that the stiffness modulus of unbound granular materials is stress dependent, but the Poisson's ratio is not or at least is to a much lesser extent and can usually be treated as a constant. A number of relationships exist to describe the stress dependency of the stiffness moduli. Seed et al. (1967) and Brown and Pell (1967) suggested the so-called K- $\theta$  model for the interdependence between the resilient modulus and the stress level. A few years later, this model was extensively described by Hicks and Monismith (1971). The K- $\theta$  model, which is the oldest and probably the most popular among the models based on non-linear elasticity, is written in its traditional form as follows:

$$M_r = k_1 \theta^{k_2}, \quad (\text{Eq. 5.2.2:1})$$

where

$M_r$	=	resilient modulus,
$\theta$	=	sum of the principal stresses when the deviator stress has its maximum value,
$k_1, k_2$	=	material parameters.

Because Equation 5.2.2:1 is not dimensionally correct, a more recommendable form of the K- $\theta$  model is presented in Equation 5.2.2:2 which is also the analogue of many commonly known modulus quantities in soil mechanics (Janbu 1967):

$$M_r = k_1 \theta_0 \left( \frac{\theta}{\theta_0} \right)^{k_2}, \quad (\text{Eq. 5.2.2:2})$$

where

$M_r$	=	resilient modulus,
$\theta$	=	sum of the principal stresses,
$\theta_0$	=	reference stress 100 kPa,
$k_1$	=	material parameter ('modulus number'),
$k_2$	=	material parameter ('stress exponent').

If the resilient modulus  $M_r$  is determined at different load levels, i.e. different stress conditions, the material parameters  $k_1$  and  $k_2$  are regression parameters and are specific for the material tested and the conditions used. The material parameter  $k_2$  describes the stress dependency. An increase of the stress dependency results in an increase of  $k_2$  (Hansson and Lenngren 2005).

This model has been very popular for describing non-linear resilient response of granular materials and it has been used in many computer programs for pavement design, for example in ILLIPAVE and ADEM by Stock and Brown (1980) and KENLAYER by Huang (1993).

Sometimes in the literature (e.g. Ooi et al. 2004) in Equation 5.2.2:2 appears the minor principal stress  $\sigma_3$  instead of the bulk stress  $\theta$ .

The simplicity of the K- $\theta$  model has made it extremely useful and widely accepted for analyzing the stress dependence of material stiffness. However, this model has several drawbacks and quite many modifications to the model have been found in the literature. The K- $\theta$  model assumes a constant Poisson's ratio  $\nu$ , which is then used to calculate radial strain. However, several studies (Hicks 1970, Hicks and Monismith 1971, Brown and Hyde 1975, Boyce 1980, Sweere 1990, Kolisoja 1997) have shown that Poisson's ratio is not a constant and varies with applied stresses. Sweere (1990), for instance, used the K- $\theta$  model and reported that the use of a constant Poisson's ratio leads to good predictions of axial strains, but rather poor predictions of radial and volumetric strains. Another drawback with the K- $\theta$  model is that the effect of stress on resilient modulus is accounted for solely by the sum of the principal stresses. Several studies have shown this to be insufficient and additional stress parameters are required for the analysis. May and Witczak (1981) noted that the in situ resilient modulus of a granular material is a function not only of the bulk stress, but also of the magnitude of the shear strain induced mainly by shear or deviator stress.

Dunlap (1963) and Monismith et al. (1967) indicated that the resilient modulus increases with confining pressure and is sensibly unaffected by the magnitude of repeated deviator stress, provided the deviator stress does not cause excessive plastic deformation. They, therefore, proposed an expression solely based on the effect of confining stress as given by Equation 5.2.2:3 or Equation 5.2.2:4:

$$M_r = k_1 \sigma_3^{k_2}, \quad (\text{Eq. 5.2.2:3})$$

$$M_r = k_1 \left( \frac{\sigma_3^{k_2}}{p_0} \right), \quad (\text{Eq. 5.2.2:4})$$

where

$M_r$	=	resilient modulus,
$k_1, k_2$	=	material parameters,
$p_0$	=	atmospheric pressure (100kPa),
$\sigma_3$	=	minor principal stress.

### 5.2.3 Resilient modulus as a function of deviatoric stress

In the models having the form of Equations 5.2.2:1 and Equation 5.2.2:2, the effect of the stress level on the resilient modulus is taken into account solely by using the sum of the principal stresses. The problem of the method is, however, that changes in the hydrostatic and deviator stress state components are not separated from each other. Since the shear strain level that depends on the level of the deviator stress has been found to have marked impact on the deformation modulus of the granular materials, it should be able to be taken into account in the modelling. An often used method is to express the resilient modulus as in Equation 5.2.3:1 (Uzan 1985):



$$M_r = k_1 \theta_0 \left( \frac{\theta}{\theta_0} \right)^{k_2} \left( \frac{q}{\theta_0} \right)^{k_3}, \quad (\text{Eq. 5.2.3:1})$$

where

$M_r$	=	resilient modulus,
$\theta$	=	the sum of the principal stresses,
$\theta_0$	=	reference stress 100 kPa,
$q$	=	deviator stress,
$k_1, k_2, k_3$	=	material parameters.

According to Uzan (1985), another way of expressing the dependence of the resilient modulus on both the bulk stress and the deviator stress is

$$M_r = k_1 \cdot \theta^{k_2} \cdot \sigma_d^{k_3}, \quad (\text{Eq. 5.2.3:2})$$

where

$M_r$	=	resilient modulus,
$k_1, k_2, k_3$	=	regression parameters determined using regression analysis,
$\theta$	=	bulk stress = 3p,
$\sigma_d$	=	deviator stress.

In the general 3-D case the deviator stress is replaced by the octahedral shear stress and Equation 5.2.3:1 assumes the form of Equation 5.2.3:3 (Uzan et al. 1992) or, equivalently, of Equation 5.2.3:4 (Uzan 1992):

$$M_r = k_1 \theta_0 \left( \frac{\theta}{\theta_0} \right)^{k_2} \left( \frac{\tau_{oct}}{\theta_0} \right)^{k_3}, \quad (\text{Eq. 5.2.3:3})$$

$$M_r = k_1 \left( \frac{I_1}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} \right)^{k_3}, \quad (\text{Eq. 5.2.3:4})$$

where

$$\tau_{oct}^2 = \frac{1}{9} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2], \quad (\text{Eq. 5.2.3:5})$$

$\sigma_{1,2,3}$	=	principal stresses,
$P_a$	=	atmospheric pressure,
$I_1$	=	first stress invariant,
$k_1, k_2, k_3$	=	material parameters.

In the case of triaxial tests, Equation 5.2.3:5 is simplified to

$$\tau_{oct} = \frac{\sqrt{2}}{3}(\sigma_1 - \sigma_3). \quad (\text{Eq. 5.2.3:6})$$

The octahedral shear stress term is believed to account for the dilation effect that takes place when the soil is subjected to a large principal stress ratio (Park and Lytton 2002). Depending on the level of stresses, the first stress invariant or bulk stress term  $I_1$  considers the hardening effect that is associated with higher modulus, while the octahedral shear stress term considers the softening effect (Park and Fernando 1998, Park et al. 2003).

This model can be used for both coarse and fine unbound granular materials. For coarse granular materials, the value of the material parameter  $k_3$  is usually small and the model collapses to the K- $\theta$  model. For fine soils, the value of the material parameter  $k_2$  is normally quite low and the resilient modulus is dependent mostly on the deviator stress  $\sigma_d$  (Flintsch et al. 2003). By setting the non-linear material properties of  $k_2$  or  $k_3$  to zero, the model can be simplified as linear-elastic, non-linear hardening or non-linear softening stress-dependent behaviours respectively as expressed by the equations reported below:

$$M_r = k_1, \quad (\text{Eq. 5.2.3:7})$$

$$M_r = k_1 \left( \frac{I_1}{P_a} \right)^{k_2}, \quad (\text{Eq. 5.2.3:8})$$

or

$$M_r = k_1 \left( \frac{\tau_{oct}}{P_a} \right)^{k_3}. \quad (\text{Eq. 5.2.3:9})$$

In order to determine the effective resilient properties of unsaturated granular materials, Lytton (1995) added a suction term to the Uzan model based on the principle of unsaturated soil mechanics as expressed below:

$$M_r = k_1 \left( \frac{I_1 - 3 \cdot \theta \cdot f \cdot h_m}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} \right)^{k_3}, \quad (\text{Eq. 5.2.3:10})$$

where

$\theta$	=	volumetric water content,
$h_m$	=	matric suction,
$f$	=	function of the volumetric water content.

The Uzan model has been shown to be superior to the K- $\theta$  model in several studies (Lade and Nelson 1987, Witczak and Uzan 1988, Kolisoja 1997). Elliot and Lourdesnathan (1989) investigated the applicability of the K- $\theta$  model for repeated deviatoric stress below and beyond the static failure condition. For pre-failure stresses,



the model showed good representation of the data. At stresses exceeding the static failure, however, the predictions were poor, as the observed resilient modulus decreased with increasing bulk modulus, while the model predicted the opposite. Elliot and Lourdesnathan then suggested that the K- $\theta$  model should be modified by incorporating a failure term, which has little or no impact until failure is approached. The modified model, known as the stress ratio model, is expressed by Equation 5.2.3:11:

$$M_r = k_1 \frac{\theta^{k_2}}{10^{mR^3}}, \quad (\text{Eq. 5.2.3:11})$$

where

$M_r$	=	resilient modulus,
$k_1, k_2, m$	=	material parameters,
$\theta$	=	bulk stress = $3p$ ,
$R$	=	stress/strength.

A modified version of Equation 5.2.3:3 is given by

$$M_r = k_1 \theta_0 \left( \frac{\theta - 3k_6}{\theta_0} \right)^{k_2} \left( \frac{\tau_{oct}}{\theta_0} \right)^{k_3}. \quad (\text{Eq. 5.2.3:12})$$

The coefficient  $k_6$  is intended to account for the pore water pressure and is a measure of the material's ability to resist tension (Amber Yau et al. 2004). According to Andrei et al. (2004),  $k_6$  is a material property related to the capillary suction in partially saturated unbound granular materials. The values for  $k_6$  are expected to be negative, and if positive, less than a third of the bulks stress (Amber Yau et al. 2004).

Another variation of Equation 5.2.3:3 has been adopted for the new national pavement design guide being finalized in NCHRP Project 1-37A:

$$M_r = k_1 \theta_0 \left( \frac{\theta - 3k_6}{\theta_0} \right)^{k_2} \left( \frac{\tau_{oct} - 1}{\theta_0} \right)^{k_3}, \quad (\text{Eq. 5.2.3:13})$$

in which the 1 in the octahedral shear stress term is to avoid numerical problems when  $\tau_{oct}$  goes to zero (Andrei et al. 2004).

Other researchers (Pezo 1993, Garg and Thompson 1997) included the deviator stress in their analysis in the following way:

$$M_r = N_1 \cdot q^{N_2} \cdot \sigma_3^{N_3}, \quad (\text{Eq. 5.2.3:14})$$

where

$M_r$	=	resilient modulus,
$N_1, N_2, N_3$	=	material parameters,

$q$  = deviator stress,  
 $\sigma_3$  = minor principal stress.

According to Tam and Brown (1988), in routine design and analysis the resilient modulus can be expressed as a simple function of stress ratio as given by Equation 5.2.3:15:

$$M_r = k_1 \left( \frac{p}{q} \right)^{k_2}, \quad (\text{Eq. 5.2.3:15})$$

where

$M_r$  = resilient modulus,  
 $k_1, k_2$  = material parameters,  
 $q$  = deviator stress,  
 $p$  = mean normal stress.

Johnson et al. (1986), however, showed that the resilient modulus is dependent on both the first invariant of stress and the stress ratio and suggested the model of Equation 5.2.3:16:

$$M_r = k_1 \left( \frac{J_2}{\tau_{oct}} \right)^{k_2}, \quad (\text{Eq. 5.2.3:16})$$

where

$M_r$  = resilient modulus,  
 $k_1, k_2$  = material parameters,  
 $J_2$  = stress invariant ( $\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3$ ),  
 $\tau_{oct}$  = octahedral shear stress.

#### 5.2.4 Influence of different factors on the equations parameters

Researchers have investigated the variation of the resilient modulus and the two empirical parameters given in Equation 5.2.2:1. The equations relating resilient modulus to soil properties have also been derived (Tian et al. 1998). The empirical parameters determined from the investigators vary significantly depending on the soil type. Further research is required in order to rely on the use of empirical equations for determining the resilient modulus (Heydinger 2003).

It is necessary to correct for seasonal effects once the parameters from Equation 5.2.2:1 are determined. The resilient modulus of coarse soils decreases when the moisture content increases due to changes in the matric suction. A theoretical model was developed for the change of the resilient modulus as a function of the temperature and the moisture content variations (Jin et al. 1994). Heydinger (2003) predicted the resilient modulus using measured monthly variations of moisture and temperature and the theoretical model. The relative effects of temperature and moisture were not



discussed. However it can be expected that changes in moisture, since they affect the soil matric suction and the bulk stress, would have a much greater effect on resilient modulus than the temperature effects. Laboratory samples were also tested at selected values of temperature and moisture. The comparison between the predicted values and the values determined in the laboratory was excellent. In addition to the excellent agreement between predicted and measured values, the results showed that the seasonal variation of the resilient modulus can be approximated very accurately using a sinusoidal equation. The maximum value of the resilient modulus was approximately 1.4 times the minimum value for the study by Jin et al. (1994). This result was most likely influenced by the type of the materials used.

The following recommendations are suggested by Heydinger (2003) for estimating the seasonal variations of the resilient modulus for aggregate soils. It is necessary to estimate the resilient modulus using Equation 5.2.2:1 and other empirical equations that depend on the soil type, the moisture content, and the density of the soils at the site. It is then necessary to adjust the resilient modulus for seasonal effects. A sinusoidal expression for the resilient modulus as a function of the day of the year is recommended. A representative value of the resilient modulus can then be computed using weighting factors (Guan et al. 1997).

Kolisoja (1997) included the effect of material density in both the K- $\theta$  and Uzan models. He expressed the modified formulations by Equations 5.2.4:1 and 5.2.4:2 where density is accounted for by the porosity of the aggregate:

$$M_r = A(n_{\max} - n)p_0 \left( \frac{\theta}{p_0} \right)^{0.5}, \quad (\text{Eq. 5.2.4:1})$$

$$M_r = B(n_{\max} - n)p_0 \left( \frac{\theta}{p_0} \right)^{0.7} \left( \frac{q}{p_0} \right)^{-0.2}, \quad (\text{Eq. 5.2.4:2})$$

where

$M_r$	=	resilient modulus,
$A, B$	=	material parameters,
$\theta$	=	bulk stress = $3p$ ,
$n$	=	material porosity,
$n_{\max}$	=	maximum porosity,
$p_0$	=	atmospheric pressure (100 kPa),
$q$	=	deviator stress.

Kolisoja stated that a reliable application of these modified models requires, in practice, at least one series of repeated load triaxial tests that covers a sufficiently large stress range performed at a given density and, obviously, the determination of the value of the maximum porosity  $n_{\max}$ . The value of the single material parameter can then be determined, after which the equations give values of the resilient modulus related to any other combination of stress and density states.

The models mentioned above are based on laboratory triaxial testing using a constant confining pressure. Karasahin (1993) conducted triaxial tests with both constant and variable confining pressure and proposed the following model

$$M_r = A \left( \frac{p_m}{p_u} \right)^B \left( \frac{p_u}{\delta p} \right)^C, \quad (\text{Eq. 5.2.4:3})$$

where

$M_r$	=	resilient modulus,
$A, B, C$	=	material parameters,
$p_u$	=	unit pressure (1 kPa),
$\delta p$	=	$p_{\max} - p_{\min}$ ,
$p_m$	=	$(p_{\max} + p_{\min})/2$ .

Karasahin used the model of Equation 5.2.4:3 to calculate the resilient modulus and reported satisfactory correlation values for both cases.

The type of confinement was varied also in a series of repeated load triaxial tests reported by Nataatmadja and Parkin (1989) and Nataatmadja (1992). The results showed differences in the stress dependence of the resilient modulus due to the type of confining pressure, and separate formulations were suggested (Eq. 5.2.4:4 and Eq. 5.2.4:5):

$$M_r = \frac{\theta}{q} (A + Bq), \quad (\text{Eq. 5.2.4:4})$$

$$M_r = \frac{\theta}{\sigma_1} (C + Dq), \quad (\text{Eq. 5.2.4:5})$$

where

$M_r$	=	resilient modulus,
$A, B, C, D$	=	material parameters,
$\theta$	=	bulk stress = $3p$ ,
$q$	=	deviator stress,
$\sigma_1$	=	major principal stress.

Equation 5.2.4:4 refers to constant confining pressure, whilst Equation 5.2.4:5 refers to variable confining pressure. Nataatmadja demonstrated that the material parameters used were sensitive to changes in grading, maximum particle size, moisture content and plasticity, and concluded that these models are suitable for ranking material performance.

As noted by Drumm and Madgett (1997), SHRP P-46 (1989) suggests reporting the resilient modulus values at a deviatoric stress of 28 kPa (4 psi) and a confining pressure of 41 kPa (6 psi). In their study and as suggested by Drumm and Madgett (1997), Naji



et al. (2003) calculated the resilient modulus at the aforementioned stress level using Equation 5.2.3:2 and correlated it with the soil suctions as well as the moisture content. A summary of the results is graphically illustrated in Figure 5.2.4:1.

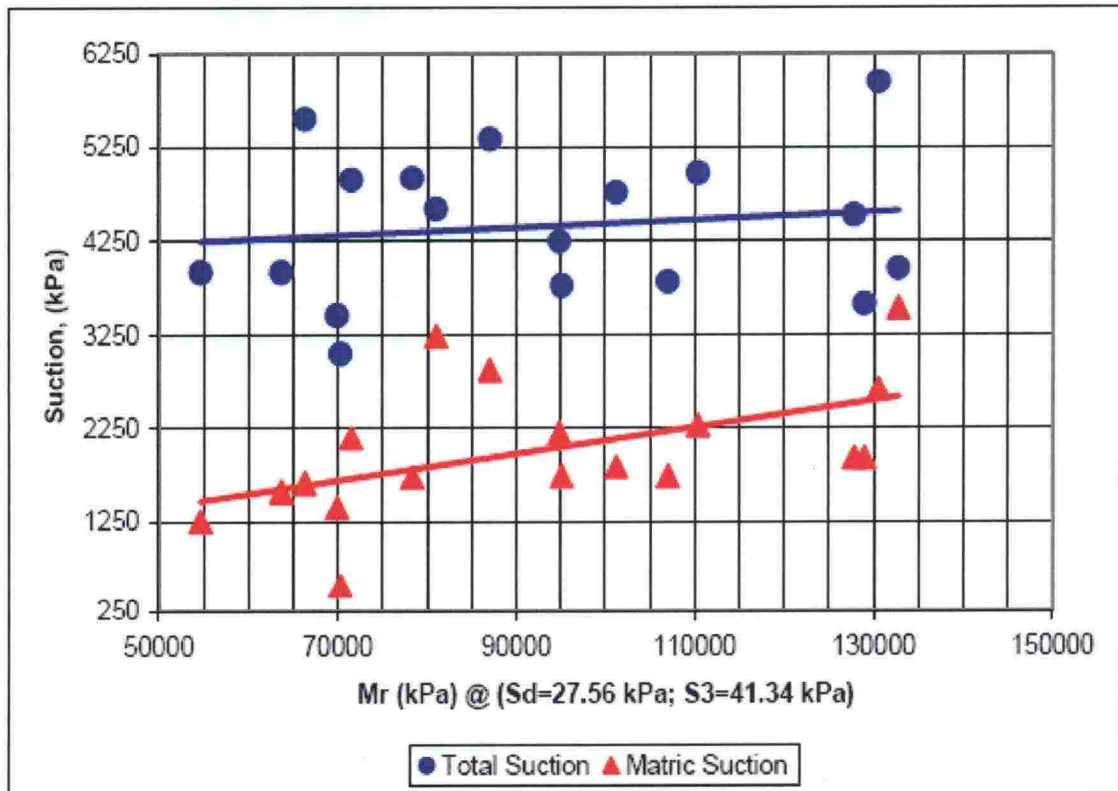


Figure 5.2.4:1 Variation of the resilient modulus with soil suction (Naji et al. 2003).

Figure 5.2.4:1 shows a plot of resilient modulus versus matric suction,  $(u_a - u_w)$ , and total suction,  $(\Psi)$ . One can see that the resilient modulus increases with increasing matric suctions. The resilient modulus shows also an increase as the total suction increases from approximately 3,000 (435 psi) to 6,000 kPa (871 psi). The increase in resilient modulus is attributed to the fact that higher soil suction should produce a stiffening of the specimens and thus a higher resilient modulus. It is also believed that higher suction increases the integrity of soil structure and, in turn, the rigidity of the soil skeleton. From Figure 5.2.4:1, it is evident that the resilient modulus exhibits a similar trend for both total and matric suctions. In other words, a change in the resilient modulus due to total suction is essentially equivalent to a change in  $M_r$  due to matric suction. This means that there are no significant changes in values of  $M_r$  due to osmotic suction (Naji et al. 2003). Consequently, one can conclude that the variation of the salt content of the tested specimens is negligible. This observation is consistent with Fredlund and Rahardjo (1993), who claim that a change of the total suction is similar to the change of the matric suction.

### 5.2.5 Poisson's ratio as a function of stress

Together with the resilient modulus, Poisson's ratio is often used as another elastic parameter to describe the resilient deformation behaviour of granular materials. However, compared to resilient modulus, very few studies have aimed at characterizing

the resilient Poisson's ratio. The Poisson's ratio of unbound granular materials is stress-dependent and that is why it should be considered with the stress-dependent modulus simultaneously in a single framework (Park and Park 2005). The determination of Poisson's ratio requires a very accurate measurement of the radial strain, something which in practice has proven to be much more difficult than measuring axial strain (Lekarp 1999). Therefore, in practical applications Poisson's ratio has typically been assumed to have a constant value, either  $n = 0.30$  (Brown 1996) or  $n = 0.35$  (Sweere 1990). In reality Poisson's ratio is not independent of the stress state but for example the results of cyclic loading triaxial tests made under constant confining pressure (CCP-test) have shown that Poisson's ratio clearly depends on the ratio between the maximum and minimum principal stresses that correspond to the load pulse (Brown and Hyde 1975). For instance, Sweere (1990) has noticed that the assumption that Poisson's ratio is constant has a very detrimental effect on the estimation of the volumetric strain of the material. Morgan (1966), who measured both axial and radial strains, noted that Poisson's ratio varied between 0.2 and 0.4 for the sands tested. However, many researchers (e.g. Park 2000) have observed that the resilient Poisson's ratio of unbound granular materials is a function of several variables, both inherent physical attributes and, particularly, the applied stress components. Poisson's ratio increases with decreasing confining pressure and increasing repeated deviator stress. This was used by Hicks and Monismith (1971), who approximated the variation of Poisson's ratio with stresses by a third-degree polynomial expression given in Equation 5.2.5:1:

$$\nu = A + B\left(\frac{\sigma_1}{\sigma_3}\right) + C\left(\frac{\sigma_1}{\sigma_3}\right)^2 + D\left(\frac{\sigma_1}{\sigma_3}\right)^3, \quad (\text{Eq. 5.2.5:1})$$

where

$\nu$	=	Poisson's ratio,
$A, B, C, D$	=	material parameters,
$\sigma_1, \sigma_3$	=	major and minor principal stress.

Karasahin (1993), who employed both constant and variable confinement in his triaxial tests, suggested that for the latter case the effect of anisotropy should be taken into consideration for calculating Poisson's ratio. As a result, Karasahin proposed separate mathematical formulations to define the stress dependence of Poisson's ratio based on the type of confining stress. Equation 5.2.5:2 was suggested for constant confinement, whereas incorporating anisotropy for variable confinement resulted in complex formulations

$$\nu = A\left(\frac{q_m}{p_u}\right)^B \left(\frac{p_u}{p_m}\right)^C \left(\frac{p_u}{\delta p}\right)^D, \quad (\text{Eq. 5.2.5:2})$$

where

$\nu$	=	Poisson's ratio,
$A, B, C, D$	=	material parameters,
$q_m$	=	$(q_{\max} + q_{\min})/2$ ,



$p_u$	=	unit pressure (1 kPa),
$\delta p$	=	$p_{\max} - p_{\min}$ ,
$p_m$	=	$(p_{\max} + p_{\min})/2$ .

The values of Poisson's ratio have sometimes been reported to be greater than 0.5 (e.g. Sweere 1990, Karasahin 1993). This apparent transgression of basic thermodynamics is a consequence of the adoption of sidering a mass composed of discrete and essentially unconnected elements. When Poisson's ratio exceeds 0.5, this indicates resilient dilation, i.e. the material transiently occupies more volume during the stress pulse.

Investigations of the non-linear elastic stress-strain behaviour of unbound granular materials have been carried out for the past 10 years at Dresden University of Technology. In this section only a short overview on the modeling of unbound granular materials can be given. Further details are available elsewhere (Gleitz 1996, Wellner 1996). Modified plate-bearings tests with cyclic loadings (Wellner 1996) were carried out on unbound granular layers. Heaving was observed at a distance range of 450-1200 mm from the load axis. At all measured stress-levels the same behaviour was observed. Linear elastic analysis did not predict this heaving and therefore repeated load triaxial tests on the same unbound granular material as used for the plate-bearings tests were conducted to investigate the non-linear behaviour. As a result of the data from the repeated load triaxial testing a new material law - the DRESDEN Model - was developed (Gleitz 1996). This non-linear elastic model is expressed in terms of the modulus of elasticity  $E$  and Poisson's ratio  $\nu$  as follows:

$$E = (Q + C \cdot \sigma_3^{Q_1}) \cdot \sigma_1^{Q_2} + D, \quad (\text{Eq. 5.2.5:3})$$

$$\nu = R \cdot \frac{\sigma_1}{\sigma_3} + A \cdot \sigma_1 + B, \quad (\text{Eq. 5.2.5:4})$$

where

$$0 < \nu < 0.5,$$

$\sigma_3$	=	minor principal stress (absolute value) [kPa],
$\sigma_1$	=	major principal stress (absolute value) [kPa],
$D$	=	constant term of modulus of elasticity [kPa],
$Q, C, Q_1, Q_2, R, A, B$	=	model parameters determined with repeated load triaxial tests.

The model includes a stress independent stiffness of 38 kPa for crushed aggregates and 30 kPa for sand and gravel (parameter  $D$ ) consequent upon the residual confining stress in-situ. The residual stress has the effect of reducing the strains at small stress levels and could be assumed by examining the modified plate-bearing test results carried out by Klemm (2001). The parameter  $D$  is mainly influenced by macroscopic parameters like the degree of compaction of the unbound granular material, the content of fines, the shape of the grains and the water content. The repeated load triaxial test results do not allow the determination of the parameter  $D$  because the residual stress needs some time to develop in reality (Werkmeister 2003). To obtain

the model parameters, the repeated load triaxial apparatus at Nottingham University was used.

To check the validity of the Dresden Model, Werkmeister et al. (2003) predicted the surface deflection induced by plate bearing tests (Figure 5.2.5:1) using the FE-program FENLAP (Werkmeister et al. 2000). A comparison was carried out to assess the accuracy of other material laws (e.g. Mayhew, Boyce and K- $\theta$  Model) by comparing the results of calculated deflections from all the models against the measured values. The best approximation is given by the Dresden-Model. The maximum deflection under the load agrees with the measured value.

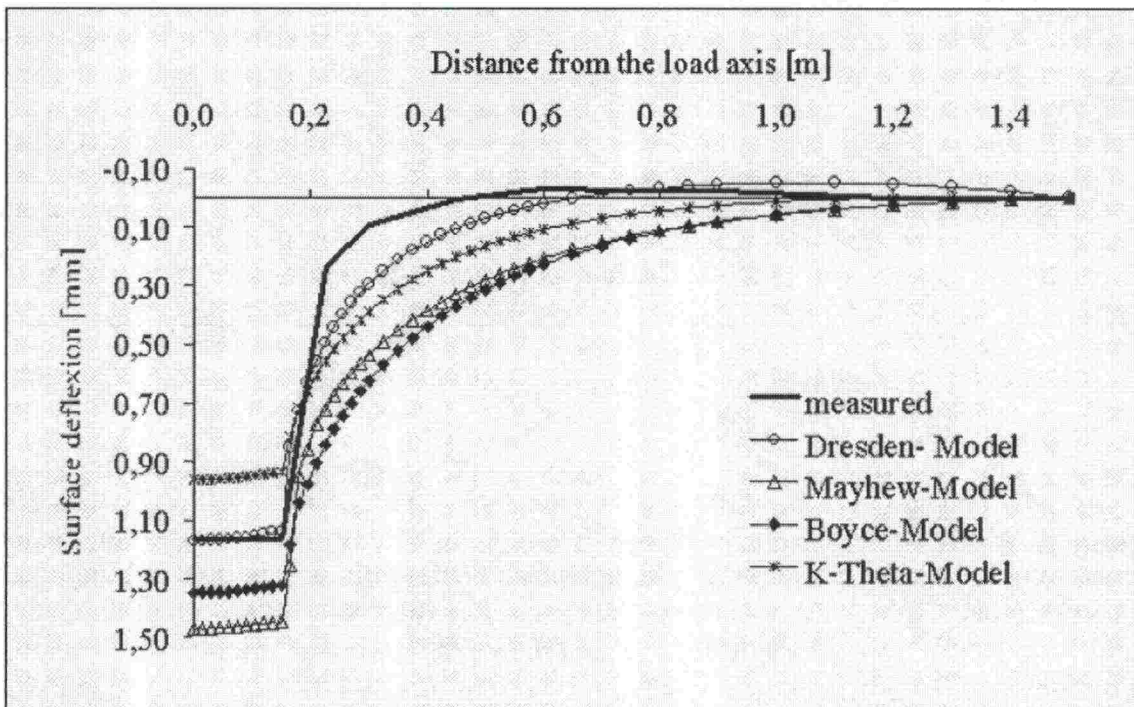


Figure 5.2.5:1 Comparison of measured and calculated resilient surface deflections (Werkmeister et al. 2000).

In their study, Werkmeister et al. (2003) investigated the resilient deformation behaviour of a Granodiorite with a maximum grain size of 32 mm at moisture contents of 4, 5, 6, and 7 %. These seem to be small steps of moisture variation, but the repeated load triaxial samples can be prepared with a very good accuracy ( $\pm 0.1\%$ ). For repeated load triaxial test results with moisture content of 6 and 7 % it was not possible to determine the material parameters. During repeated load triaxial tests at 7% (= optimum water content) water was draining out of the sample during the test, which means there will be inhomogeneous conditions during testing. However, the repeated load triaxial test results at moisture contents of 4 and 5% were taken into account for modeling. Figure 5.2.5:2 clearly shows a high dependency of the resilient deformation behaviour and, hence, the model parameters on the moisture content. Increasing the water content resulted in a significant reduction in the stiffness, a result also observed by others (Dawson et al. 1996, Barksdale and Itani 1989, Hicks and Monismith 1971, Heydinger et al. 1996, Smith and Nair 1973, Raad et al. 1992, Vuong 1992).



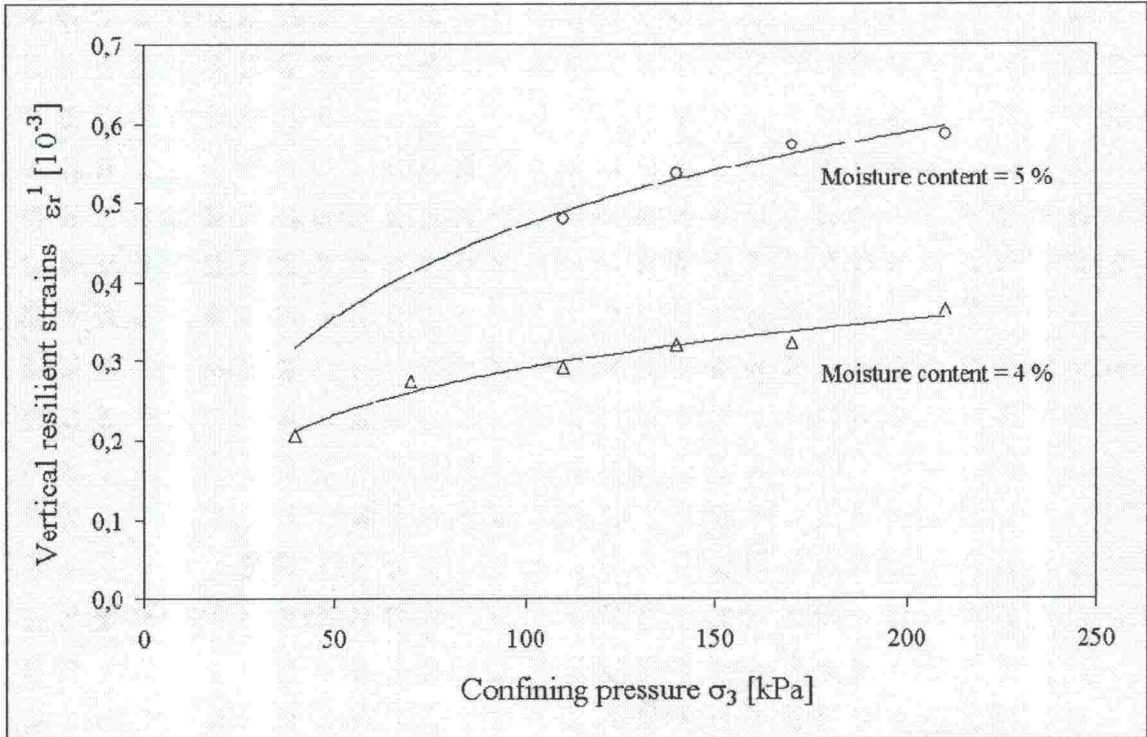


Figure 5.2.5:2 Vertical resilient strain versus confining pressure at different moisture contents ( $\sigma_{1max} / \sigma_3 = 2$ ) (Numrich 2003).

Lade and Nelson (1987) derived a relationship between Poisson's ratio and the resilient modulus based on a thermodynamic constraint. The derivation of the equation is based on the principle of the conservation of energy and the path independence of the energy density function. After the study by Lytton et al. (1993) and Liu (1993), an expression that relates the stress state and the rate of change of Poisson's ratio with a changing stress state was derived taking into account the resilient modulus and the thermodynamic constraints. This relationship between Poisson's ratio and the stress state is as follows:

$$\frac{2}{3} \frac{\partial v}{\partial J_2} + \frac{1}{I_1} \frac{\partial v}{\partial I_1} = v \left[ \frac{1}{3} \frac{k'_3}{J_2} + \frac{k_2}{I_1^2} \right] + \left[ -\frac{1}{6} \frac{k'_3}{J_2} + \frac{k_2}{I_1^2} \right], \quad (\text{Eq. 5.2.5:5})$$

where

$v$	=	Poisson's ratio,
$k_1, k_2, k_3$	=	material parameters,
$I_1$	=	normalized first stress invariant,
$J_2$	=	normalized second invariant of the deviatoric stress.

### 5.3 Resilient modulus using shear-volumetric approach

In order to characterize the stress-strain relationship in unbound granular materials, a different approach can be used. The approach consists of decomposing both stresses and strains into volumetric and shears components. In this case, resilient modulus and

Poisson's ratio would be replaced by bulk modulus and shear modulus. The definitions of the basic stress and strain parameters used in this approach are

$$p = \frac{1}{3}(\sigma_1 + 2\sigma_3), \quad (\text{Eq. 5.3:1})$$

$$q = \sigma_1 - \sigma_3, \quad (\text{Eq. 5.3:2})$$

$$\varepsilon_{v,r} = \frac{2}{3}(\varepsilon_{1,r} + 2\varepsilon_{3,r}), \quad (\text{Eq. 5.3:3})$$

$$\varepsilon_{s,r} = \frac{2}{3}(\varepsilon_{1,r} - \varepsilon_{3,r}), \quad (\text{Eq. 5.3:4})$$

$$K = \frac{p}{\varepsilon_{v,r}}, \quad (\text{Eq. 5.3:5})$$

$$G = \frac{q}{3\varepsilon_{s,r}}, \quad (\text{Eq. 5.3:6})$$

where

$p$	=	mean normal stress,
$q$	=	deviator stress,
$\varepsilon_{v,r}$	=	recoverable volumetric strain,
$\varepsilon_{s,r}$	=	recoverable shear strain,
$K$	=	bulk modulus,
$G$	=	shear modulus.

Pioneering research on that type of modelling approach has been done at the University of Nottingham in England (e.g. Brown and Hyde 1975). According to Brown and Hyde (1975), there are three advantages in using bulk and shear moduli for non-linear materials: no assumptions of linear-elastic behaviour are needed in the calculations, the shear and volumetric components of stress and strain are treated separately, and they have a more realistic physical meaning in a 3-dimensional stress regime than resilient modulus and Poisson's ratio.

One of the oldest and meanwhile one of the most well-known material models of this type is the so called Boyce's model (Boyce 1980). Boyce used a mechanistic approach constrained by the theorem of reciprocity, i.e. no net loss of strain energy, developing a theoretical non-linear elastic model for the stress-strain relationship of granular materials. He was able to use this approach to model his own data, and it has been employed by many other researchers, particularly in mainland Europe (Paute et al. 1993, Dawson et al. 1996). In the Boyce model, given by Equation 5.3:7 and Equation 5.3:8, the material is assumed to be isotropic, which enables the model to express the response moduli as functions of the stress invariants



$$\varepsilon_v = \frac{1}{K_1} p^A \left( 1 - \beta \frac{q^2}{p^2} \right), \quad (\text{Eq. 5.3:7})$$

$$\varepsilon_s = p^A \frac{1}{3G_1} \left( \frac{q}{p} \right), \quad (\text{Eq. 5.3:8})$$

where

$\varepsilon_v$	=	resilient volumetric strain,
$\varepsilon_s$	=	resilient shear strain,
$p$	=	mean normal stress,
$q$	=	deviator stress,
$A, G_1, K_1$	=	model parameters,
$\beta$	=	$(1-A)K_1/6G_1$ .

The model is also elastic following the application of the theorem of reciprocity. This imposes a certain relationship between the shear and volumetric strains and, consequently, limits the number of material parameters to three, making the model relatively simple to define (Lekarp 1999). Allaart (1992) showed that Boyce's solution was the simplest of a family of possible solutions. However, the assumption of elasticity is an important disadvantage of the Boyce model in dealing with the inelastic response of unbound granular materials. When a granular material is subjected to repeated loading, such as those applied in repeated load triaxial tests, the loading and unloading part of the stress-strain curve do not coincide, and energy is dissipated. This is, by definition, an inelastic response and inaccurate predictions should be expected from an elastic model such as the Boyce model.

Many other researchers (Jouve et al. 1987, Mayhew 1983, Sweere 1990) have suggested and verified a similar type of material models that fit better to the test results than the Boyce's model does. Sweere (1990), for instance, showed an unsatisfactory prediction of strains by the Boyce model with large discrepancies between predicted and measured values of the volumetric strain component. Sweere stated that the solution to the problem lies in removing the coupling of volumetric and shear strains by the theorem of reciprocity, in other words the strict prerequisite of the model being elastic. By doing so, volumetric and shear strains can be related to the stresses independently. Sweere reported quite satisfactory predictions of both shear and volumetric strains using basically the same equations given by Boyce but keeping the relationships between shear and volumetric strains with stresses independent from each other. Instead of three material parameters, as in the Boyce model, the modified model would then contain four or five independent parameters.

The method of decomposing the stress-strain response into volumetric and shear components was also used by Brown and Pappin (1985) in developing the so-called contour model. This model, which resembles Boyce's model and was developed at the University of Nottingham, is a non-linear relationship that is capable of taking into account effective mean and deviatoric stresses and stress path dependence. The resilient volumetric and shear strains may be expressed as contours in  $p$ - $q$  stress space, and the magnitudes of strains are derived as the change in contour values from the initial to the

final stress state. Based on the contour model, the volumetric strain depends only on the maximum values of the stresses, whereas the shear strain depends not only on the maximum stresses but also on the length of the stress path:

$$\varepsilon_v = \delta \left\{ \left( \frac{p}{A} \right)^B \left[ 1 - C \left( \frac{q}{p} \right)^2 \right] \right\}, \quad (\text{Eq. 5.3:9})$$

$$\varepsilon_s = D \delta \left( \frac{q}{p + E} \right) \left( \frac{\sqrt{p_r^2 + q_r^2}}{p_m} \right)^F, \quad (\text{Eq. 5.3:10})$$

where

$\varepsilon_v$	=	resilient volumetric strain,
$\varepsilon_s$	=	resilient shear strain,
$p$	=	mean normal stress,
$q$	=	deviator stress,
$A, B, C, D, E, F$	=	model parameters,
$\delta$	=	change in,
$q_r$	=	$q_{\max} - q_{\min}$ ,
$p_r$	=	$p_{\max} - p_{\min}$ ,
$p_m$	=	$(p_{\max} + p_{\min})/2$ .

The volumetric and shear strains described by Equation 5.3:9 and Equation 5.3:10 can be presented as contours - which give the name for the model - plotted in the  $p$ - $q$  stress plane (Fig. 5.3:1). An example of the strain contours for a crushed rock material is presented in Figure 5.3:2 and Figure 5.3:3, where the unit of strain is  $1 \cdot 10^{-6}$  (micro strain).

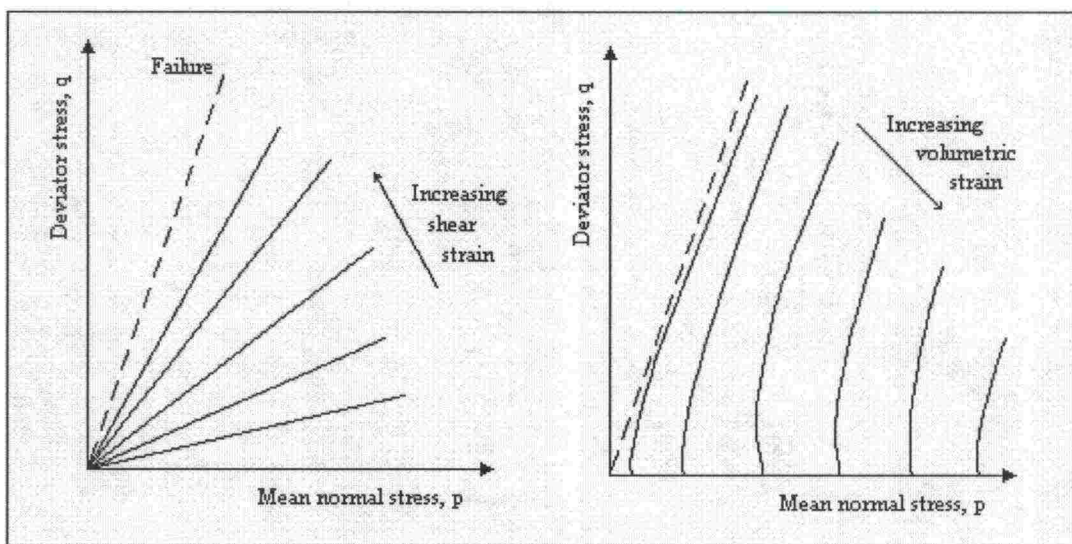


Figure 5.3:1 Contour Model in  $p$ - $q$  Stress Space (Brown and Pappin 1985).



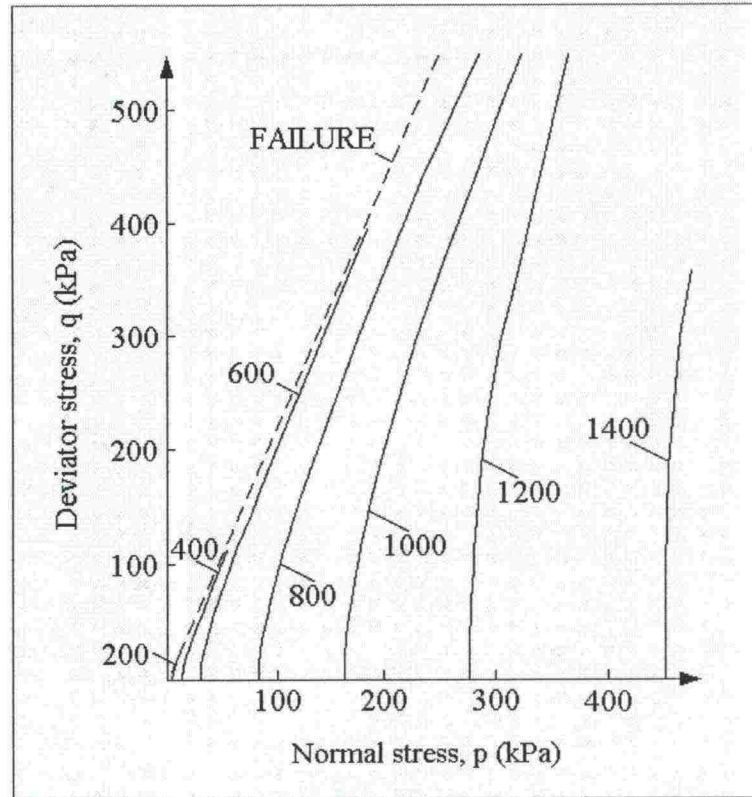


Figure 5.3:2 An example of the contour plots of the resilient volumetric strain plotted in the  $p$ - $q$  stress plane (Pappin and Brown 1980).

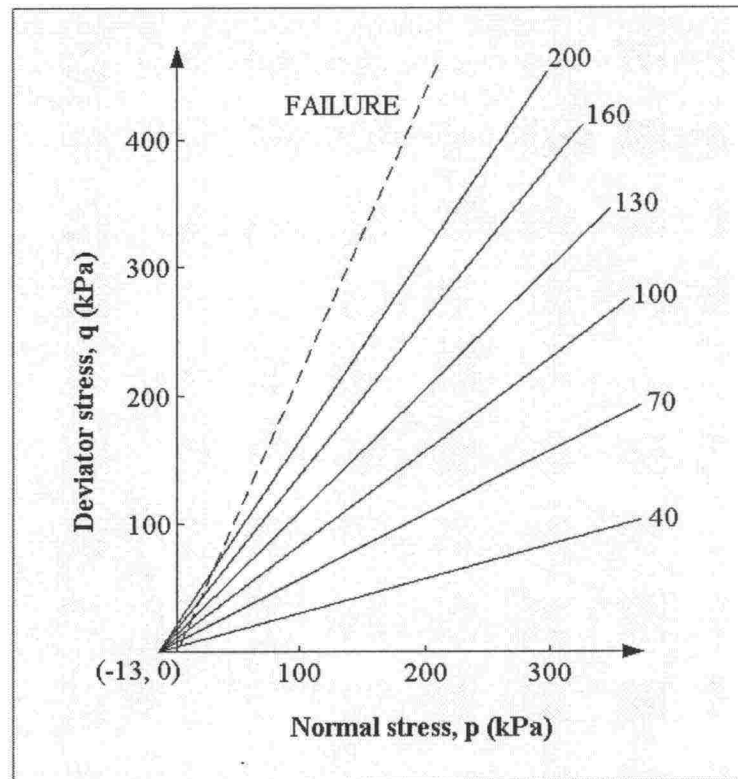


Figure 5.3:3 An example of the contour plots of the normalised resilient shear strain plotted in the  $p$ - $q$  stress plane (Pappin and Brown 1980).

Even though the contour model, to some extent, resembles the Boyce model, there are major differences between the two. One of the main differences is that the contour model predicts the shear strain by taking into account the length of the stress path corresponding to the load cycle. Moreover, the material parameters in the two parts of the contour model are independent from each other, leading to better agreement between predicted and measured values of the strains due to an advantageous curve-fitting. Independent material parameters also mean that the contour model, unlike the Boyce model, is inelastic.

A similar curve-fitting approach, as in the contour model, was used by Mayhew (1983), who plotted volumetric and shear strains in the  $p$ - $q$  stress space to model the results of repeated load triaxial tests on granular materials. Mayhew, however, showed that the stress path length which is included in the contour model had no significant impact on the shear strain response. By removing the stress path dependence, the equations of the contour model turn into the inelastic modified version of the Boyce model suggested by Sweere (1990). The mathematical expression of the Mayhew model, rewritten by Brown and Selig (1991), is given by the following equations:

$$\varepsilon_v = p^A \frac{1}{K_1} \left[ 1 - C \frac{q^2}{p^2} \right], \quad (\text{Eq. 5.3:11})$$

$$\varepsilon_s = p^B \frac{1}{3G_1} \left( \frac{q}{p} \right), \quad (\text{Eq. 5.3:12})$$

where

$\varepsilon_v$	=	resilient volumetric strain,
$\varepsilon_s$	=	resilient shear strain,
$p$	=	mean normal stress,
$q$	=	deviator stress,
$A, B, C, G_1, K_1$	=	model parameters.

Thom (1988) developed a non-linear elastic model to predict the results of triaxial and hollow cylinder tests by separating the volumetric and shear strain components in the same manner as in the Boyce and contour models. Thom, however, included the major and minor principal stress components as well as the 2-dimensional stress invariants  $r$  and  $S$  in the equations. This approach has not found much application, although it may be of value for materials undergoing stress rotation as explained by Thom. Karasahin (1993) attempted to verify this model, but found it difficult to obtain the material parameters by a non-linear regression analysis.

Elhannani (1991), as reported by Karasahin (1993), introduced anisotropy into the original Boyce model. Karasahin compared different models and reported better correlation between the observed values and the values predicted by the Elhannani model. The anisotropic response of granular materials was also taken into account in a recent study conducted by Hornyk et al. (1998). The results of repeated load triaxial tests were first fitted into the Boyce model, but the correlation found was quite poor, apparently because of the cross-anisotropic behaviour of granular materials. The original Boyce model was then modified



by incorporating a coefficient of anisotropy, which resulted in a much better agreement between measured and predicted values. Examples of the results reported by Hornyh et al. are illustrated in Figure 5.3:4.

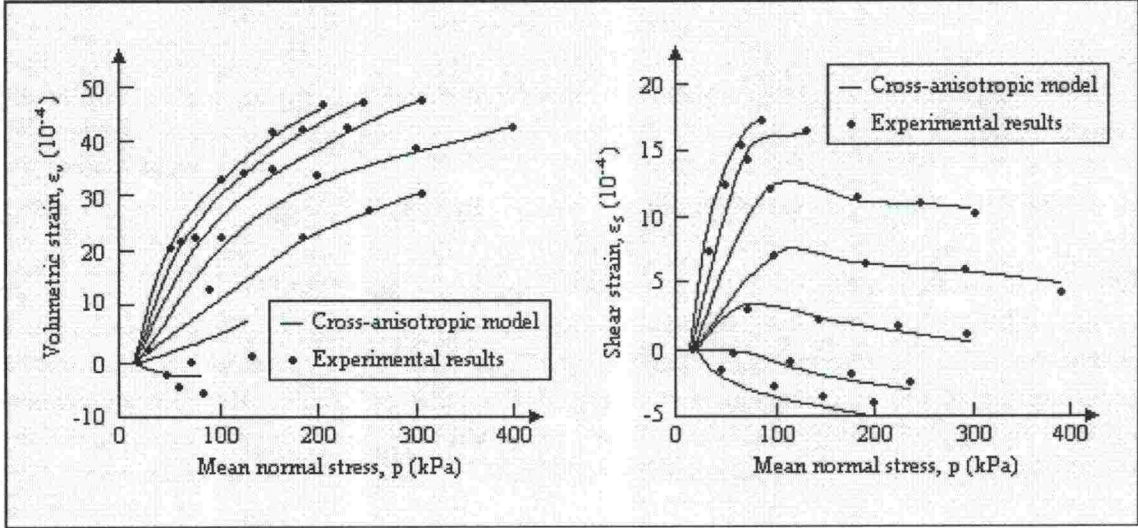


Figure 5.3:4 Examples of fit using the cross-anisotropic model (Hornyh et al. 1998)

The relationships developed by Hornyh et al. (1998) are

$$\varepsilon_v^* = \frac{1}{K_a} \frac{p^{*n}}{p_a^{n-1}} \left[ 1 + \frac{(n-1)K_a}{6G_a} \left( \frac{q^*}{p^*} \right)^2 \right], \quad (\text{Eq. 5.3:13})$$

$$\varepsilon_q^* = \frac{1}{3G_a} \frac{p^{*n}}{p_a^{n-1}} \frac{q^*}{p^*}, \quad (\text{Eq. 5.3:14})$$

where

$$p^* = \frac{\gamma\sigma_1 + 2\sigma_3}{3}, \quad (\text{Eq. 5.3:15})$$

$$q^* = \gamma\sigma_1 - \sigma_3, \quad (\text{Eq. 5.3:16})$$

$$\varepsilon_v^* = \frac{\varepsilon_1}{\gamma} + 2\varepsilon_3, \quad (\text{Eq. 5.3:17})$$

$$\varepsilon_q^* = \left( \frac{\varepsilon_1}{\gamma} - \varepsilon_3 \right), \quad (\text{Eq. 5.3:18})$$

$K_a, G_a, n, \gamma$  = model parameters.

Correia et al. (1999) compared the adequacy of different models in describing the non-linear resilient behaviour of unbound granular materials. The anisotropic Boyce model of

Hornych et al. (1998) was shown to be superior in predicting both resilient volumetric and shear strains during repeated load triaxial tests with variable confining pressure. The superiority of this model was shown to remain even when the parameters were determined by triaxial tests with constant confining pressure.

In another recent study, Hoff et al. (1999) introduced a new model based on the concept of hyper elasticity, taking into account the effect of resilient dilatancy in unbound granular materials. The hyper elastic model was said to give a more realistic distribution of the stresses in granular layers than that obtained using a linear elastic model.

The research work of the above-mentioned models has mainly been based on the results of triaxial tests, where cyclic axial load and either constant or cyclic confining pressure have been used. As the loading method used in the triaxial test - the principal stress directions do not change - is not exactly equivalent to the real loading condition (Chan 1990, Chan and Brown 1994), some modelling approaches have utilised measurement data about the mechanical behaviour of the material collected using a hollow cylinder apparatus which facilitates the rotation of the principal stress directions. An example is the Thom's model (1988), which also includes five independent material parameters.



## 6 FACTORS AFFECTING PERMANENT DEFORMATION

### 6.1 Introduction

A railway embankment is exposed to a large number of load applications during its service life. Although the permanent deformation is normally a fraction of the total deformation produced by each load repetition, the gradual accumulation of a large number of these small plastic deformation increments could lead to eventual failure. Failure and also excessive permanent deformations of the embankment must of course be prevented. To do so the knowledge of the plastic behaviour of unbound granular materials is very important.

Most of the research carried out over the years has concentrated on the resilient behaviour of granular materials. In comparison to the resilient behaviour, less research has been devoted to the plastic response and permanent deformation development in unbound granular materials. This is perhaps due to the practical difficulties in studying permanent deformation behaviour. While resilient tests are fairly quick and each laboratory specimen can be studied for a great number of stresses, the monitoring of the build-up of permanent deformations in unbound granular materials and permanent deformation tests are very time-consuming, and usually separate specimens are required for each set of stresses. As a result, greater advances have been made in understanding the resilient response than the long-term performance of granular materials.

The permanent strain development in granular materials is affected by several factors: stress level, stress history, number of load applications, principal stress rotation, moisture content, density, grading and aggregate type, and physical properties of aggregate particles.

- The importance of the applied stress level is strongly emphasized in the literature. Permanent strain is said to be related directly to the deviator stress and inversely to the confining pressure (Barksdale 1972). However, several studies have suggested that permanent strain is principally governed by some form of stress ratio consisting of both deviatoric and confining stresses (e.g. Brown and Hyde 1975).
- A number of limited investigations found in the literature (Chan 1990) indicate that reorientation of principal stresses due to moving traffic significantly increases the amount of plastic strains in granular materials.
- The growth of permanent strain in granular materials is a gradual process during which each load application contributes a small increment to the accumulation of strain. The significance of the number of load repetitions has been recognized many times in the literature (e.g. Sweere 1990, Paute et al. 1993).
- The amount of permanent strain in granular materials is also influenced greatly by the presence of water. At high levels of saturation, deformation resistance in the material reduces quite rapidly, probably as positive pore water pressure is generated. Proper drainage in granular materials is, therefore, a necessity for improving performances. The reduction in fines content is likely to achieve this but

may conflict with the desire to provide a more deformation-resistant mixture by increasing the fines to promote better packing.

- The effect of stress history on permanent strain development has been recognized in the literature but hardly investigated. The study by Brown and Hyde (1975) clearly shows that permanent strain is reduced notably as the result of stress history. However, if previous loading has loosened the material (e.g. by dilating it), the effect of stress history is the opposite. In laboratory permanent strain tests, the effect of stress history is eliminated by using a new specimen for each set of stresses applied.
- Density, described by the degree of compaction, is believed to have a pronounced impact on the long-term behaviour of granular materials (Thom and Brown 1988, Barksdale 1991). Resistance to permanent strain in these materials is highly improved as the result of increased density.
- The effect of fines content on the permanent deformation behaviour of unbound granular materials was investigated by Barksdale (1972), Thom and Brown (1988), and Barksdale (1991), who concluded that the permanent deformation resistance of granular materials is reduced as the amount of fines increases.
- The effect of particle size distribution, or grading, is a disputed subject and different views are found in the literature.
- As for the effect of aggregate type, it has been suggested that crushed, angular materials undergo smaller permanent deformations compared to materials such as gravel with rounded grains (Barksdale and Itani 1989).

## 6.2 Stress level

The literature available shows that the stress level is one of the most important factors affecting the development of permanent deformations in unbound granular materials. The magnitude of the permanent deformations developed strongly depends on the stress level and increases with rising deviator stress and decreasing confining stress (Werkmeister 2003).

Early repeated load triaxial tests reported by Morgan (1966) clearly showed that the accumulation of axial permanent strains is directly related to the deviator stress and inversely related to the confining pressure. Morgan studied the behaviour of sands under repeated loading with an increasing number of load cycles and considered the influence of deviator stress and confining pressure on the accumulation of permanent axial strain. He found a direct dependency between the accumulated permanent axial strain after any given number of load cycles and the deviator stress at a particular level of confining pressure. On the other hand, by maintaining the deviator stress at a constant level, Morgan found that the permanent axial strain was inversely proportional to the confining pressure.

Barksdale (1972) comprehensively studied the behaviour of several unbound granular materials using repeated load triaxial tests with constant confining pressure and  $10^5$  load



cycles. He drew the general conclusion that permanent axial strain strongly depends on the applied load and increases with decreasing confining pressure and increasing deviator stress. A summary of the plastic stress-strain characteristics of some of the materials investigated by Barksdale is illustrated in Figure 6.2:1. Barksdale, then, used a rather complicated hyperbolic stress-strain expression to relate the development of permanent axial strains and the ratio of repeated deviator stress to constant confining pressure.

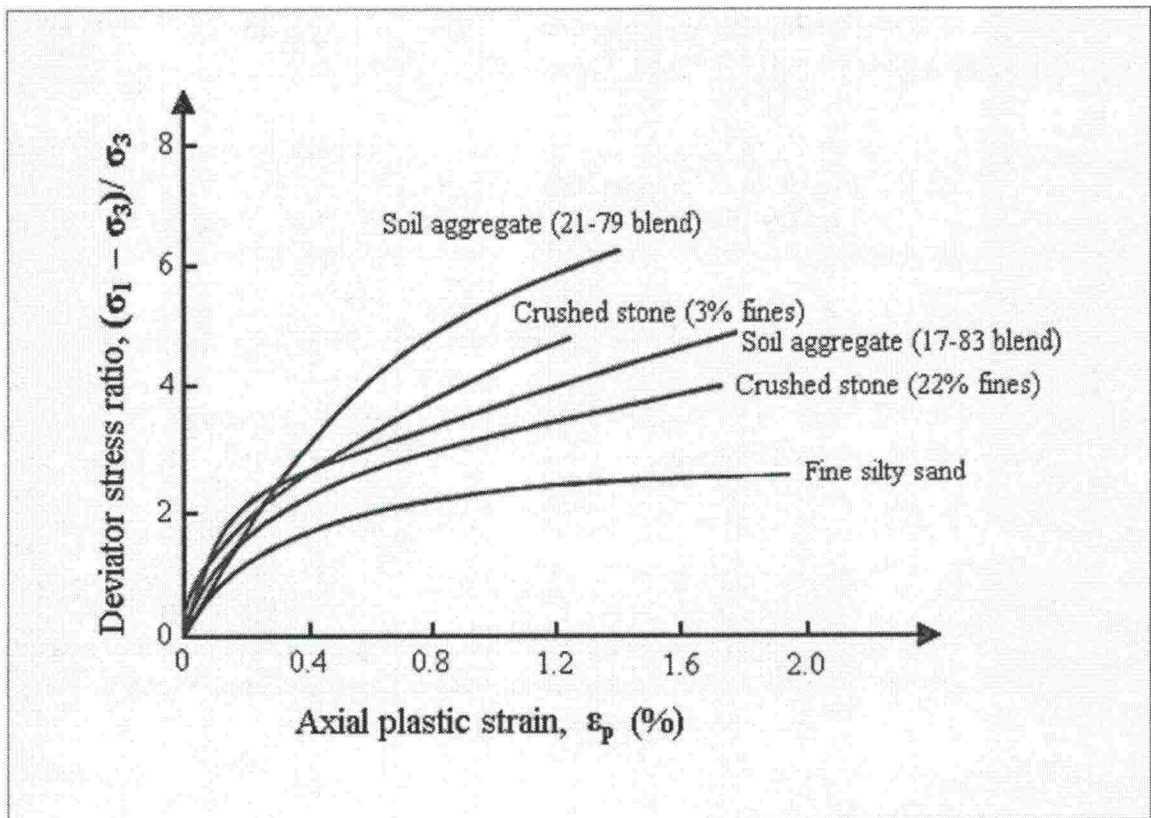


Figure 6.2:1 Plastic stress-strain characteristics of some of the material investigated by Barksdale (Barksdale 1972).

Several other researchers, who carried out repeated load triaxial tests on unbound granular materials, have reported that permanent deformation behaviour in granular materials is principally governed by some form of stress ratio consisting of both deviatoric and confining stresses. Lashine et al. (1971) conducted repeated load triaxial tests on a crushed stone in partially saturated and drained conditions and found that the measured permanent axial strain settled down to a constant value directly related to the ratio  $q_{\max}/\sigma_{3,\max}$ , where  $q_{\max}$  and  $\sigma_{3,\max}$  are the maximum deviator stress and the maximum confining pressure respectively. Similar results were reported by Brown (1974), Hyde (1974), and Brown and Hyde (1975), who studied the response of crushed stone under repeated triaxial loading conditions with constant confining pressure. Brown and Hyde further stated that similar results are obtained from tests with variable confining pressure if the mean value of the applied confining stress is used in the analysis.

Other researchers (Raymond and Williams 1978, Pappin 1979, Thom 1988, Paute et al. 1993) have attempted to explain permanent strain behaviour under repeated loading

using the ultimate shear strength of the material. In this approach, the static failure line is considered as a boundary for permanent strain under repeated loading. This has been questioned by Lekarp and Dawson (1998), who argue that failure in granular materials under repeated loading is a gradual process and not a sudden collapse as in static failure tests. Therefore, the ultimate shear strength and stress levels that cause sudden failure are of no great interest for the analysis of the material behaviour when the increase in permanent strain is incremental.

Pappin (1979) conducted repeated load triaxial tests on a well-graded limestone using cyclic deviator and confining stresses. He concluded that the permanent axial shear strain can be expressed as a function of the length of the stress path in the  $p$ - $q$  space and the stress ratio  $(q/p)_{\max}$ , where  $q$  and  $p$  are the applied deviator stress and the mean normal stress respectively. His observation further revealed that the resistance to permanent deformation decreases when the applied stress approaches static failure, i.e. the accumulated permanent strains increase as the deviator stress increases. This is illustrated in Figure 6.2:2, in which the accumulation of permanent shear strain in repeated load triaxial tests is given for two different stress paths A and B.

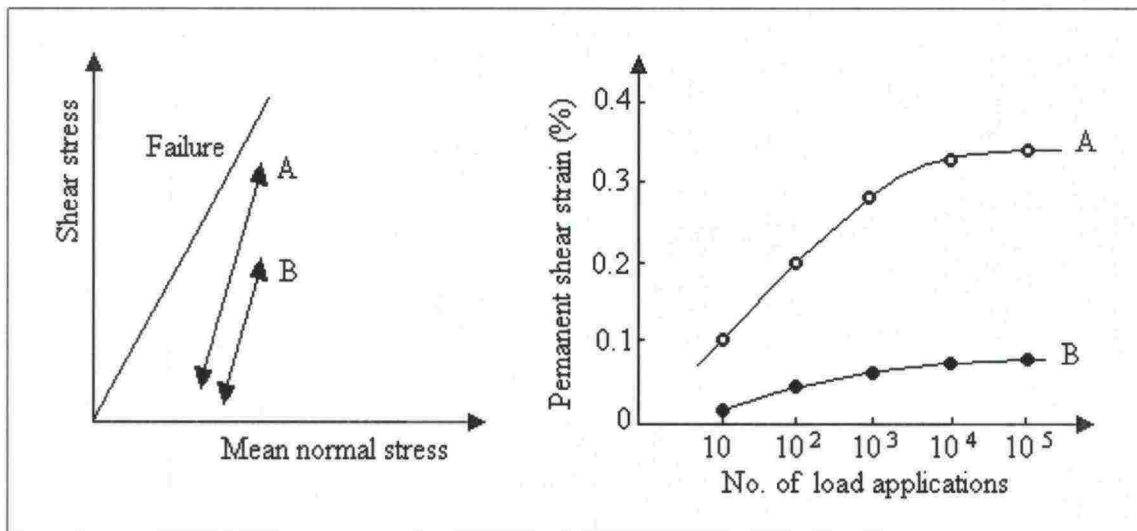


Figure 6.2:2 Permanent strain and stresses (Brown 1985).

The importance of the static failure stress has also been noted by others. Barret and Smith (1976) investigated the behaviour of a crushed dolerite (5 mm maximum particle size) with a 10% clay binder under simulated traffic loadings using cyclic triaxial tests at constant stress ratio. They found that the static Mohr-Coulomb envelope could be considered a natural boundary to both axial and horizontal permanent strain. Raymond and Williams (1978) studied the behaviour of ballast and suggested that at the same density and confining pressure there is a relationship between the plastic strain response and the static failure of the material. They also showed that the permanent shear strain can be estimated using the stress ratio  $q_{\max}/q_{\text{failure}}$ , in which  $q_{\max}$  is the maximum deviator stress and  $q_{\text{failure}}$  the deviator stress at failure. The same stress ratio was used later by Dawson and Kolisoja (2004) and Kolisoja et al. (2004).

Dawson and Kolisoja (2004) and Kolisoja et al. (2004) used an equation of the form  $\epsilon_p = a \cdot N^b$  to model the development of plastic deformation. Here, the total plastic



accumulated strain,  $\epsilon_p$ , is controlled by the number of load applications,  $N$ , and two material parameters,  $a$  and  $b$ . From the test data collected, the equation above was fitted to the results using the assumption that  $a = 100$ . Hence, it was possible to derive the parameter  $b$  as a function of the magnitude of the stress applied to the aggregate. This is given in Figure 6.2:3 where the stress ratio  $q/p$  is a measure of how close to the static failure condition the stress applied to the aggregate is (values of  $q/p$  of around 2 are typical for aggregates at static failure). The particular data refers to a Finnish aggregate with a fines content of 10.7 %, which was subjected to a freeze-thaw cycle prior to testing under repeated loading.

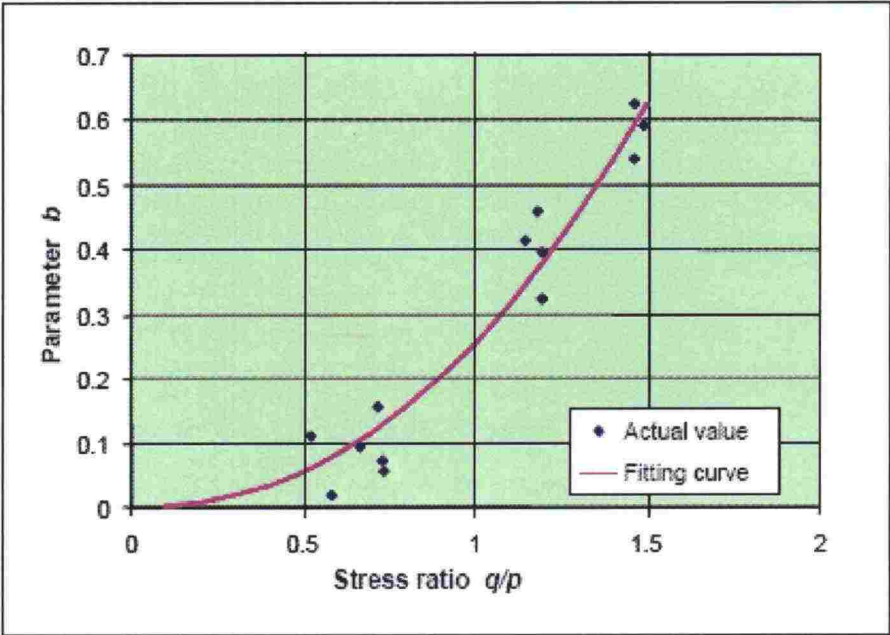


Figure 6.2:3 Values of permanent deformation equation parameter  $b$  as a function of the stress applied (Dawson and Kolisoja 2004).

From these data it appears that a value of  $b = 0.15$  is generally acceptable but that higher values of  $b$  are usually associated with excessive and/or rapid plastic deformation (Dawson and Kolisoja 2004). One very important observation concerning the way of presenting the results shown in Figure 6.2:3 is that the parameter  $b$  appears as the exponent in the equation considered and is therefore not a linear measure of the accumulation rate of permanent deformation. Thus, the accumulation rate of permanent deformation per load cycle is in fact increasing much faster as a function of the applied cyclic peak stress ratio  $q/p$  than it is shown in Figure 6.2:3 (Kolisoja et al. 2004).

Further data of this kind are presented in Table 6.2:1. The first four specimens were subjected to monotonic failure testing, while the last four, made as far as possible to replicate the first four specimens, were subjected to a series of repeated loads. The results of the failure testing (first four lines) are presented as  $q/p$  values. The plastic deformation data were fitted to the model of the equation  $\epsilon_p = a \cdot N^b$ , and the value of the parameter  $b$  determined. The values of  $q/p$  when  $b = 0.15$  were then estimated. The rate of plastic deformation build-up should then not be excessive. The values of  $(q/p)_{b=0.15}/(q/p)_{failure} = q_{b=0.15}/q_{failure}$  are listed in the last column.

Table 6.2:1 Stress sensitivity of Quarriebraes materials (Dawson and Kolisoja 2004).

Material Description	Code	Moisture Content (%)	Fines Content (%)	q/p at failure	q/p for $b=0.15$ in Eqn. 1	% of q/p at failure
Quarriebraes Moist	Q1f	4.0	4.4	1.65		
Quarriebraes Saturated	Q2f	14.0	2.5	1.65		
Quarriebraes Moist, High Fines	Q3f	5.7	15.3	0.76		
Quarriebraes Saturated, High Fines	Q4f	15.1	11.6	0.45		
Quarriebraes Moist	Q1m	3.7	4.7		1.1	67
Quarriebraes Saturated	Q2m	16.1	2.7		1.1	67
Quarriebraes Moist, High Fines	Q3m	5.1	13.2		0.55	72
Quarriebraes Saturated, High Fines	Q4m	16.6	12.0		0.25	56

Thom (1988), on the other hand, studied the behaviour of several granular materials using repeated load triaxial and hollow cylinder tests and concluded that permanent shear strain could be calculated better if the stress ratio  $(q_{\text{failure}} - q_{\text{max}})/q_{\text{failure}}$  was used.

### 6.3 Stress history

The permanent deformation behaviour of soils and granular materials at any instant is directly related to the stress history, i.e. the order of the application of loads. If these materials were initially subjected to low stresses, this would reduce the effect of following stress of higher level. If, however, the initial applied stresses are higher than the following stresses, little permanent strain will develop due to the subsequent stresses (Barksdale 1991). When repetitive loads are applied, the effect of stress history appears as a result of gradual material stiffening by each load application, causing a reduction in the proportion of permanent to resilient strain during subsequent loading cycles (Lekarp 1997 and Lekarp 1999).

Brown and Hyde (1975) studied the effect of stress history on the behaviour of granular materials. The results shown in Figure 6.3:1 indicated that permanent strain development was clearly dependent on the loading order. As shown in the figure, the permanent strain resulting from a successive increase in the stress level is considerably smaller than the strain that occurs when the highest stress level is applied immediately.



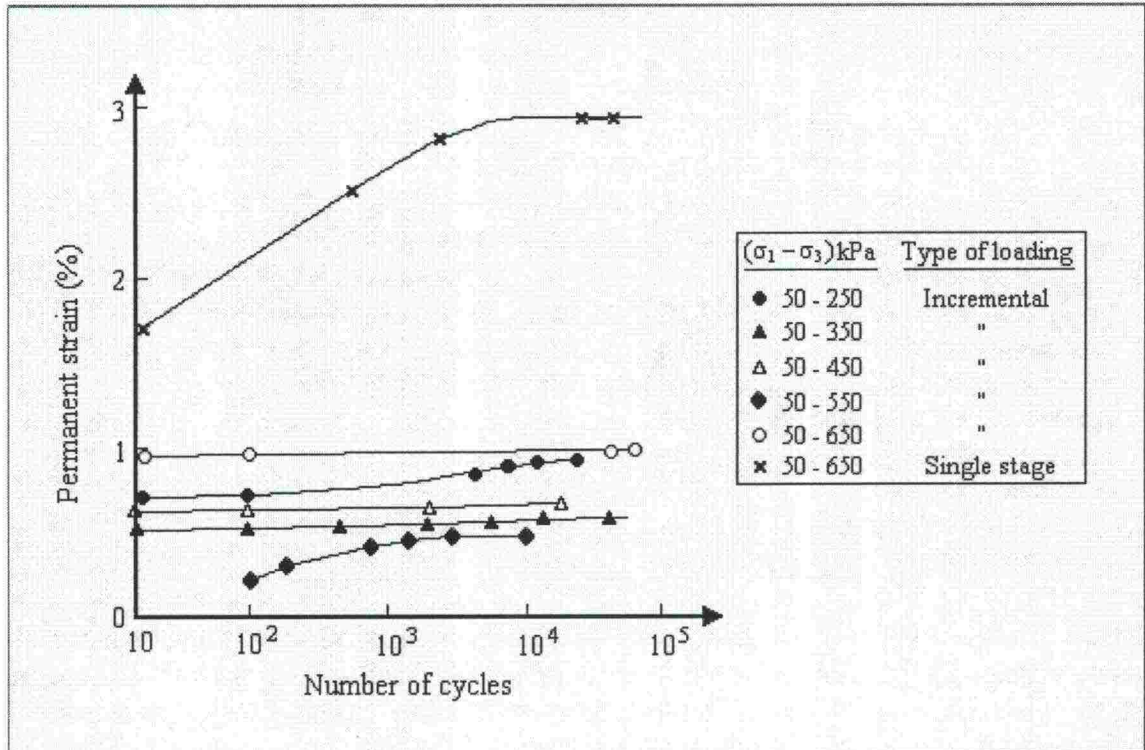


Figure 6.3:1 Effect of loading history on permanent strain (Brown and Hyde 1975).

In a more recent work, Hoff (1999) studied the development of permanent deformations as a function of load history using a loading procedure where the number of load steps and the maximum load are not fixed, but dependent on the materials resistance against permanent deformations. A procedure with 3,000 pulses for each load step was chosen. Figure 6.3:2 illustrates the load pulses applied to a sample in a typical test. This procedure applies quite high stresses on the sample at the end of each load series where rapid development of permanent strains starts. After the high stresses the sample is unloaded and a new series of load steps are applied for a different confining pressure. The process of loading to high stress levels seems to have a limited influence on the resilient behaviour of the sample for the second, third, and forth load series. No conditioning load was applied to the sample before testing. A conditioning load would have been at a lower level of mobilized shear strength than the load steps approaching the failure state and would have made the interpretation of the results more difficult.

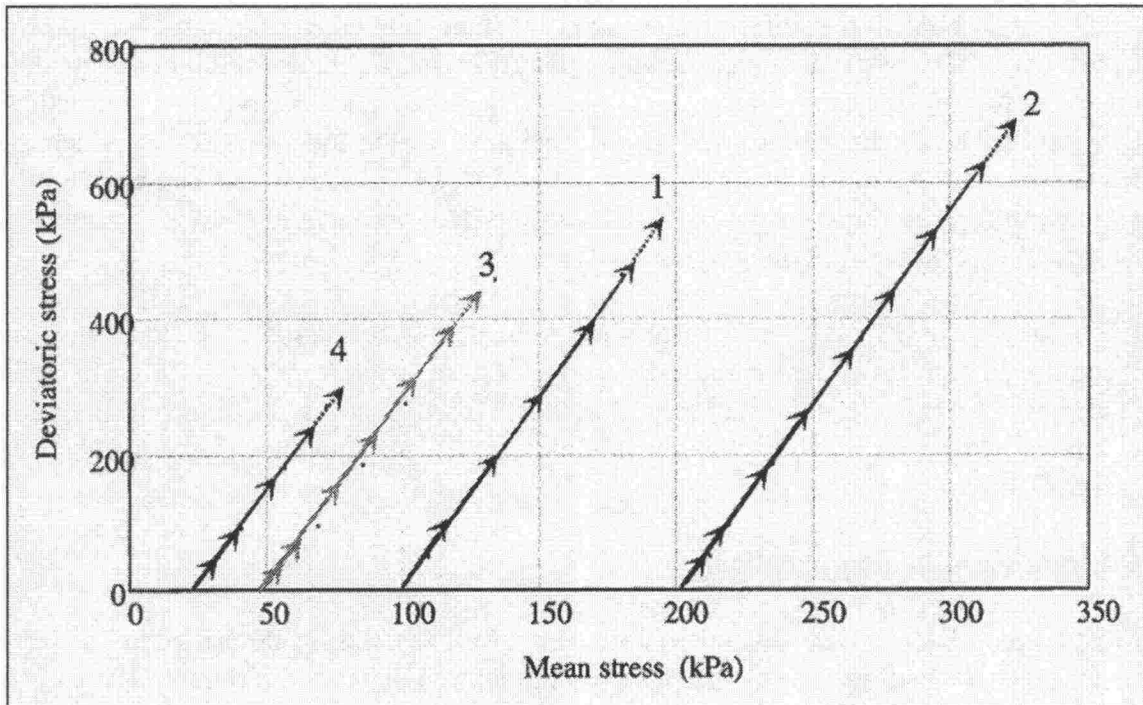


Figure 6.3:2 Load series for permanent deformation testing (Hoff 1999).

Figure 6.3:3 illustrates the development of permanent deformations as a function of loading cycles.

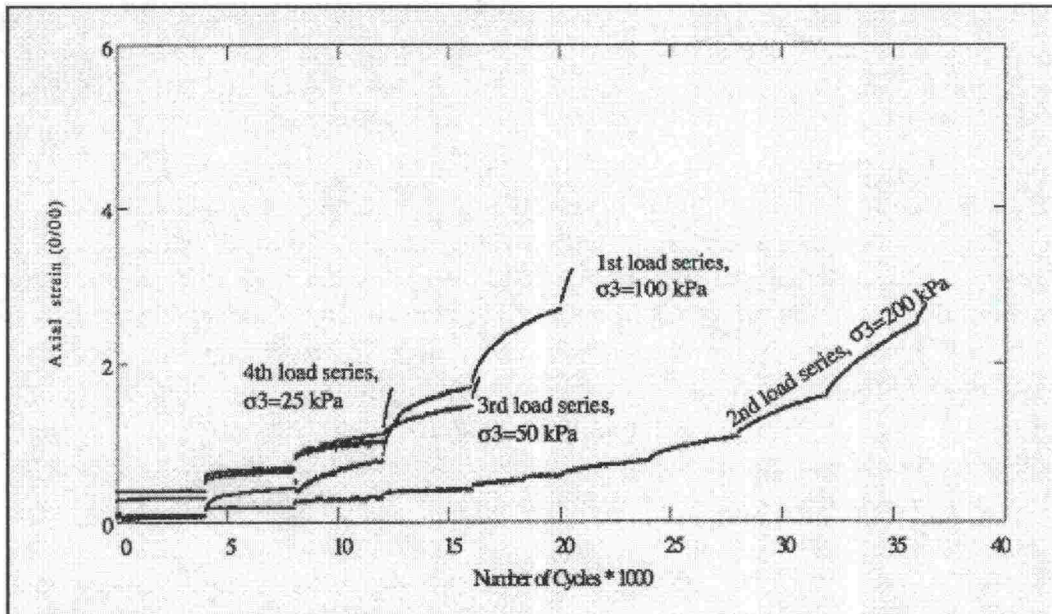


Figure 6.3:3 Development of permanent deformations as a function of loading cycles (Hoff 1999).

As we can see from Figure 6.3:3, some permanent deformations develop during the entire test. The permanent deformations at low stresses are however limited and develops in the beginning of a load step. At higher stresses the permanent deformations do not stabilise at the end of the step. When this occurs, the material is no longer able to withstand the loads in the long run and a failure condition is reached. This “failure-



load" is not equal to the failure loads found by static triaxial testing, but describes the behaviour of the materials under cyclic loading. The failure loads are highly dependent on the confining pressure. Figure 6.3:3 shows also that more permanent deformations develop in the first load series than in the beginning of the following series. When the load approaches the failure line, permanent deformations will start to increase even for the second, third, and forth load series. This means that the development of permanent deformations is dependent on both the stress level and the load history.

Even though the effect of stress history on permanent deformation behaviour has been recognised, very limited research appears to have been done to study this effect. In laboratory permanent deformation tests, the effect of stress history are normally eliminated by using a new specimen for each stress path applied (Lekarp 1997 and Lekarp 1999).

#### **6.4 Number of load applications**

Most elasto-plastic material models assume that all the plastic deformations develop under the first loading and that further loading below this level are purely elastic. This does not fit well with the behaviour of unbound aggregates, which tend to develop some additional permanent deformations for each repeated load step.

The number of load cycles is one of the most important factors to consider in the analysis of the long-term behaviour of granular materials with repeated load triaxial tests because it is a prerequisite e.g. for the estimation of how much the railway embankment will deform. The number of load cycles is important also from a practical testing point of view (time required for completing the tests and hence overall experimental costs). For instance, 80,000 load cycles are set by the prEN-standard (prEN 13286-7 2002). Sometimes this number of load repetitions is not adequate. In addition, the number of load applications is important also because the growth of permanent deformation in granular materials under loading is a gradual process during which each load application contributes to the accumulation of strain by a small increment. If the intensity of the applied loading is not too high, the accumulation of permanent deformations on a certain stress path is normally assumed to stabilise as the number of load repetitions increases. The curve representing the accumulated permanent deformation approaches asymptotically a limiting value, i.e. the permanent deformation rate per load cycle tends towards zero. Increasing stress ratios lead to a progressive rise of the accumulating permanent deformations. The significance of the number of load repetitions has been mentioned many times in the literature.

Some researchers (Morgan 1966, Barksdale 1972, Sweere 1990) have reported continuously increasing permanent strain under repeated loading. On the other hand, Brown (1974) and Brown and Hyde (1975), who investigated the behaviour of a well-graded crushed granite (5 mm maximum particle size), noted that an equilibrium state was established after approximately 1000 load applications.

Two types of sands were investigated by Morgan (1966) in a series of repeated load triaxial tests at both constant and variable confining pressures. Morgan applied up to  $2 \times 10^6$  load application and noted that the permanent strain was still increasing at the end of the tests. After an initial period of up to  $2 \times 10^5$  cycles, the rate of increase of

permanent strain with number of load cycles became linear. He mentioned, however, that the actual value of this rate increase was so small that it could be neglected for practical purposes.

Barksdale (1972) studied the behaviour of unstabilized base course materials using repeated load triaxial tests at constant confining pressure and repeated deviator stress. He applied  $10^5$  load applications and concluded that permanent axial strain in untreated granular materials accumulates linearly with the logarithm of the number of load cycles. Barksdale also noted that for very low deviator stresses the rate of accumulation of plastic strains tends to decrease with the number of load applications. As the deviator stress increases beyond a particular threshold, the rate of permanent strain development tends to increase with an increasing number of load repetitions. His test results further indicated that after a relatively large number of load repetitions the rate of plastic strain accumulation might undergo a sudden increase. Figure 6.4:1 illustrates the relationship, for granite gneiss, between the axial plastic strain and the number of load applications at different deviator stresses.

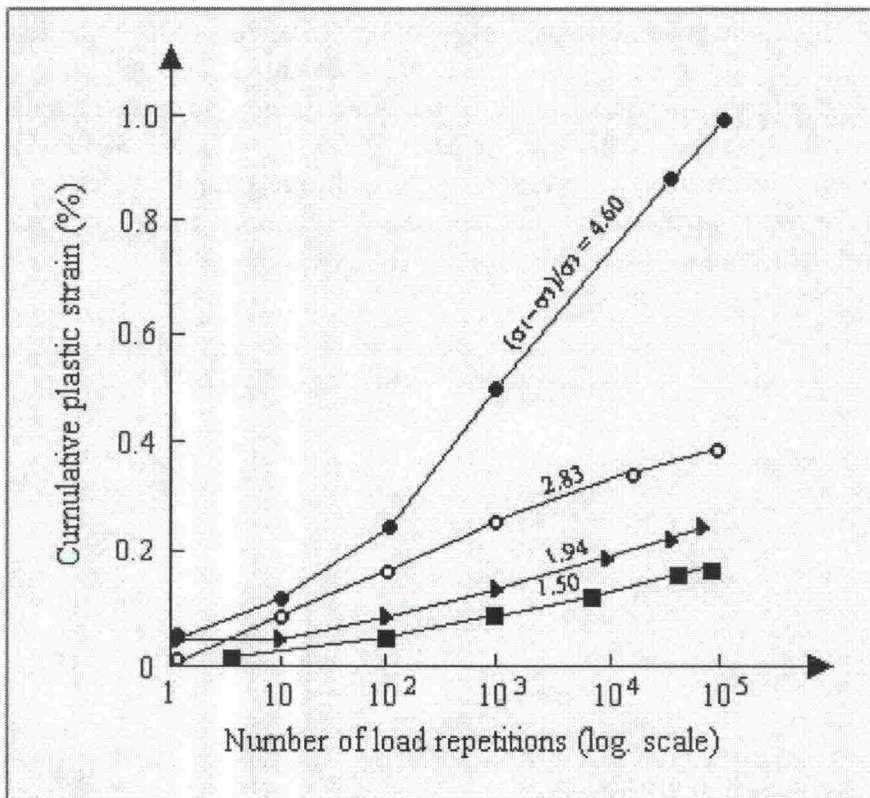


Figure 6.4:1 Influence of the number of load repetitions and deviator stress ratio on plastic strain in a granite gneiss (Barksdale 1972).

Sweere (1990) carried out repeated load triaxial tests on granular materials and observed a distinct increase in the rate of accumulation of permanent strain at a high number of load applications. He then suggested that this type of behaviour could be dealt with if the relationship between the logarithm of both permanent strain and number of load cycles was considered.



Chan (1990) noticed, in his studies, that when the applied shear stress ratio,  $q_{\max}/p_{\max}$ , was increased to high values close to the static strength of the material, permanent strains increased rapidly as a result of dilation. According to Chan, the relationship between permanent strain and the logarithm of the number of cycles tended, in this situation, to follow a hyperbolic function rather than the linear function suggested by Barksdale (1972).

Triaxial tests were also used by Paute et al. (1993), who studied the permanent strain behaviour of crushed limestone and micro granite. Based on the test results, it was argued that the rate of increase of permanent axial strain in granular materials under repeated loading decreases constantly to such an extent that it is possible to define a limit value for the accumulation of permanent strain. According to Lekarp (1997) and Lekarp and Dawson (1998), stabilization is achieved only when the applied stresses are low. High stresses, on the other hand, would result in a continuous increase of permanent strain and gradual deterioration.

A work by Kolisoja (1998) involving very large numbers of cycles reveals that the development of permanent deformation may not be expressible as a simple function, since material which appears to be approaching a stable condition may then become unstable once again under further loading. Thus, the material response must be treated as complex. Kolisoja found that specimens can apparently stabilise after 80,000 load cycles (degressive curve linearity). However, with additional load cycles, a progressive linearity in the shape of the strain plot can be observed following further development of permanent deformations (Figure 6.4:2).

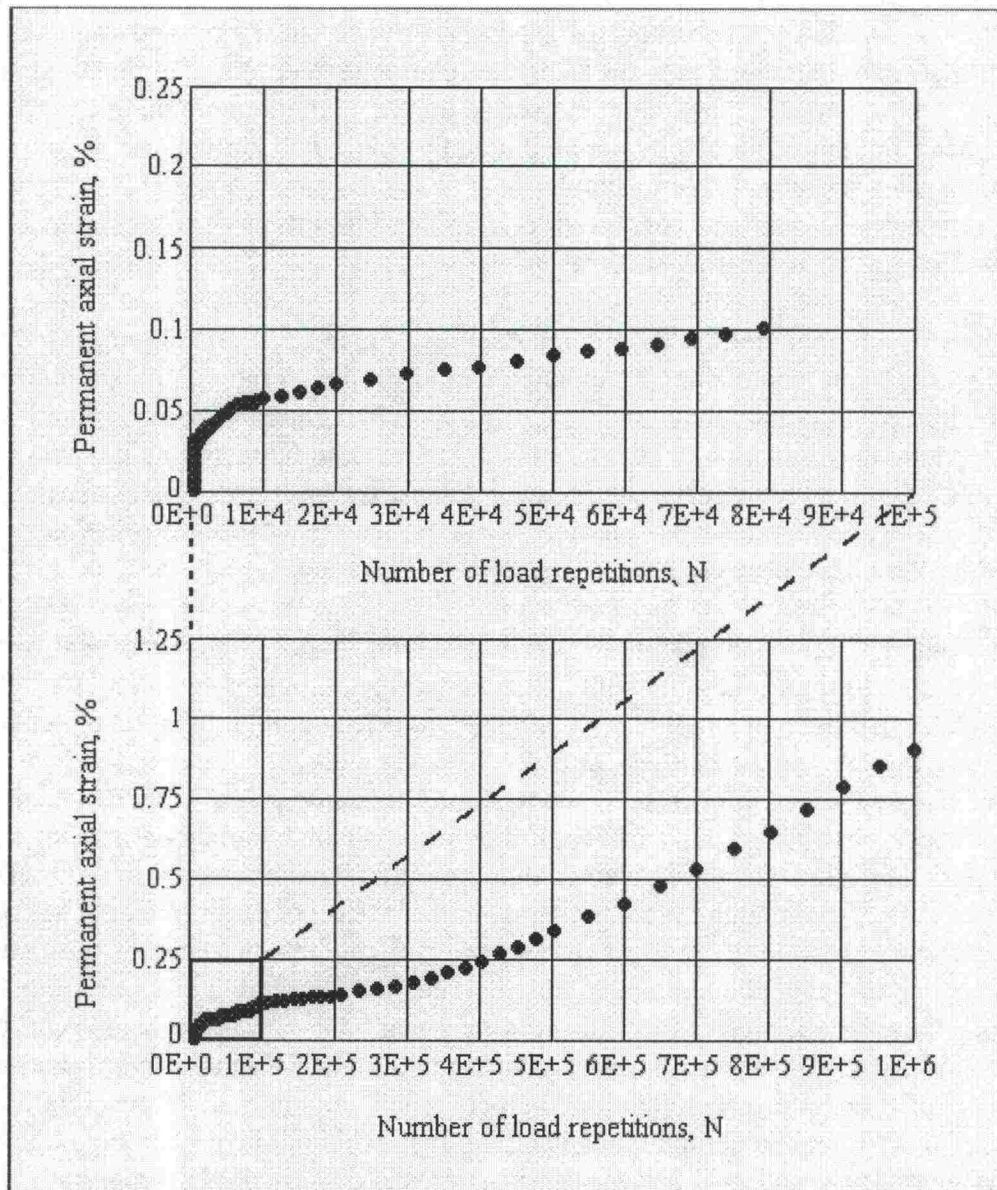


Figure 6.4:2 An example of the effect of the number of load applications on the permanent axial strain of a repeated load triaxial test specimen (Kolisoja 1998).

### 6.5 Principal stress rotation

Reorientation of principal stresses during shear is a feature of the induced stresses associated with many field loading situations. The influence of reorientation on soil strength and stress-strain response needs to be considered for a reliable prediction of in situ behaviour. The effect of principal stress rotation on the permanent strain behaviour of granular materials under repeated loading is not yet fully understood. This is probably due to the fact that repeated load triaxial testing, the most common means of reproducing traffic conditions in a laboratory, fails to provide for the continuous change in the direction of principal stresses. Nevertheless, the literature available indicates that the stress reorientation in granular materials under traffic loading results in larger permanent strains than those predicted by cyclic triaxial testing.



Youd (1972) studied the behaviour of sands in a cyclic shear box and found that the density markedly increased as a result of the rotation of the principal stress axes. The density increase was then shown to be directly related to the magnitude of the observed cyclic shear. Similar observations were reported by Ansell and Brown (1978) for crushed limestone. They performed cyclic simple shear tests on a single-sized crushed limestone of 3 mm particle size and observed a considerable influence of principal stress reorientation on the volumetric strains.

A limited number of hollow cylinder tests were carried out by Thom and Dawson (1989) to study the behaviour of a crushed rock with 4 mm maximum grain size. The results showed that the change in the direction of principal stresses significantly influenced the accumulation of permanent strain. It should be mentioned, however, that Thom and Dawson used different specimen densities and stress paths, which most certainly influenced the results. Therefore, the effect of the rotation of principal stresses alone might not have been completely isolated.

Hollow cylinder tests were also used by Chan (1990) to study the permanent response of a crushed dolomitic limestone with 5 mm maximum particle size. In order to observe the influence of principal stress rotation, Chan conducted some of his tests under triaxial conditions during which no shear stresses were applied. The tests with shear stress reversal showed much higher permanent strains in comparison with those obtained under triaxial conditions. The results further revealed that higher permanent strains, both axial and horizontal, were obtained due to bidirectional compared to unidirectional shear reversal. Unidirectional shear reversal is the stress condition under a wheel load moving in one direction. Bidirectional shear reversal, on the other hand, is the change in the direction of principal stresses when a wheel load moves back and forth. Chan mentioned, however, that when the applied shear stress was higher, there was a significant difference between the permanent strain obtained in these two cases and the influence of principal stress rotation on permanent strains was more significant at stresses close to failure. On the other hand, at low levels of shear ratio  $q/p$ , where  $q$  is the deviator stress and  $p$  the mean normal stress, and when the magnitude of the reversed shear stress was small compared with the normal stresses, small differences were found between the strain response under the condition with reversed shear stress and the triaxial condition. The results of one of the test series reported by Chan are given in Figure 6.5:1.

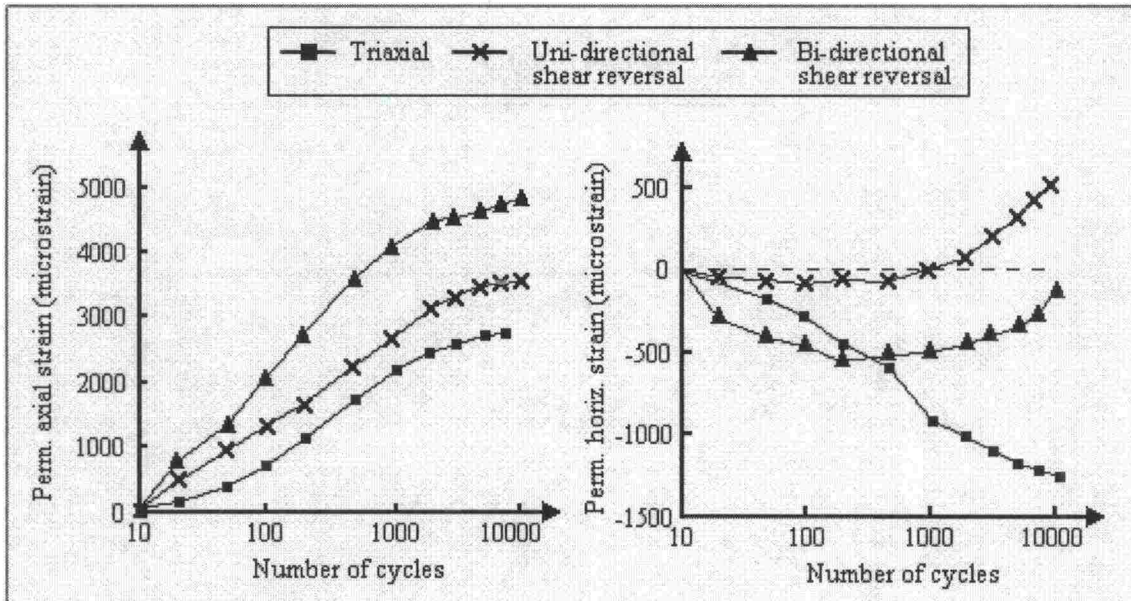


Figure 6.5:1 Variation of permanent strains with the number of stress cycles (Chan 1990).

The difference between moving wheel load, stationary wheel load, and cyclic wheel load for plastic strains in the structure has been clearly noted in the experiments performed on the test tract at the University of Nottingham. Figure 6.5:2 displays the surface profiles for two different gradings of the tested material for moving and stationary cyclic load after 6,000 loading experiments. One of the materials represented the rock material of an unbound layer in accordance with the British standards and the other one represented a bit more well-graded rock material to attain the optimal grading with a high compaction degree. In both cases, a 75 mm wide ring made of hard rubber was used when the average contact pressure between the ring and test structure was 750 kPa.



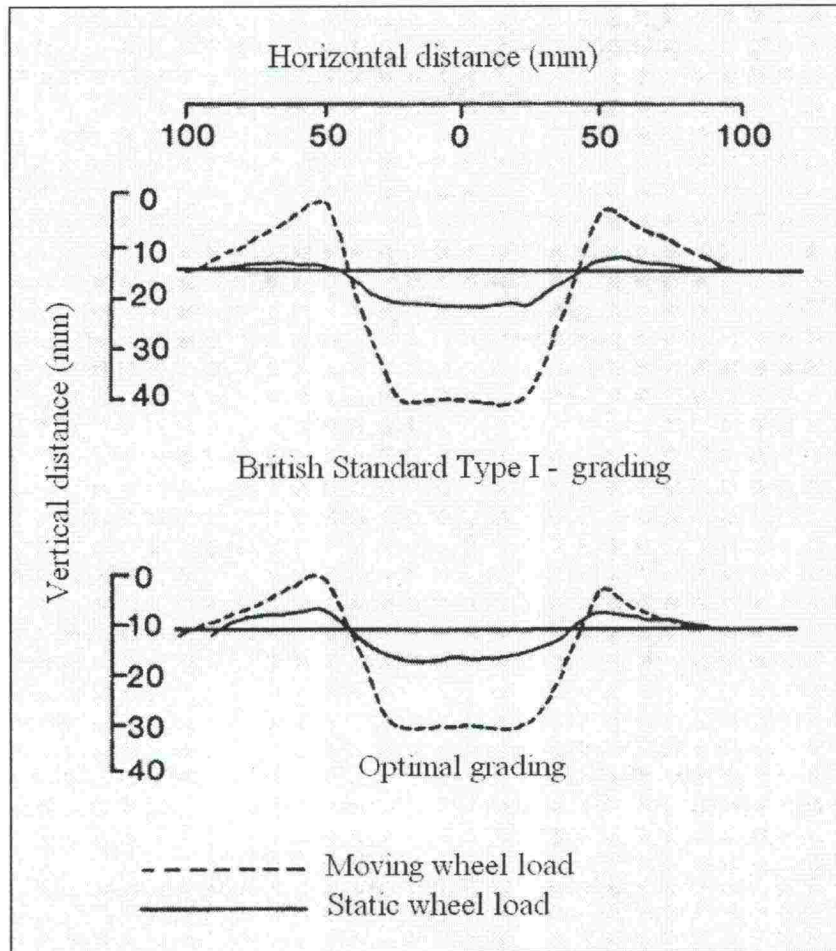


Figure 6.5:2 Comparison of plastic strains caused by a moving and a stationary cyclic wheel load (Chan 1990).

The permanent strain caused by the moving load was about the triple compared to the strain caused by a stationary cyclic load in measurements taken from the surface of the structure. By comparing the development velocities (Figure 6.5:3) of the formation of permanent strains we can also note that with the stationary load the permanent strain grew very slowly after about one thousand loading tests, but with the moving load the increase of permanent strains was continuous. Therefore, the difference between the loading cases would have been emphasised if the experiment had been continued (Kolisoja 1993).

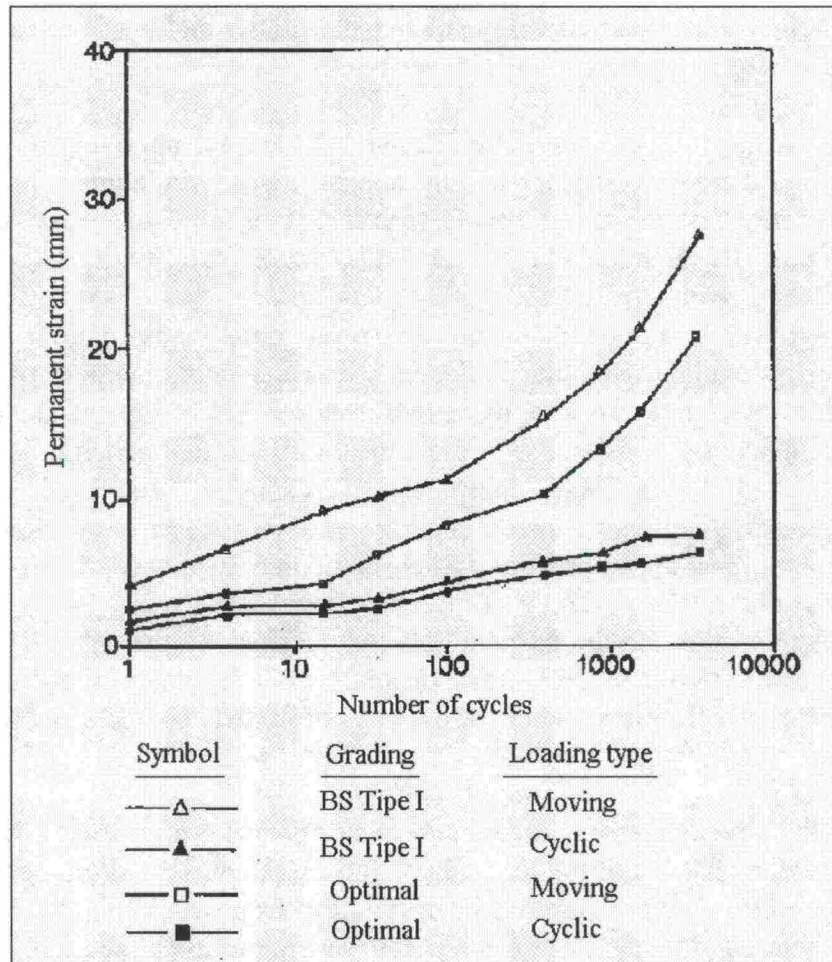


Figure 6.5:3 The development of plastic strains caused by a moving and a stationary cyclic wheel load (Chan 1990).

Within the same experiments in Nottingham, loads that move back and forth and in one direction were compared for permanent strains. In these experiments, it was noted that in general the permanent strains in course-graded and unbound material are larger for a two directional load than for a one directional load (Kolisoja 1993).

## 6.6 Moisture content

In field situations, water is always present in soils and unbound granular layers. Moisture content has a complex effect on the deformation behaviour of granular materials. The magnitude of the influence depends not only on the moisture content but also on the grain size distribution of the material and the electrochemical properties of the material that are based on the mineralogical composition of the aggregate (Kolisoja 1997).

The water film on the surface of the grains influences the shear resistance. In addition, the presence of a relatively low amount of moisture has a positive influence on the strength and the stress-strain behaviour of unbound granular materials even if the maximum grain size of the material is relatively high. When Thom (1988), for instance, made cyclic loading triaxial tests for crushed rock specimens having the maximum grain size of 10 mm, he observed that the effect was higher on the aggregates including



ample fine-grained fractions than on the aggregates consisting mostly of coarse grained fractions and having a very open structure.

However, as the moisture content increases and saturation is approached, positive pore water pressure may develop under rapidly applied loads, resulting in diminishing the permanent deformation resistance of the material. This type of behaviour has been observed by many researchers in both field and laboratory conditions.

At the particle level, the development of excess pore water pressure in granular materials can be explained by the deflections and the rearrangement of the mineral skeleton as it is exposed to external loads (Kolisoja 1997). As a result, the total volume of the pores tends to decrease. If the permeability of the material is too low in comparison with the loading rate, the pore water cannot flow away from the loaded region fast enough. As a result, excess pressure is developed in the pore water and the water becomes almost incompressible when compared with the mineral skeleton. In addition, according to the principle of effective stresses (Terzaghi 1936), the effective stresses transmitted through the particle skeleton decrease by the amount of the excess pore water pressure (Kolisoja 1997). The decrease in the contact forces, in turn, assists the development of deformations both at the level of particle contacts and at the level of the entire particulate system.

The risk for the development of excess pore water pressure increases if the material is exposed to sudden changes of the loading conditions. A special risk in the development of the excess pore water pressure has been found to exist when the loading applied to the mineral skeleton of the aggregate is repeated several times (Figure 6.6:1).

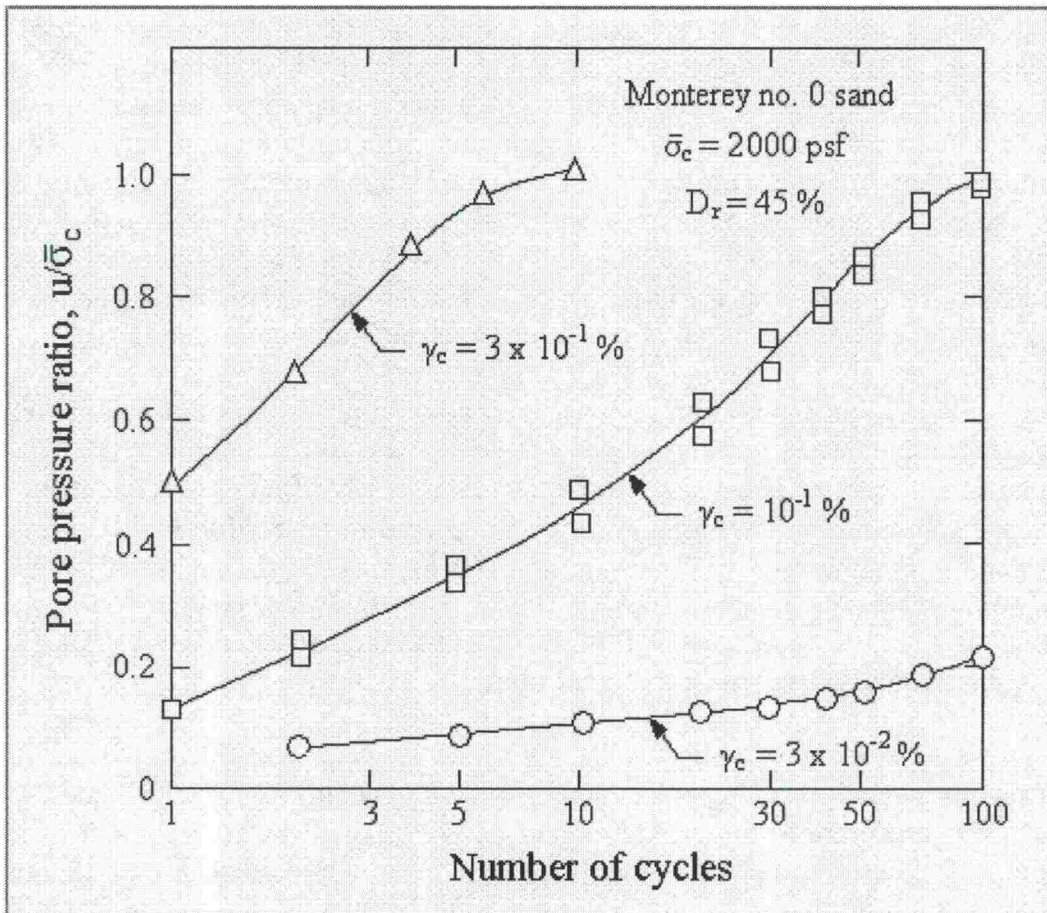


Figure 6.6:1 Pore pressure build-up during cyclic strain-controlled undrained triaxial tests on a saturated sand (Dorby et al. 1982).

The literature available reveals that researchers who have studied the effect of water content in granular layers in the laboratory and in the field believe that the combination of a high degree of saturation and low permeability due to poor drainage leads to excessive pore water pressure, low effective stress and, consequently, low stiffness and low deformation resistance (Haynes and Yoder 1963, Barksdale 1972, Maree et al. 1982, Thom and Brown 1987, Dawson et al. 1996). Thus a high water content within an unbound granular layer causes a reduction in the stiffness and, hence, in the deformation resistance of the layer.

In a study conducted by Haynes and Yoder (1963), the total permanent axial strain rose by more than 100 % as the degree of saturation increased from 60 % to 80 %.

Different granular materials were tested by Barksdale (1972), who found up to 68 % greater permanent axial strain development in soaked samples compared with those tested in partially saturated conditions. He mentioned, however, that since the tests were performed in drained conditions, which allowed the free flow of water out of the specimens, high positive pore water pressure could not have been generated during the tests.

Thom and Brown (1987) studied the influence of the water content on the permanent deformation behaviour of dolomitic materials. In particular, they studied the behaviour



of a dolomitic limestone with a maximum particle size of 10 mm using repeated load triaxial tests. The results showed a substantial increase in the strain rate as a result of increased moisture content. It was further noted that a relatively small increase in the water content could trigger a dramatic strain rate increase. It is important to note that Thom and Brown, like Barksdale (1972), observed a large increase in permanent strain even though no apparent pore pressure was generated. It was argued that such a behaviour could be due to the fact that the presence of water in an aggregate assembly practically "lubricates" the particles, hence increasing both resilient and, particularly, permanent deformation. This has also been observed by Maree et al. (1982).

Holubec (1969) conducted drained repeated load triaxial tests on crushed gravel and rock base course materials using a cyclic deviator stress of approximately 30 kPa. The permanent deformation behaviour was then investigated at different water contents. The test results showed that any increase in the water content resulted in higher permanent deformation. For the crushed rock base at 1,000 load cycles the measured permanent axial strain increased by approximately 300 % when the water content was increased from 3.1 % to 5.7 %. Similarly, for the crushed gravel the permanent axial strain rose by 200 % for an increase in moisture content from 3 % to 6.6 %.

Other researchers who investigated the influence of water content on the permanent deformation behaviour of granular materials have generally drawn the same type of conclusions as described above. Dawson et al. (1996) studied the behaviour of micro granite, soft limestone, and hard limestone with a maximum particle size of 28 mm and observed a high sensitivity of permanent strain development to moisture content. Paute et al. (1994) performed repeated load triaxial tests on several different aggregate types; all tested at a variety of moisture contents below the optimum, and showed that the amount of permanent shear strain markedly increased with increasing moisture content.

Lashine et al. (1971) compared the results of undrained and drained repeated load triaxial tests and observed a much greater permanent strain development in undrained tests. They argued that such a notable difference in behaviour was due to the build-up of excessive pore pressure during undrained test conditions.

The stress-strain behaviour of soils and granular materials can be improved significantly by draining the system. An example of the positive effect of drainage on permanent strain development in granular materials is illustrated in Figure 6.6:2 for triaxial tests with different drainage conditions. Figure 6.6:1 shows the results of repeated load triaxial tests where both samples started at the same moisture content but one was allowed to drain and become drier like in a real unbound granular layer, while the other one stayed at the same moisture content and experienced a much larger amount of permanent deformation.

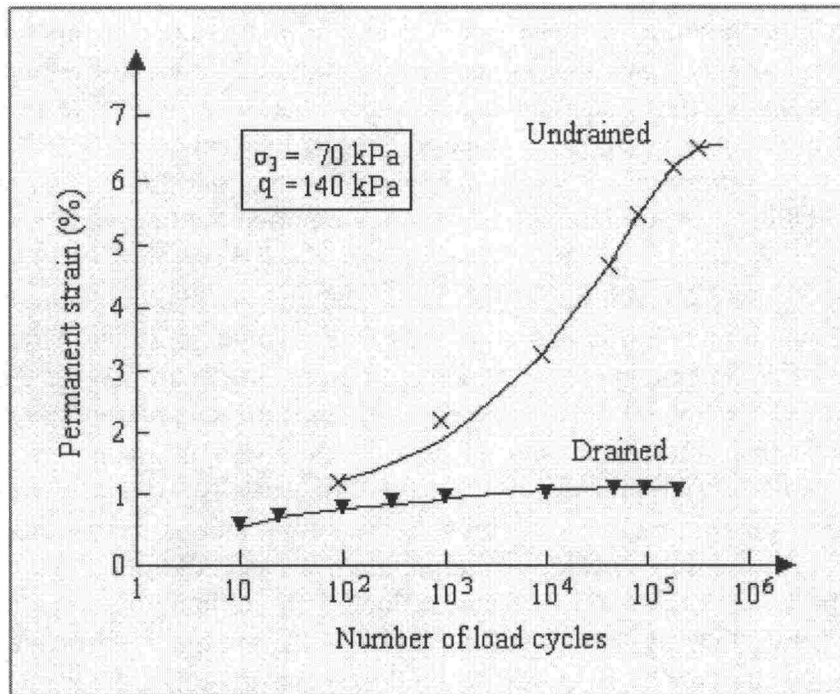


Figure 6.6:2 Influence of drainage on permanent deformation development (Dawson 1990).

In his work, Hoff (1999) found that the moisture content influences the permanent deformation properties and tends to lift the failure limit (Figure 6.6:3). However, his number of tests was too small to cover the inherent variations and in addition give a detailed interpretation of the effects of moisture on the permanent deformation properties.

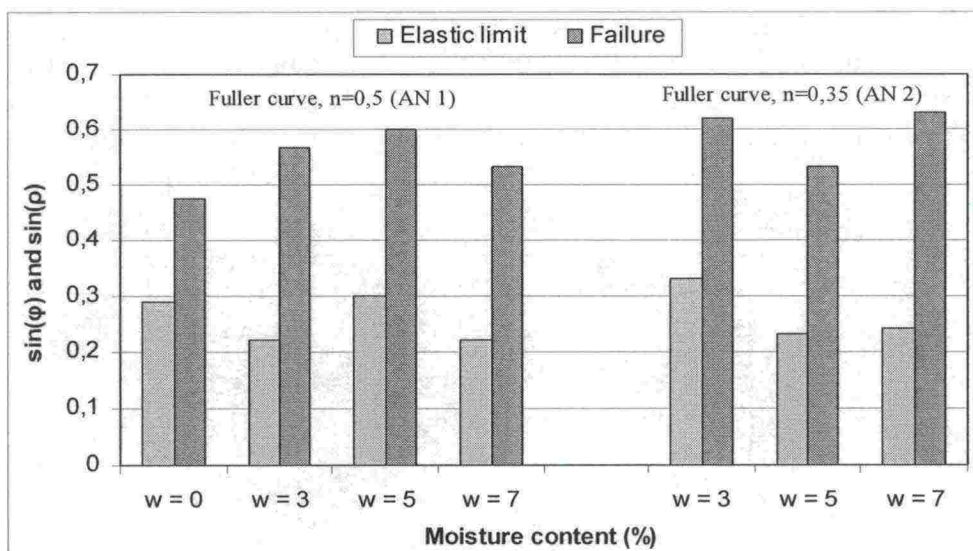


Figure 6.6:3 Elastic limits and failure angles for different moisture levels for two different well-graded materials from Åndalen (Hoff 1999).

As discussed in section 4.7, Uthus et al. (2005) studied the influence of water and fines on the behaviour of unbound granular materials under traffic loading. The basis is the



Fuller curve (Equation 4.6.2:3), where two different grading coefficients ( $n = 0.5$  and  $n = 0.35$ ) are used to investigate the influence of grading, in particular the finer part of the curve. In this study, two materials were tested: one hard very fine grained gneiss (Gneiss 1) from a quarry in Askøy outside Bergen in Norway ( $LA = 17.1$ ) and one weaker fine to medium grained gneiss (Gneiss 2) with about 30 % mica taken from a quarry near Göteborg in Sweden ( $LA = 24$ ). They reported the following results:

- Gneiss 1 (Figure 6.6:4 and Figure 6.6:5) → The material with high amounts of fines ( $n = 0.35$ ) shows a significant drop in the friction angle on a shear strength basis ( $\rho$  and  $\phi$ ) as the water content was increased from 3.0 % to 5.9 %. An additional increase in water content did not seem to influence the shear strength so much. This tendency is the same in the elastic as well as in the ultimate stress stage. The material with low amounts of fines ( $n = 0.50$ ) does not show such a tendency, as an increased water content rather seems to increase the mobilised angle of friction ( $\rho$ ) on a Mohr Coulomb envelope interpretation basis. Additional increased water content from 4.8 % to 6.8 % did not influence the ultimate shear strength for the specimens significantly. However, in the design stress stage / elastic strain range an increase in water content from 4.8 % to 6.8 % seems to reduce the friction angle.
- Gneiss 2 (Figure 6.6:6 and Figure 6.6:7) → For the unbound material with high fines contents ( $n = 0.35$ ) and the lowest water content ( $w = 4.0$  %) the mobilised friction  $\rho$  is somewhat lower than the corresponding value for Gneiss 1, and drops when the water content is slightly increased to 5.2 %. The aggregate with the smallest amount of fines shows a reduced angle of friction ( $\rho$  and  $\phi$ ) when the water content increases from 3.7 % to 4.9 % and increases again for  $w = 6.1$  %. This might be explained by the change in density that has the same variation. Remarkable is also the flat Modified Proctor curve and the high optimum water content  $w = 7.6$  % in this case. In the ultimate stress stage (near plasticity stage) the water content seems to have a significant influence on the friction angle ( $\phi$ ) for the aggregate with high amounts of fines ( $n = 0.35$ ), but not so pronounced as for Gneiss 1. The material with less fines ( $n = 0.50$ ) is also in this case less sensitive to variations in water contents, and the variations have the same pattern as in the elastic stress stage.

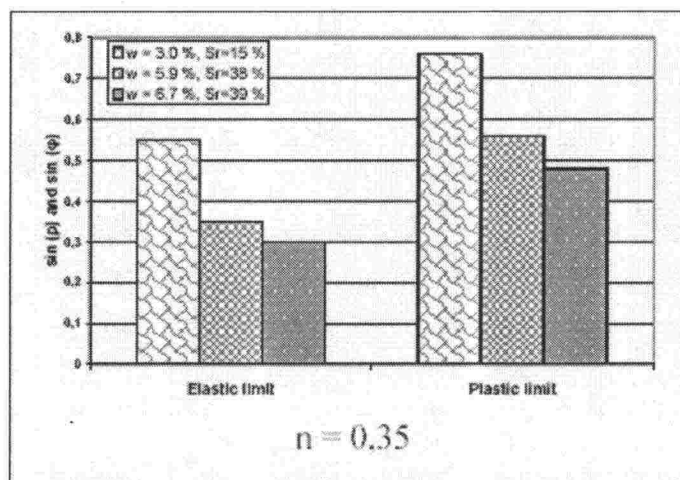


Figure 6.6:4 Elastic and plastic threshold limits on shear stress basis of Gneiss 1 with parameter  $n = 0.35$  (Uthus et al. 2005).

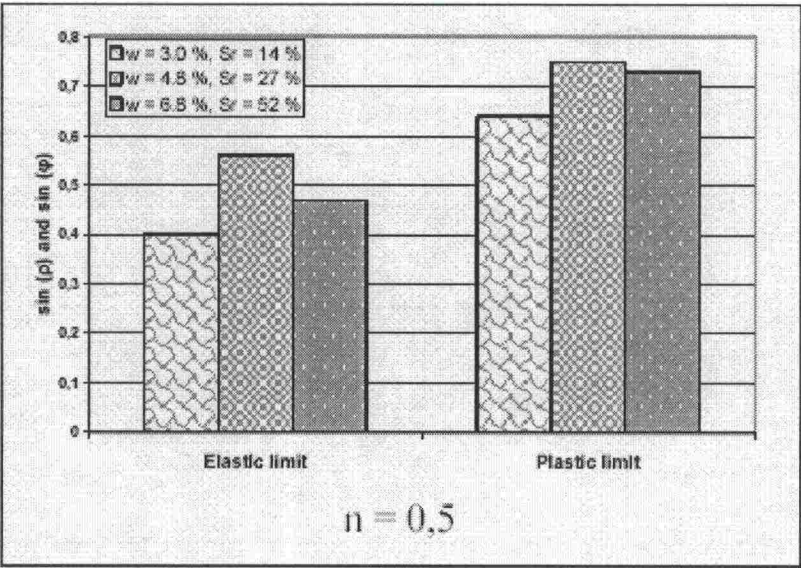


Figure 6.6:5 Elastic and plastic threshold limits on shear stress basis of Gneiss 1 with parameter  $n = 0.5$  (Uthus et al. 2005).

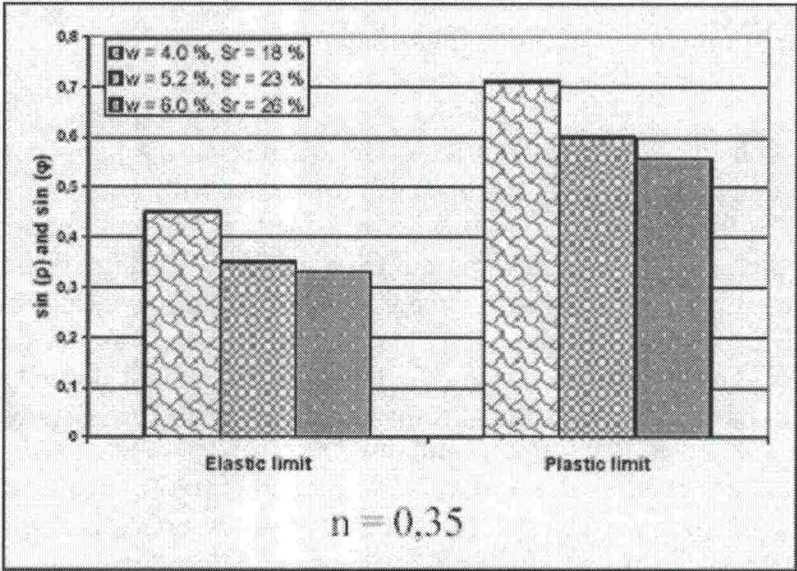


Figure 6.6:6 Elastic and plastic threshold limits on shear stress basis of Gneiss 2 with parameter  $n = 0.35$  (Uthus et al. 2005).



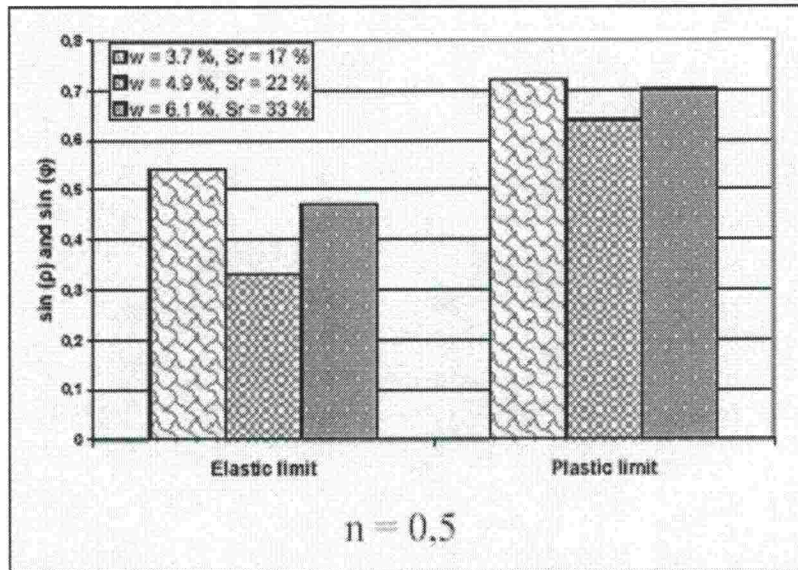


Figure 6.6:7 Elastic and plastic threshold limits on shear stress basis of Gneiss 2 with parameter  $n = 0.5$  (Uthus et al. 2005).

In Figure 6.6:4 the degree of saturation corresponding to a water content of 6.7 % is 39 % as reported by Uthus et al. (2005). This value seems to be too low. A more realistic value would be 49 % or 59 %.

Dawson and Kolisoja (2004) studied the effect of frost on the permanent deformation behaviour of unbound aggregate materials. Cold weather will cause water in the ground to freeze. As the frozen front moves downwards due to a long cold period, suctions are established which sucks water towards the freezing front. By this means excess water collects in the ground as ice. When thawing commences in the spring, this water tends to be trapped in the pores of the aggregate and cannot leave as the drainage system remains frozen. This freezing and thawing phenomenon should never be allowed in a railway embankment. More detailed information about the frost susceptibility of railway embankments can be found from Nurmikolu (2005). A possible remedy is a coarser aggregate in which suction is less easily developed. In particular, coarse aggregate material below the granular material can act as a capillary break, cutting off the supply of water to the freezing front. Figure 6.6:8 shows the deformation rate as a function of the moisture content. The data are for several aggregates, with different fines contents and changing water contents. It can be seen that the resistance to permanent deformation dramatically decreases when the moisture content of the material increases because of thawing.

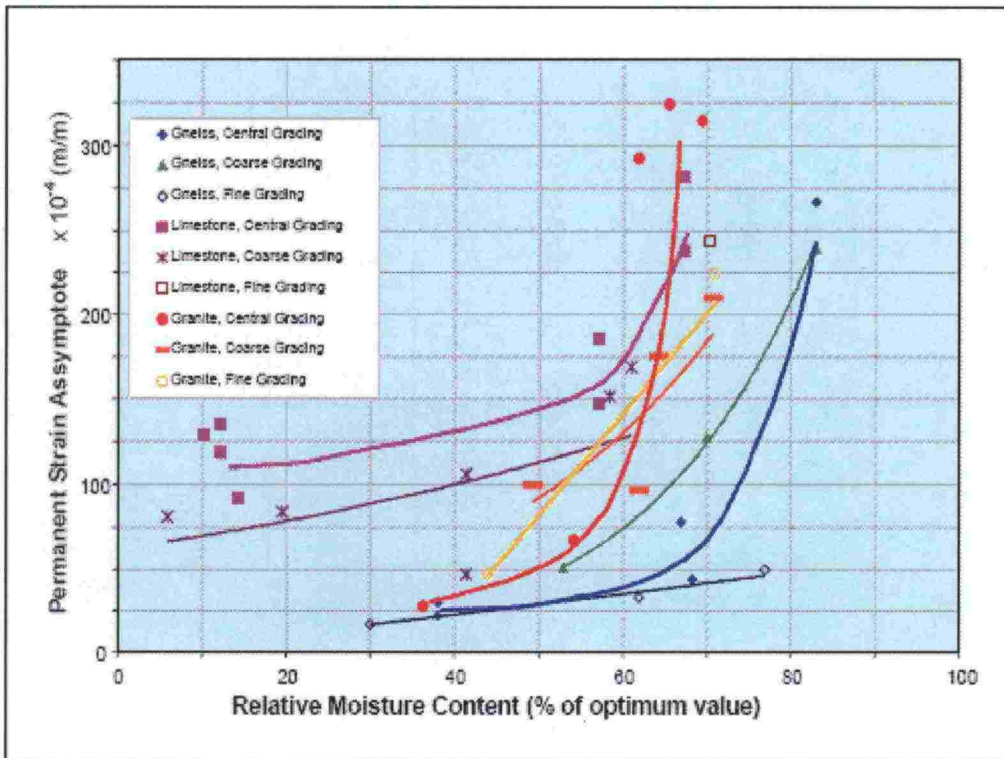


Figure 6.6:8 Permanent strain rate as a function of moisture content for 9 granular materials (Dawson and Kolisoja 2004).

## 6.7 Density

The effect of density, as described by the degree of compaction, has been regarded in previous studies as one of the most important factors influencing the long-term behaviour of granular materials and the development of permanent deformations (Holubec 1969, Barksdale 1972, Allen 1973, Marek 1977, Thom and Brown 1988, Barksdale 1991). Resistance to permanent deformation under repetitive loading appears to be highly improved as a result of increased density. Therefore, the same stress path leads to smaller permanent strains for a specimen with a high density than one with a low density.

The effect of density on the stiffness of a material skeleton of a granular material can be easily explained qualitatively by means of particle level considerations: the denser the granular material is packed, the more distinct particles have contact points with adjacent particles (Figure 6.7:1).



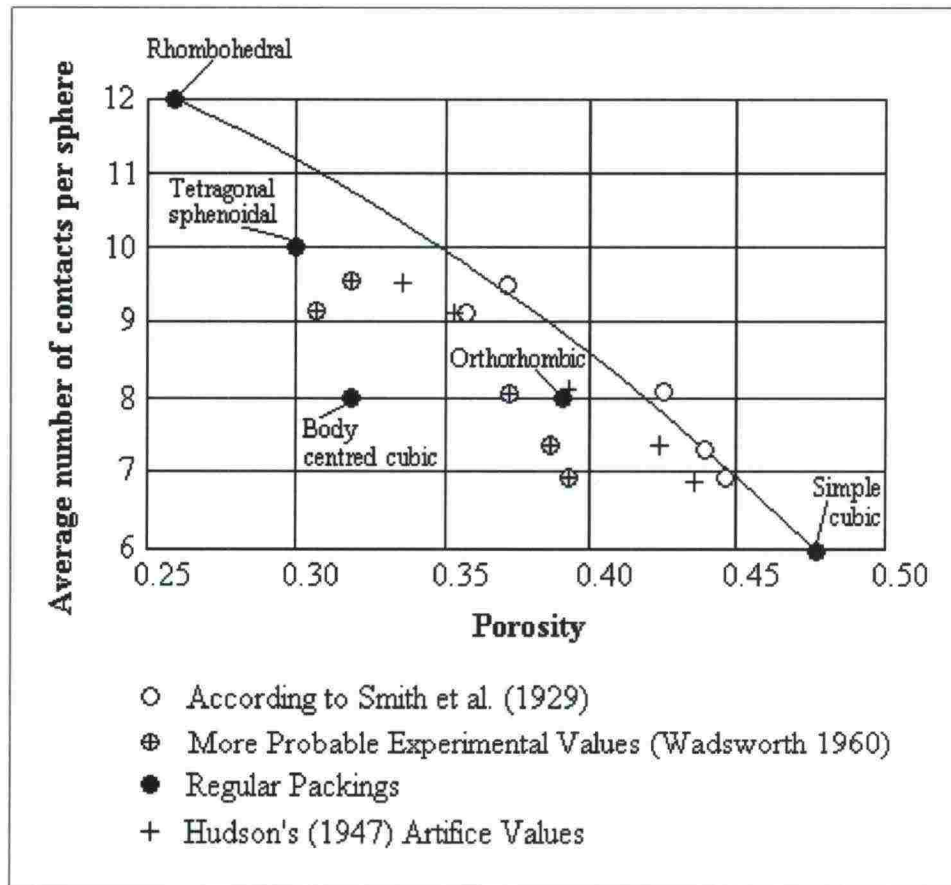


Figure 6.7:1 Dependence of the number of particle contacts on porosity for spherical particles packed in different ways (Cumberland and Crawford 1987).

When the number of particle contacts increases, the force exerted on a single particle contact by a certain external load most evidently decreases. When the load level is low, the deformations are elastic and occur mainly in the neighbourhood of the particle contacts. As a result of the smaller contact force, the deformations are smaller. Correspondingly, when the load level is high, the tangential force component in some of the particle contacts will have already reached the maximum value of the interparticle frictional force. If the mineral skeleton is now densely packed, the particulate system cannot significantly be rearranged, because each particle contact limits the directions of free movements of the particles. In fact, a significant rearrangement of the mineral skeleton consisting of very densely packed particles requires an increase in the total volume of the material, i.e. dilation. Dilation, in turn, requires work to be done against the external load, which is observed from outside of the mineral as a higher stiffness value.

Barksdale (1972, 1991) studied the behaviour of unbound granular materials using repeated load triaxial tests at constant confining pressures. The investigation of the effect of density on plastic strain behaviour indicated, for all the materials tested, an average of 185 % increase in permanent axial strain if the material was compacted at 95 % instead of 100 % of the maximum compaction density. Figure 6.7:2 shows the dramatic influence of density on permanent strain behaviour, as reported by Barksdale. Similar results have been reported by Marek (1977).

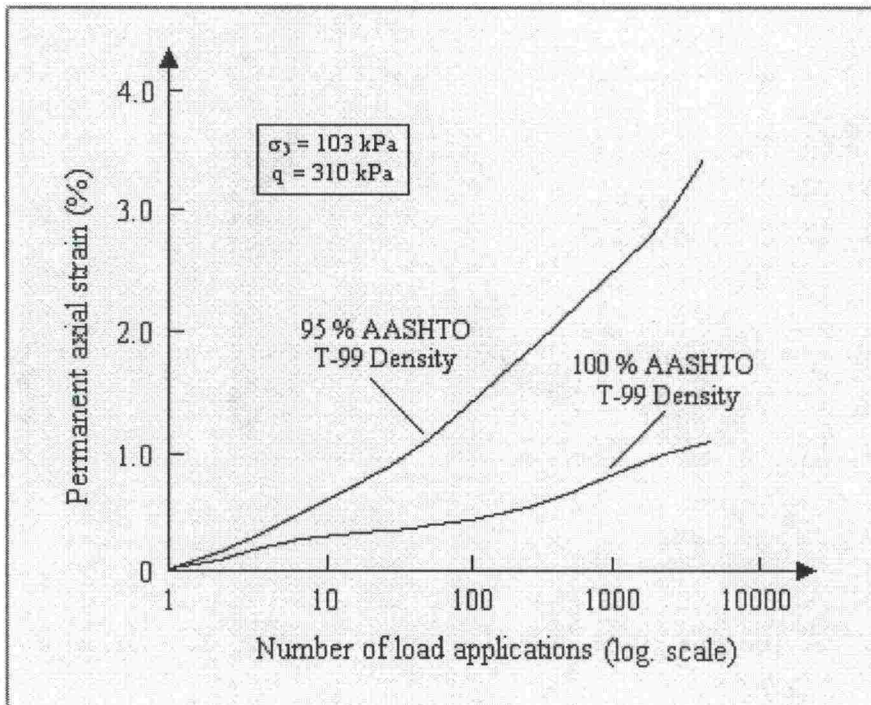


Figure 6.7:2 Effect of density on permanent strain (Barksdale 1991).

Repeated load triaxial tests were conducted by Allen (1973) on samples of crushed limestone and siliceous gravel. The specimens were prepared at different densities and subjected to identical stresses and an equal number of load applications. Although the permanent deformation behaviour was not the main objective of his studies, the test results clearly showed that the amount of plastic strain was highly dependent on the density of the material. Allen reported an 80 % reduction in total plastic strain for crushed limestone and a 22 % reduction for gravel as the specimen density was increased from Proctor density to modified Proctor density.

Hoff (1999) stated that permanent deformation is closely connected to the level of compaction. In fact, compaction is the primary method used to give a granular material a relatively dense grain structure that limits the amount of further decrease in volume due to compressive stresses. However, shear stresses that mobilise a high relative portion of the shear strength of the material under its given stress regime may give permanent deformations due to some further volume decrease and shear deformations in the granular layer. Figure 6.7:3 shows elastic limit angles and failure angles for four different levels of compaction of metamorphic and well-graded gneiss from Åndalen in Norway. As can be seen from the figure a very clear connection between the level of compaction and the development of permanent deformations is observed. A similar tendency for the failure friction angle was found in the case of a limestone with very weak grains from Visnes in Norway: the elastic limit angle was not improved by heavier compaction (Figure 6.7:4).



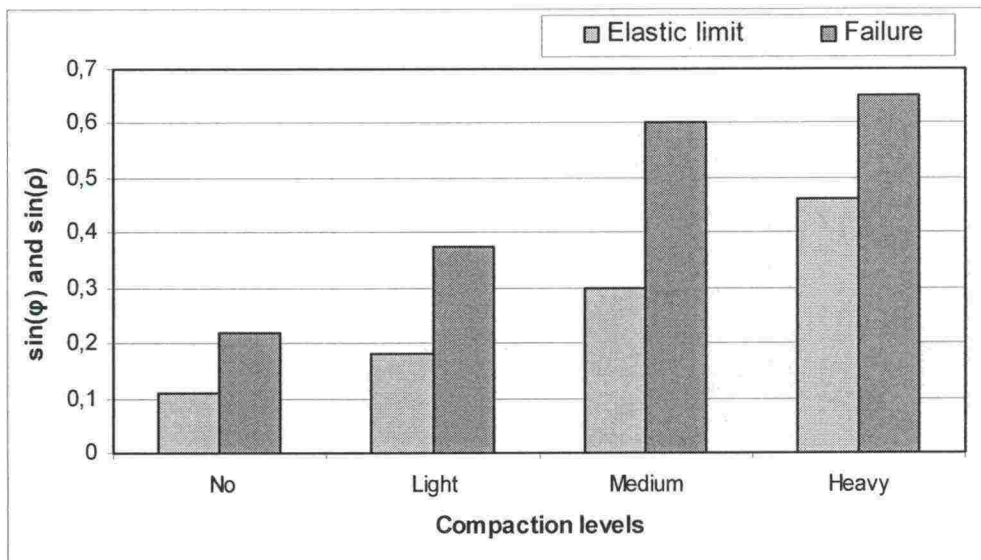


Figure 6.7:3 Elastic and failure limits for different compaction levels of the Åndalen material (Hoff 1999).

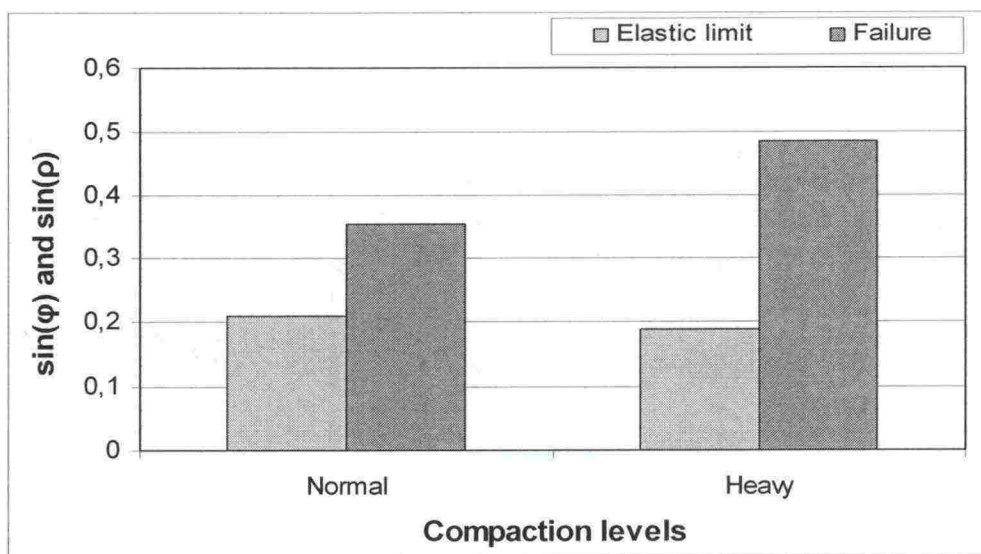


Figure 6.7:4 Elastic and failure limits for two different compaction levels of the Visnes limestone (Hoff 1999).

Thom and Brown (1988) performed a series of triaxial tests on a dry, crushed dolomitic limestone of moderate strength at different gradings. In order to investigate the effect of density on mechanical properties, specimens were prepared at each grading using different compactive efforts. The plastic strain development showed a great sensitivity to the degree of compaction (Figure 6.7:5). For well-compacted samples there seems to be a small increase in strains with the grading parameter  $n$  up to a value of one and a decrease again for very open-graded materials. Uncompacted samples, however, are strongly influenced by grading.

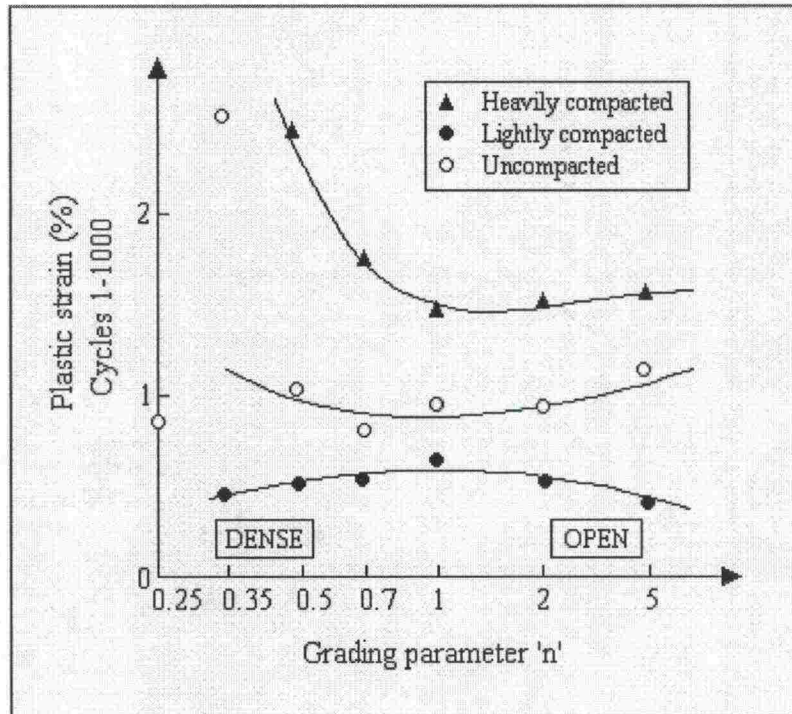


Figure 6.7:5 Effect of grading and compaction on plastic strain under cyclic load (Thom and Brown 1988).

The reduction of plastic strain due to increased density has been suggested by Holubec (1969) as being particularly large for angular aggregates, providing there is no accompanying increase in transient pore pressure during repetitive loading. For rounded aggregates, however, this decrease in strain with increasing density is not considered to be significant, as they are initially of higher relative density than angular aggregates for the same compactive effort.

## 6.8 Fines content and grading

### 6.8.1 Fines content

The effect of fines content was investigated by Barksdale (1972), Thom and Brown (1988), and Barksdale (1991), who concluded that the permanent deformation resistance of unbound granular materials is reduced as the amount of fines increases. Barksdale (1972) stated that increasing fines content leads to a significantly higher permanent strain. This is illustrated in Figure 6.8.1:1 for crushed biotite granite gneiss. The figure also shows that the difference in permanent strain due to a change in fines content is greater at larger applied stress ratios. Similar results were later found by Kolisoja (1998) and Belt (1997). They stated that significantly higher permanent strains may be expected for aggregates containing extremely high fines content.



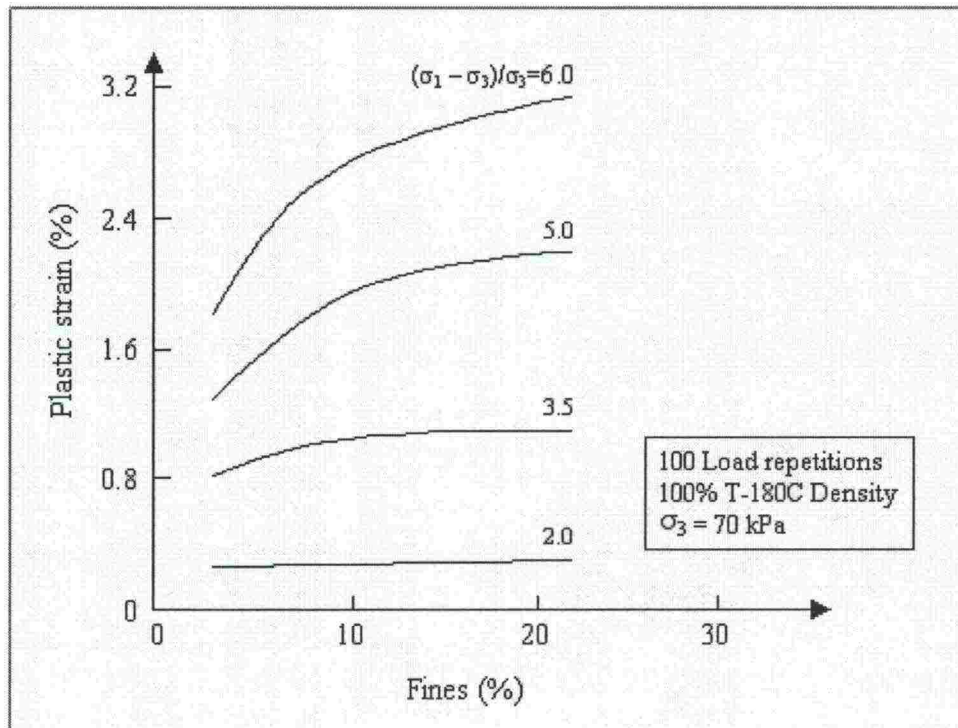


Figure 6.8.1:1 Influence of fines on plastic strain for a crushed granite gneiss base (Barksdale 1972).

Thom and Brown (1988) investigated the behaviour of a crushed limestone at different gradings, and concluded that permanent deformation resistance is reduced as the fines content increases. This could be explained by the assumption that the entire fines fraction do not necessarily fit into the pore spaces between the large particles. Therefore, a skeleton of larger particles in full contact does not exist. As a result, the resistance against permanent deformation and stiffness decreases. However, in regard to the effect of grading on accumulation of plastic strain, they discussed that the effect was unclear since it varied with the compaction level. For the uncompacted specimens, the uniform grading showed the least permanent strain, whereas the resistance to plastic strain was similar for all gradings when heavily compacted. These findings have been questioned by Dawson et al. (1996), who found the effect of grading on permanent deformation to be more significant than the degree of compaction, with the highest plastic strain resistance for the densest mix. Kamal et al. (1993) reported similar observations. This difference may be a consequence of the extremely wide range of densities and gradings adopted by Thom and Brown (1988), which far exceeded the range likely to be experienced in reality.

### 6.8.2 Grading

Hoff (1999) reported that different types of materials are likely to have different resistances against development of permanent deformations. He tested with the same procedure six different materials and used both well-graded and open graded materials. The elastic limit angle and the failure friction angle for the different materials were determined and are shown in Figure 6.8.2:1 for the well-graded materials and in Figure 6.8.2:2 for the open-graded materials.

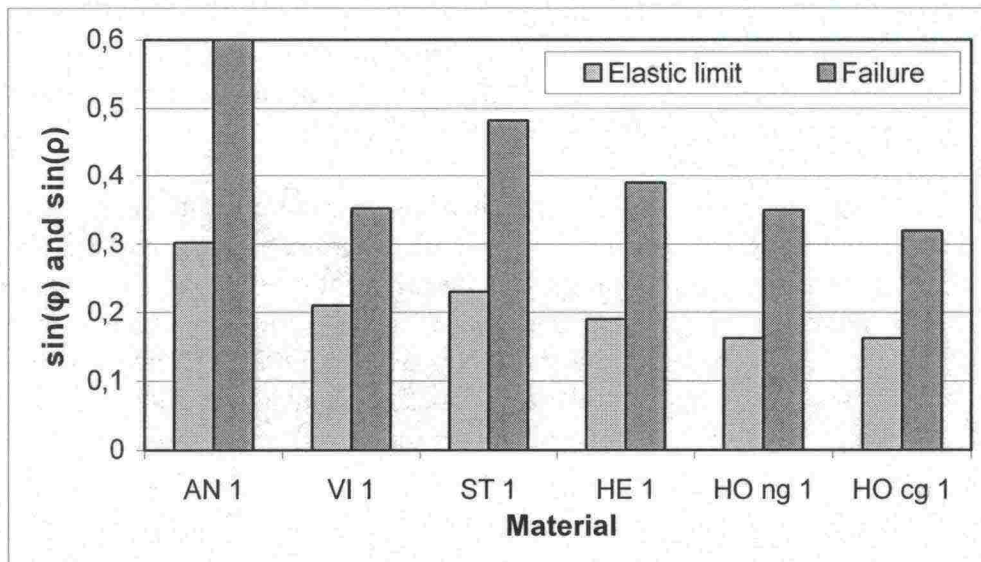


Figure 6.8.2:1 Elastic limit angles and failure angles for the well-graded materials (Hoff 1999).

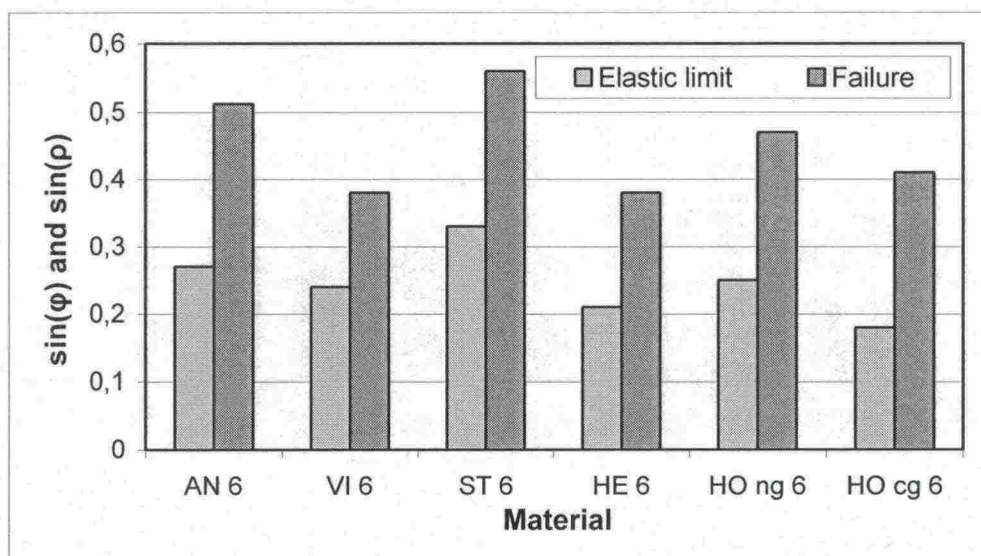


Figure 6.8.2:2 Elastic limit angles and failure angles for the open-graded materials (Hoff 1999).

In Figure 6.8.2:1 and Figure 6.8.2:2 the materials from Åndalen (AN), Steinskogen (SN), Hedrum (HE), and Visnes (VI) are rock materials, while the materials from Hovinmoen (HO) are gravel materials (ng = natural gravel and cg = crushed gravel). For all these materials  $d_{\max} = 22$  mm. The grain size distributions are illustrated in Figure 4.6.2:3 (distributions 1 and 6). From these tests the well-graded gravel materials seem to have a slightly lower resistance against permanent deformations than crushed materials. The same tendency is found for the open graded materials. However, the two rock materials with the lowest grain strength show lower values for this grading. The low resistance against permanent deformations for the materials from Visnes (VI) and Hedrum (HE) is probably caused by crushing of particle edges and corners. The well-graded samples will have more contact points and thereby lower contact point stresses. This may



explain why the differences in the elastic limit angle are smaller for the well-graded samples. For the higher stresses found closer to the failure situation, the crushing of particles is important. The highest resistance against permanent deformations is found for the two crushed rock materials with high grain strength.

Hoff (1999) tested also nine different grain size distributions of the Åndalen (AN) material (see Figure 4.6.2:3 for the grain size distributions). Figure 6.8.2:3 shows the elastic limit angle and the failure angle for the different gradings. As can be seen, the open graded materials (Figure 6.8.2:2) have lower resistance against permanent deformations than the more well-graded samples (Figure 6.8.2:1). The very open-graded materials (AN7 and AN9) have a quite low resistance against permanent deformations. The three gradings with similar shape of the curve but different grain size (AN4, AN5, and AN6) shows no improvement for the largest grains (AN6) as is the case for the resilient behaviour. The best performance is found for a material with Fuller curve grading parameter  $n = 0.5$  (AN 1).

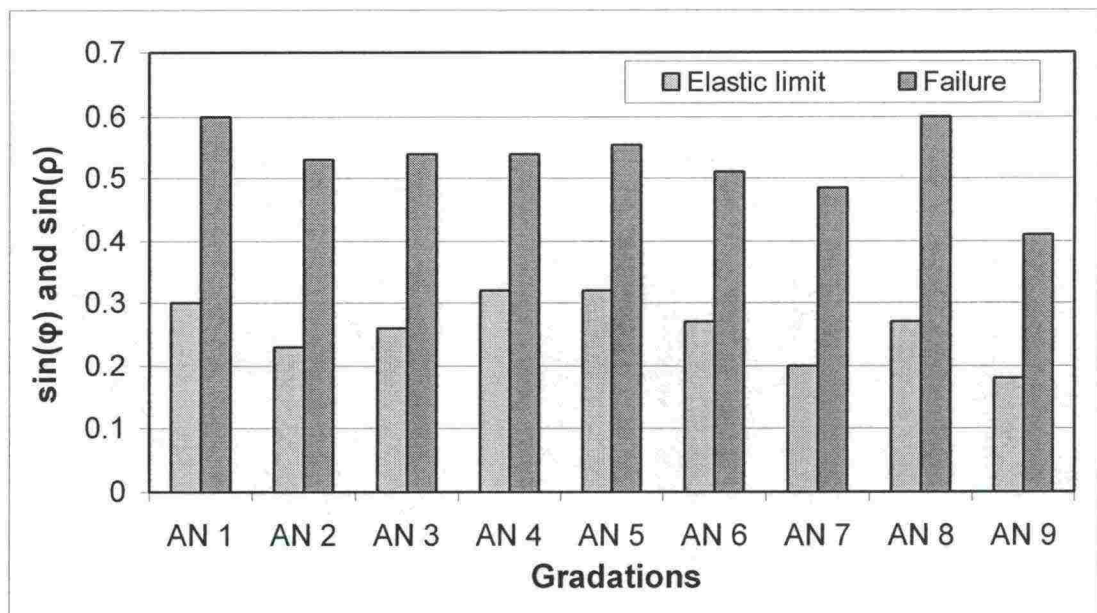


Figure 6.8.2:3 Elastic limit angles and failure angles for different gradations of the Åndalen material (Hoff 1999).

It has also been argued that if a change in grading produces an increase in relative density for the same compactive effort, then the permanent strain will decrease (Dunlap 1966). Van Niekerk (2002) recognised that unbound granular materials with a more balanced grading perform better than the more uniformly graded materials.

It is worth emphasising that the experimental observations of the effects of the grain size distribution presented above are mainly based on the results of test series performed with repeated load triaxial testing equipment. Therefore, aggregates having an open-graded grain size distribution do not necessarily behave as well in the real structure as could be expected on the basis of the permanent deformation resistance measured using the cyclic loading triaxial test (Belt et al. 1997). One major reason for that is the rotation of the principal stress directions during the real loading condition. This has been shown

to markedly affect the permanent deformation behaviour of unbound granular materials (Brown and Chan 1996, Chan 1990).

## **6.9 Physical properties of aggregate particles**

### ***6.9.1 Introduction***

The physical properties of aggregate particles, like grain shape, particle surface roughness, elastic properties, and electro-chemical properties depict one of the least observed parameters affecting the permanent deformation behaviour of unbound granular materials.

The grain shape and surface roughness of particles affect the deformation behaviour of coarse-grained granular materials and depend on the mineralogical composition of the aggregate. In addition, the mineralogical composition has a significant effect on the electro-chemical properties of the aggregate particles.

### ***6.9.2 Grain type and shape***

The grain shape apparently affects most significantly the deformation behaviour of coarse-grained aggregates indirectly by means of their compactibility, since the grain shape clearly affects the compactibility of aggregates. Aggregates having a very schistose or elongated grain shape are also naturally more sensitive to particle defects. As a result, the amount of fines fraction increases, the proportion of coarse grains decreases, and the amount of permanent deformations due to the rearrangement of the defected particles increases (Kolissoja 1997).

With respect to grain shape, two general groups can be formed: the first group is composed of natural sands and gravel, and the second group is composed of crushed materials. In the first group particle contacts are between two smooth surfaces, whilst in the second group the particle edges can be very sharp. However, the sharpest edges are evidently somewhat rounded when the crushed material is handled and compacted in the eventual structure. Therefore, at micro level the difference between the crushed material and natural aggregate is not necessarily as large as could be expected based on the visual inspection of the material immediately after crushing. The difference between the natural aggregate having rounded grains and the crushed aggregate having sharp-edged grains is of most significance in the long term and permanent deformation behaviour of unbound granular materials (Werkmeister 2003). The crushed material is likely to have more grain abrasion than the natural aggregate, especially at high stresses, and therefore more fines fraction is produced. However, the test series normally used to determine the mechanical behaviour of aggregates - by applying thousands or even tens of thousands of loading cycles - do not measure this effect. Therefore, if the mechanical behaviour of unbound granular materials - especially crushed aggregates - is evaluated based on the modulus values only, the materials can act under long-term loading conditions worse than expected.

Allen (1973) compared the plastic strain development under repetitive loading in samples of crushed limestone and gravel. He observed that, for the same density, the amount of permanent strain in the crushed stone was much lower than in the gravel. It



was then argued that the variation in plastic strains for different aggregate types at the same density is related to the surface characteristics of the aggregate. The crushed stone particles are rougher and more angular and therefore exhibit a higher angle of shearing resistance due to a better particle interlock. Consequently, angular materials undergo smaller plastic deformations compared to the materials, such as gravel, with rounded particles.

The type of aggregate is also said to play an important role in the material behaviour. Barksdale (1972, 1991) concluded in his studies that the accumulation of permanent axial strain is strongly dependent on the aggregate type.

### ***6.9.3 Particle Surface Roughness***

Many investigations have addressed the effect of macro and micro roughness of the particles on the deformation behaviour (Cheung 1994, Kolisoja 1997, Thom and Brown 1989). The ability of the material to resist permanent deformations has been found to correlate better with the macro level surface roughness, i.e. the visible roughness of the individual particles based on the number of protrusions in the surface (Thom and Brown 1989, Brown and Selig 1991).

### ***6.9.4 Electro-chemical properties***

Among the factors depending on the electro-chemical properties, in turn, the interactions between aggregate particles and the pore water located in the pore spaces are of great significance. The finer the particles the material is composed of, the greater is the significance.

Electro-chemical properties of aggregate are assumed to have an especially wide effect on the moisture content sensitivity of the material. Thus, a material that adsorbs plenty of water may retain relatively high degree of saturation in even fairly coarse-grained aggregate if water has been available even temporarily. Therefore, that kind of material is liable to the development of excess pore water pressure in rapid loading conditions, such as traffic loading. Excess pore water pressure causes stiffness to decrease, which in turn lays the basis for the development of permanent deformations. A method for recognising this type of problem-aggregates is to measure the dielectricity of the material. This method has been applied, for example, in Finland and in the USA (Saarenketo and Scullion 1996).

## **6.10 Temperature**

Temperature changes above 0 °C apparently do not affect much the deformation behaviour of unbound coarse-grained aggregates. However, if the pore water located between the soil particles freezes, the stiffness of the material increases considerably. This has been experimentally proven even with very low moisture content both with laboratory and in-situ measurements (Huhtala and Pihlajamäki 1986).

At the particle level, the reason for the strengthening caused by freezing are the very strong bonds between the soil particles produced by ice. The bonds efficiently prevent the movement of the particles. Because water in partly saturated materials is

concentrated at the contact points between the particles, relatively low moisture content is sufficient to markedly increase the stiffness. Due to the viscose properties of ice, the strengthening effect is apparently stronger the shorter the loading time and the lower the material temperature.

On the regions of seasonal frost, the annual temperature changes have many unfavourable effects on the mechanical behaviour of the unbound coarse-grained materials (see Section 6.6). However, it should be noted that the effects are not caused by the temperature or temperature changes as such but changes in the state of the material as well as environmental circumstances produced by the freezing and frost action.

If granular materials are fully saturated when they freeze, they can at least in principle loosen due to the freezing expansion of water. As the freeze melts in the spring, permanent deformations may be developed in the loosened layer as the material is recompacted by the traffic loading (Kolisoja 1997).

If uneven frost heave occurs, large local shear strains that loosen the materials can result. Moreover, when the heave recovers unevenly during the thawing period, shear strains are also produced.



## 7 MODELLING OF PERMANENT DEFORMATION BEHAVIOUR

### 7.1 Introduction

Most of the research carried out to study the mechanical properties of granular materials deals with the resilient behaviour of these materials. In comparison, the amount of work on plastic behaviour is relatively limited. The available models of the permanent deformation behaviour of unbound granular materials are, consequently, much less developed than those of the resilient deformation behaviour. According to Lekarp (1997), this is probably due to the fact that monitoring long term behaviour is a very time-consuming and cumbersome process when very large numbers of load applications need to be employed ( $10^5$  to  $10^6$  cycles). Furthermore, each specimen can only be subjected to a single stress path since permanent deformation behaviour is strongly influenced by stress history.

In modelling the long-term behaviour of granular materials, it is essential for the analysis to take into account the gradual accumulation of permanent strain with the number of load applications and the important role played by stress conditions. Hence, one of the main objectives of the research on the long-term behaviour of unbound granular materials is establishing constitutive relationships, which allow accurate predictions of the permanent strain at any number of cycles and at a given stress level (Lekarp 1997 and Lekarp 1999).

Over the years, several researchers have attempted to outline procedures for predicting permanent strain in unbound granular materials and the permanent deformation of such materials has been modelled in a variety of ways. Some of these are logarithmic with respect to number of loading cycles (e.g. Barksdale 72, Sweere 90), whilst others are hyperbolic, tending towards an asymptotic value of deformation with increasing numbers of load cycles (e.g. Wolf and Visser 1994, Paute et al. 1993). This chapter aims to describe the state of the art in the modelling of permanent deformation behaviour based on the literature available to date.

The computational procedures for the prediction of permanent strains in granular materials are generally based on the effect of the number of load applications and also the applied stresses. The gradual accumulation of permanent strain is normally defined as a function of the number of load repetitions. The amount of accumulated permanent strain after a certain number of load cycles is then defined as a function of the stress components. Some of the models found in the literature, such as those suggested by Khedr (1985) and Paute et al. (1988), are based on the assumption that the accumulation of permanent strain gradually levels off, resulting in a final response that is totally resilient. Other models, however, predict a continuing increase in permanent strain with no signs of stabilization in material behaviour. Examples of such models are the so-called log-normal model (Barksdale 1972) and the log-log model (Sweere 1990). There are studies (e.g. Barksdale 1972 and Chan 1990) indicating that the long-term performance of granular materials depends on the level of applied stresses, so that low stresses result in a final equilibrium state and high stresses lead to a rapid growth of permanent strains and eventual failure. This has been used by some researchers (e.g. Lekarp 1997) for developing models similar to those based on the shakedown concept (Collins et al. 1993), suggesting that stabilizing and deteriorating behaviours of granular materials are

separated by a certain stress level, called the shakedown load. This stress level may be useful as a design criterion.

There is a clear need for developing more general and theoretically sound computational models for the prediction of the permanent strain response of granular materials. For routine analysis, however, simplicity of the modeling procedures is important and should be taken into consideration. This calls for more intensive research into this area in the coming years.

## 7.2 Correlation between static and dynamic loading tests

Several research workers have attempted to correlate repetitive loading and simple static loading test results. However, this approach has received mixed support since the behaviour of granular materials is generally regarded as very complex and the cyclic and static loadings do not necessarily induce the same structural response (Lekarp 1997 and Lekarp 1999).

Lentz and Baladi (1980 and 1981) used the static stress-strain results of sand samples to predict the cumulative permanent strain of identical samples tested under repeated loading conditions. They suggested that, provided the samples used in the static and repeated loading tests are identical in every respect, the permanent strain under repeated loading can be expressed by a constitutive expression of the form

$$\varepsilon_{1,p} = \varepsilon_{0.95S} \ln(1 - q/S)^{-0.15} + \left\{ \frac{a(q/S)}{[1 - b(q/S)]} \right\} \ln(N), \quad (\text{Eq. 7.2:1})$$

where

$\varepsilon_{1,p}$	=	accumulated permanent strain after N load cycles,
$\varepsilon_{0.95S}$	=	static strain at 95 % of static strength,
S	=	static strength,
q	=	deviator stress,
N	=	number of load cycles,
a, b	=	regression parameters.

Lentz and Baladi obtained a good correlation between measured and calculated values in their investigation. However, they pointed out that the equation was based on the results from a single sub grade material and additional research was needed. The validity of the equation above was later investigated by Sweere (1990) for both sands and granular base course materials and rejected.

Others have suggested that the amount of permanent strain is determined by the closeness of the applied repeated stresses to the static failure stress. Gerrard et al. (1975) suggested a new approach by relating the applied stress level to the static shear strength given by the Mohr-Coulomb static failure envelope. In this case, the static failure envelope is plotted on a normal Mohr-Coulomb diagram as shown in Figure 7.2:1a. The Mohr circles of the stresses applied in each repetitive loading test are then drawn and the permanent strains marked on the circles at the point nearest the static failure envelope.



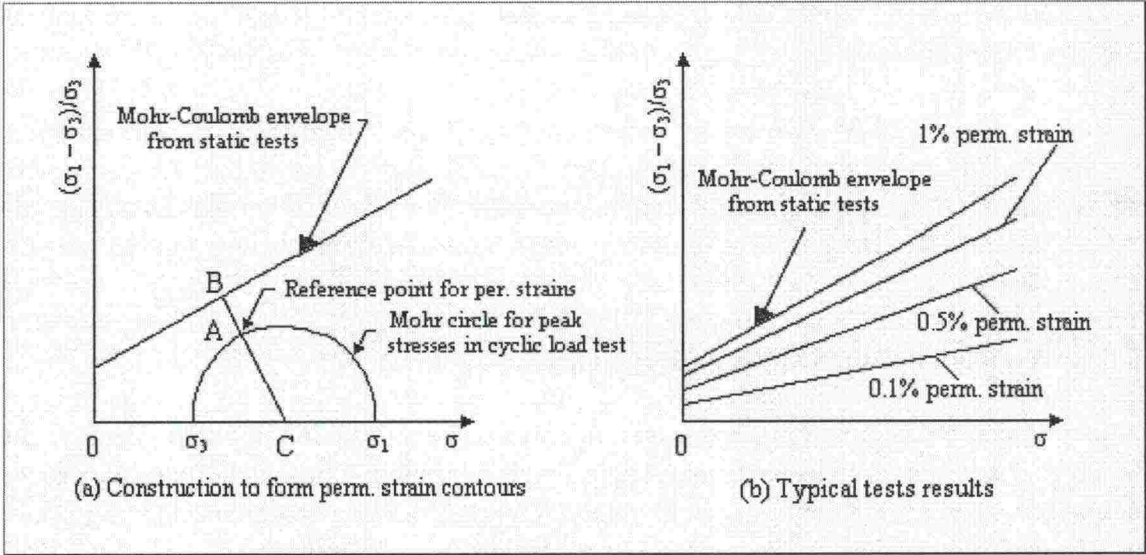


Figure 7.2:1 Mohr-Coulomb representation of permanent strains (Gerrard et al. 1975).

The applied stress level during the cyclic triaxial test is referred to by the ratio of the cyclic shear stress to the corresponding static shear strength, i.e. the ratio CA/CB in Figure 7.2:1a. Permanent strains after a certain number of load cycles are then drawn as contours for the corresponding ratios of the applied stress to the shear strength as shown in Figure 7.2:1b. Whilst this approach allows the relationship of permanent strain to stress to be estimated, it makes no prediction of the development with the number of load applications. Berret and Smith (1976) used this approach to express the permanent strain response of a crushed dolerite with a clay binder and reported good correlation with observations.

7.3 Correlation between resilient and plastic behaviour

Veverka (1979) studied both the resilient and plastic behaviour of granular materials and found a correlation between the two. He proposed a simple relationship between permanent and resilient strains given by

$$\epsilon_p(N) = a \cdot \epsilon_r \cdot N^b,$$
 (Eq. 7.3:1)

where

- $\epsilon_p(N)$  = accumulated permanent strain after N cycles,
- $\epsilon_r$  = resilient strain,
- N = number of load cycles,
- a, b = material parameters.

The equation suggested by Veverka is very simplistic, requiring considerable testing as the stress dependency is not explicit, and has not been confirmed by other researchers. Sweere (1990) studied the behaviour of unbound base course materials and sands and found no such relation as suggested in the equation above.

Other two models falling into this category are the VESYS model (Kenis 1978, Kenis and Wang 1997) and the Three-Parameter model (Tseng and Lytton 1989). The VESYS model is a linear model, whereas the Three-Parameter model is a non-linear model. Both the models were used by Park and Park (2005) in their work.

The VESYS model states that the ratio of vertical plastic strain per cycle,  $d\varepsilon^p / dN$ , to the resilient strain,  $\varepsilon_r$ , is an exponent function of the number of load cycles,  $N$ :

$$\frac{1}{\varepsilon_r} \frac{d\varepsilon^p}{dN} = \mu N^{-\alpha}, \quad (\text{Eq. 7.3:2})$$

where

$\varepsilon_p$	=	permanent deformation,
$\varepsilon_r$	=	elastic or resilient deformation,
$N$	=	number of load applications,
$\mu$	=	parameter representing the constant of proportionality of strains,
$\alpha$	=	parameter indicating the rate of decrease.

The non-linear Three-Parameter model states that the plastic strain has a limit,  $\varepsilon_0^p$ , and a logarithmic rate of work hardening. The model also states that the plastic strain increases with the number of load applications,  $N$ :

$$\varepsilon_p = \varepsilon_0^p \cdot e^{\left(\frac{\rho}{N}\right)^\beta}, \quad (\text{Eq. 7.3:3})$$

where

$\varepsilon_p$	=	permanent deformation,
$N$	=	number of load applications,
$\varepsilon_0^p, \rho, \beta$	=	material parameters.

#### 7.4 Permanent deformation moduli

Most of the work on modelling permanent deformation behaviour deals with axial and horizontal stresses and strains. According to Jouve et al. (1987), the build-up of plastic strain in granular materials is best treated by decomposing the stresses and strains into volumetric and shear components. In this way, in a similar manner to elasticity theories, plastic deformation moduli can be defined by

$$K_p(N) = \frac{p}{\varepsilon_{v,p}(N)}, \quad (\text{Eq. 7.4:1})$$

$$G_p(N) = \frac{q}{3\varepsilon_{s,p}(N)}, \quad (\text{Eq. 7.4:2})$$



where

$K_p(N)$	=	bulk modulus with respect to permanent deformation,
$G_p(N)$	=	shear modulus with respect to permanent deformation,
$\varepsilon_{v,p}(N)$	=	permanent volumetric strain for $N > 100$ ,
$\varepsilon_{s,p}(N)$	=	permanent shear strain for $N > 100$ ,
$p$	=	mean normal stress,
$q$	=	deviator stress,
$N$	=	number of load cycles.

The experimental results were then used to express the permanent deformation moduli as functions of the number of load cycles,  $N$ , by

$$G_p = \frac{A_2 \cdot \sqrt{N}}{\sqrt{N} + D_2}, \quad (\text{Eq. 7.4:3})$$

$$\frac{G_p}{K_p} = \frac{A_3 \cdot \sqrt{N}}{\sqrt{N} + D_3}, \quad (\text{Eq. 7.4:4})$$

in which the parameters  $A_2$ ,  $A_3$ ,  $D_2$ , and  $D_3$  are functions of the stress ratio  $q/p$ . The parameter  $A_2$  is dependent on the stress ratio by a relationship of the type  $a/x^b$  while  $A_3$ ,  $D_2$ , and  $D_3$  are connected linearly with this ratio.

No other verification of this shear-volumetric approach was found in the literature.

### 7.5 Permanent strain and number of cycles

Barksdale (1972) performed a comprehensive study of the behaviour of several different base course materials, using repeated load triaxial tests with  $10^5$  load applications. For a given stress condition, he found that the accumulated permanent axial strain was proportional to the logarithm of the number of load cycles and expressed the results by a log-normal expression of the form

$$\varepsilon_{1,p} = a + b \cdot \log(N), \quad (\text{Eq. 7.5:1})$$

where

$\varepsilon_{1,p}$	=	total permanent axial strain,
$N$	=	number of load cycles,
$a, b$	=	constants for a given deviator stress and confining pressure.

The long-term deformation behaviour of granular materials was also investigated by Sweere (1990) in a series of repeated load triaxial tests. After applying  $10^6$  load cycles, he observed that the log-normal approach did not fit his test results and suggested that for a large number of load repetitions a log-log approach should be employed and expressed the results (without an explicit stress dependency definition) by

$$\varepsilon_{l,p} = a \cdot N^b, \quad (\text{Eq. 7.5:2})$$

where

$\varepsilon_{l,p}$	=	accumulated permanent axial strain,
$N$	=	number of cycles,
$a, b$	=	regression parameters.

The applicability of the log-log model presented by Sweere was later challenged by Wolff and Visser (1994) who performed full-scale Heavy Vehicle Simulator (HVS) testing with several million load applications. They described the build-up of permanent deformation behaviour as consisting of two phases. In the HVS test results, an initial phase of up to 1.2 million load repetitions was observed with a rapid development of permanent deformation and a constantly diminishing rate of increase. During the second phase, permanent strain development seemed much slower and the rate of permanent strain development approached a constant value. The log-log model failed to give reliable estimates of the permanent strain at large numbers of load cycles, as shown in Figure 7.5:1. Wolff and Visser suggested an improved stress-strain model given by

$$\varepsilon_{l,p} = (m \cdot N + a) \cdot (1 - e^{-bN}), \quad (\text{Eq. 7.5:3})$$

in which  $a$ ,  $b$ , and  $m$  are regression parameters.

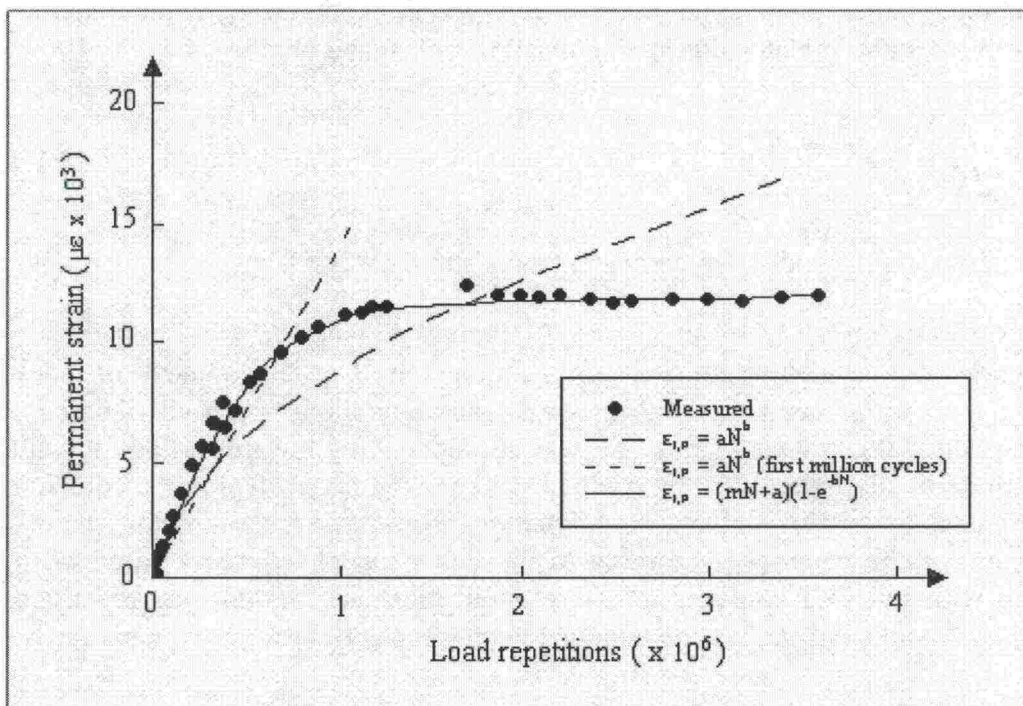


Figure 7.5:1 Different functions for modelling permanent strain development (Wolff and Visser 1994).

It is interesting to note that all the equations described above imply that the accumulation of permanent strain with the number of load cycles proceeds indefinitely. Yet, several other researchers (Chan 1990, Morgan 1966, Pappin 1979, Paute et al.



1993)) have reported that at least at certain levels of applied stresses the induced permanent strain eventually levels off, resulting in an equilibrium condition.

Khedr (1985) studied the permanent strain behaviour of a crushed limestone, using repeated load triaxial tests with variable confining pressure. He came to the conclusion that the rate of permanent strain accumulation decreases logarithmically with the number of load repetitions according to an expression of the form

$$\frac{\varepsilon_p}{N} = A \cdot N^{-m}, \quad (\text{Eq. 7.5:4})$$

where

$\varepsilon_p$	=	accumulated permanent strain,
$N$	=	number of load cycles,
$m$	=	material parameter,
$A$	=	material and stress-strain parameter given as a function of shear stress ratio and resilient modulus.

While Kehdr reported correlation coefficients quite close to unity for all tested samples, no other verification of this model was found in the literature.

Paute et al. (1988) suggested that permanent strain increases gradually towards an asymptotic value. They expressed the relationship between permanent axial strain (ignoring that accumulated during the first 100 cycles) and number of cycles by

$$\varepsilon_{1,p}^* = \frac{A \cdot \sqrt{N}}{\sqrt{N} + D}, \quad (\text{Eq. 7.5:5})$$

in which  $\varepsilon_{1,p}^*$  is the additional permanent axial strain for  $N > 100$ .

In a separate study, Paute et al. (1993) proposed a new approach to express the influence of the number of load applications on the development of permanent deformation in unbound granular materials. The modelling approach is based on the assumption that permanent strain increases asymptotically towards a limiting value that is mathematically represented by one of the regression parameters in the equation. This limit value, in turn, is defined as a function of stresses related to the static failure condition of the material. According to the Paute model, which is based on triaxial testing with repeated confining pressure application and 80,000 load repetitions, the total permanent axial strain in an unbound granular material can be expressed by

$$\varepsilon_{1,p}(N) = \varepsilon_{1,p}(100) + \varepsilon_{1,p}^*(N), \quad (\text{Eq. 7.5:6})$$

where

$\varepsilon_{1,p}(N)$	=	total permanent axial strain,
$\varepsilon_{1,p}(100)$	=	accumulated permanent axial strain during the first 100 cycles,

$\varepsilon_{1,p}^*(N)$  = additional permanent axial strain for  $N > 100$ ,  
 $N$  = number of cycles.

In this case, the accumulated permanent axial strain after an initial period of 100 cycles is given by

$$\varepsilon_{1,p}^*(N) = A \cdot \left[ 1 - \left( \frac{N}{100} \right)^{-B} \right], \quad (\text{Eq. 7.5:7})$$

where

$A, B$  = regression parameters.

When  $B$  is positive,  $\varepsilon_{1,p}^*(N)$  goes toward a limit value (equal to  $A$ ) as  $N$  increases toward infinity. The parameter  $A$  is therefore considered the limit value for accumulated permanent axial strain. For practical purposes the limit value  $A$  is chosen as the lesser of the value calculated in order for Equation 7.5:7 to fit the test data and the value of two times  $\varepsilon_{1,p}^*(20,000)$ .

This approach was later investigated by Lekarp et al. (1996). On the basis of the test results presented, the authors found the Paute model to be generally successful in predicting the gradual accumulation of permanent strain with number of load cycles. In fact, their experimental data showed a decrease in the accumulation rate of permanent strain with the number of load repetitions, and the total permanent axial strain, in most cases, seemed to increase asymptotically towards a limit value.

Lekarp (1997) and Lekarp and Dawson (1998) reported test results suggesting that the equation suggested by Paute et al. is valid only at low levels of applied stress.

Bonaquist and Witczak (1997) developed a constitutive model based on the flow theory of plasticity. As these authors explain, the constitutive models based on the flow theory are incremental. Further, the total strain increment consists of a reversible resilient strain increment and an irreversible plastic strain increment. The plastic strain increment is assumed to be a function of the current states of stress and strain, and the incremental change in stress. The flow theory describes plasticity by introducing three concepts. The first is the yield function, which separates the material response into elastic and plastic regions. The second is the flow rule, which specifies the incremental stress-strain relationship in the plastic region. Finally, the third concept is the hardening rule, which defines the change in the yield function caused by plastic strains. Bonaquist and Witczak combined the hierarchical approach proposed by Desai et al. (1986) and the bounding surface concept proposed by Mroz et al. (1978) and developed a new permanent strain model for repeated loading of soils and granular materials. The mathematical expression of this new model is given by

$$\varepsilon_{1,p} = \sum \varepsilon_N = \sum \frac{1}{N^h} \varepsilon_i, \quad (\text{Eq. 7.5:8})$$

where



$\varepsilon_{1,p}$	=	accumulated permanent strain after N load cycles,
$\varepsilon_N$	=	permanent strain for load cycle N,
$\varepsilon_i$	=	permanent strain for first load cycle ,
N	=	number of load cycles,
h	=	repeated load hardening parameter, a function of stress to strength ratio.

The basic hierarchical model defines the magnitude of the permanent strain occurring during the first cycle of loading. Under repeated loading, the cyclic hardening behaviour of the material is modeled by expressing the permanent strain for any load cycle as a power function of the permanent strain during the first load cycle. The accumulated permanent strain is then calculated as the sum of the permanent strain in each cycle.

Other researchers tried to relate the cyclic permanent deformations to both the applied stresses and the number of load cycles. Gidel et al. (2001) proposed a comprehensive relationship, taking into account the number of load cycles and the maximum applied cyclic stresses  $p_{\max}$ ,  $q_{\max}$ :

$$\varepsilon_1^p(N) = \varepsilon_{10}^p \cdot \left[ 1 - \left( \frac{N}{N_0} \right)^{-B} \right] \cdot \left[ \frac{L_{\max}}{p_a} \right]^n \cdot \frac{1}{\left( m + \frac{s}{p_{\max}} - \frac{q_{\max}}{p_{\max}} \right)}, \quad (\text{Eq. 7.5:9})$$

where

$L_{\max}$	=	$\sqrt{p_{\max}^2 + q_{\max}^2}$ ,
$p_a$	=	100 kPa,
$\varepsilon_{10}^p$ , B, n	=	model parameters,
m, s	=	parameters of the failure line of the material of equation $q = m \cdot p + s$ .

The empirical model of Gidel, which is written as the product of a function of the number of load cycles by a function of the maximum stresses, of the form  $\varepsilon_j^p(N) = f(N) \cdot g(p_{\max}, q_{\max})$  was studied by El abd et al. (2004). Comparisons with their experimental results indicated that the function g proposed by Gidel gave satisfactory predictions, but that the function f(N) could not be used, because the tests performed in their study did not show a complete stabilization of permanent strains. It was therefore replaced by the function proposed by Sweere (1990),  $f(N) = AN^B$  which gave better results.

This empirical model has been implemented in the program first because of its simplicity. One of its drawbacks is that it describes only the variation of permanent axial strains. Work is also under way to implement in the programme a more accurate, incremental, elasto-plastic model, also used at LCPC for unbound granular materials, the model of Chazallon (2000).

An example of prediction of results of a permanent deformation test with the model of Gidel is shown on Figure 7.5:2. The test is a variable confining pressure test (VCP) and includes 4 sequences of loading of 50000 cycles each, with the same stress ratio  $q/p = 2$  and with increasing stress amplitudes.

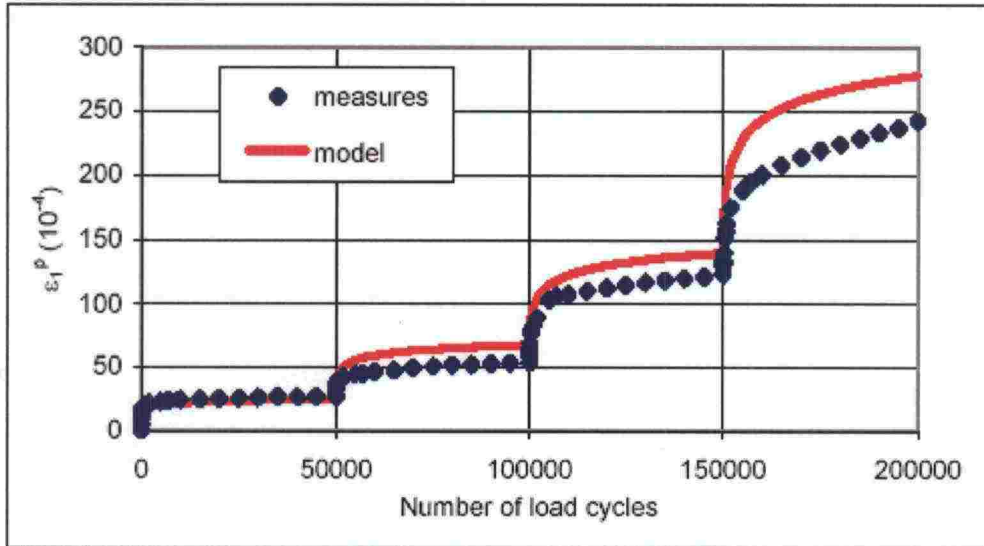


Figure 7.5:2 Example of the prediction of a permanent deformation test with the model of Gidel (El abd et al. 2005).

## 7.6 Permanent strain and its relationship to stresses

Previous research indicates that the stress level has a significant influence on the development of permanent deformation. Several researchers who carried out repeated load triaxial tests on unbound granular materials have found that permanent deformation behaviour is governed by some form of shear stress ratio.

Lashine et al. (1971) carried out repeated load triaxial tests on a crushed stone in partially saturated and drained conditions. They reported that after about 20,000 load applications, the measured permanent axial strain settled down to a constant level, which was directly related to the ratio of the applied stresses and given by

$$\varepsilon_f = 0.9 \cdot \frac{q_{\max}}{\sigma_3} \% , \quad (\text{Eq. 7.6:1})$$

where

$\varepsilon_f$	=	total accumulated permanent axial strain,
$q_{\max}$	=	maximum deviator stress,
$\sigma_3$	=	constant confining pressure.

Similar results have been reported by Brown and Hyde (1975) who studied the response of a crushed stone under cyclic triaxial condition with constant confining pressure. Brown and Hyde further stated that the same results could be obtained from tests with variable confining pressure if the mean value of the applied confining stress was used in the analysis.



Barksdale (1972) conducted repeated load triaxial tests on granular base course materials and used the results to relate the permanent axial strain to the ratio of repeated deviator stress and constant confining pressure. He used the general complex hyperbolic expression given by Duncan and Chang (1970) for static triaxial tests and found a close fit to the plastic stress-strain curves obtained from repeated load test results. Barksdale suggested that for a given number of load applications the variation of permanent axial strain with stresses can be expressed by

$$\varepsilon_a = \frac{(\sigma_1 - \sigma_3) / (K \cdot \sigma_3^n)}{1 - \left[ \frac{R_f \cdot (\sigma_1 - \sigma_3) / 2 \cdot (C \cdot \cos \varphi + \sigma_3 \cdot \sin \varphi)}{(1 - \sin \varphi)} \right]}, \quad (\text{Eq. 7.6:2})$$

where

$\varepsilon_a$	=	permanent axial strain,
$K \sigma_3^n$	=	relationship defining the initial tangent modulus as a function of confining pressure, $\sigma_3$ (K and n are constants),
$\sigma_1$	=	major principal stress,
$\sigma_1 - \sigma_3$	=	deviator stress,
C	=	cohesion,
$\varphi$	=	angle of internal friction,
$R_f$	=	a constant relating compressive strength to an asymptotic stress difference.

The hyperbolic expression of the equation above was reported to agree closely with the experimental results obtained from the triaxial tests with  $10^5$  load repetitions. Examples of the comparison between the calculated hyperbolic plastic stress-strain and the experimental curves reported by Barksdale are given in Figure 7.6:1.

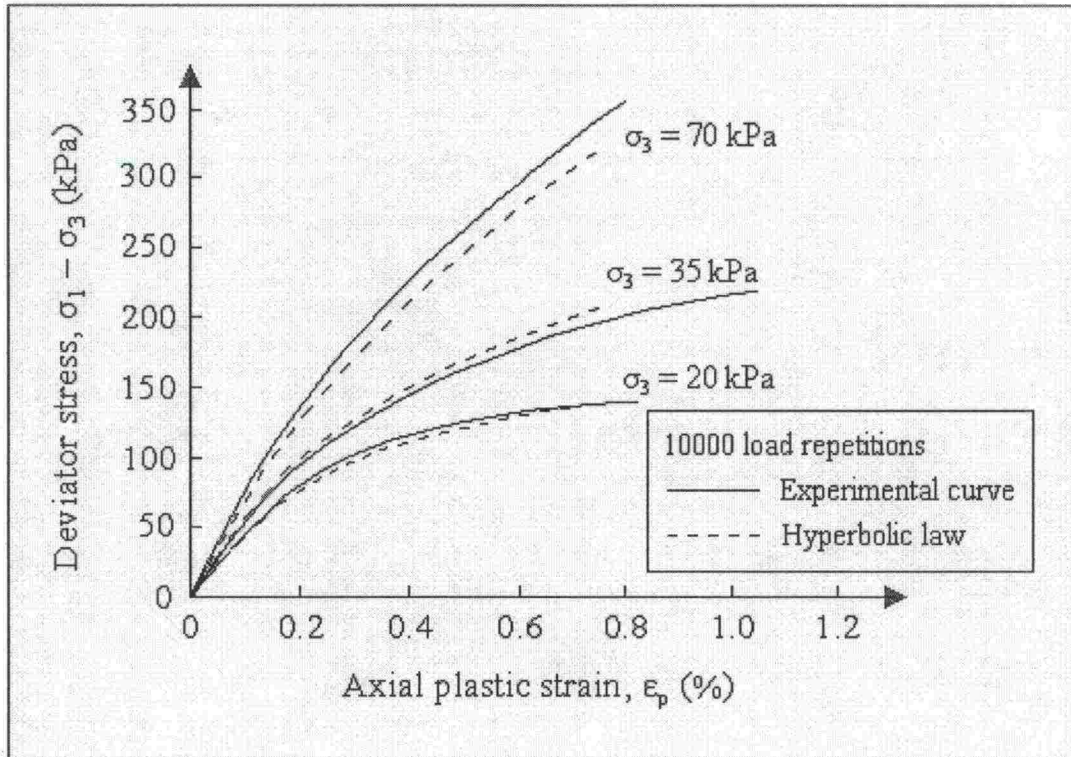


Figure 7.6:1 Comparison between the calculated hyperbolic plastic stress-strain curves with cyclic loading results (Barksdale 1972).

Pappin (1979) performed triaxial tests with variable confining pressure on specimens of a well-graded crushed limestone and calculated a shape factor for the variation of permanent strain with the number of load cycles. He then modified the applied stresses and expressed the total permanent shear strain rate as a function of the length of the stress path in the  $p$ - $q$  space and the applied shear stress ratio by an expression of the form

$$\varepsilon_p = (fnN) \cdot L \cdot \left( \frac{q^\circ}{p^\circ} \right)_{\max}^{2.8}, \quad (\text{Eq. 7.6:3})$$

where

$\varepsilon_p$	=	accumulated permanent strain,
$fnN$	=	shape factor,
$L$	=	stress path length,
$q^\circ$	=	modified deviator stress,
$p^\circ$	=	modified mean normal stress.

Pappin stated that, unless the material was stressed close to the static failure limit, large permanent strains did not occur. However, the mathematical expression suggested by Pappin is not asymptotic to failure and predicts finite permanent strain even at or beyond the static failure stress, as shown in Figure 7.6:2. Pappin found no satisfactory relationship for permanent volumetric strain.



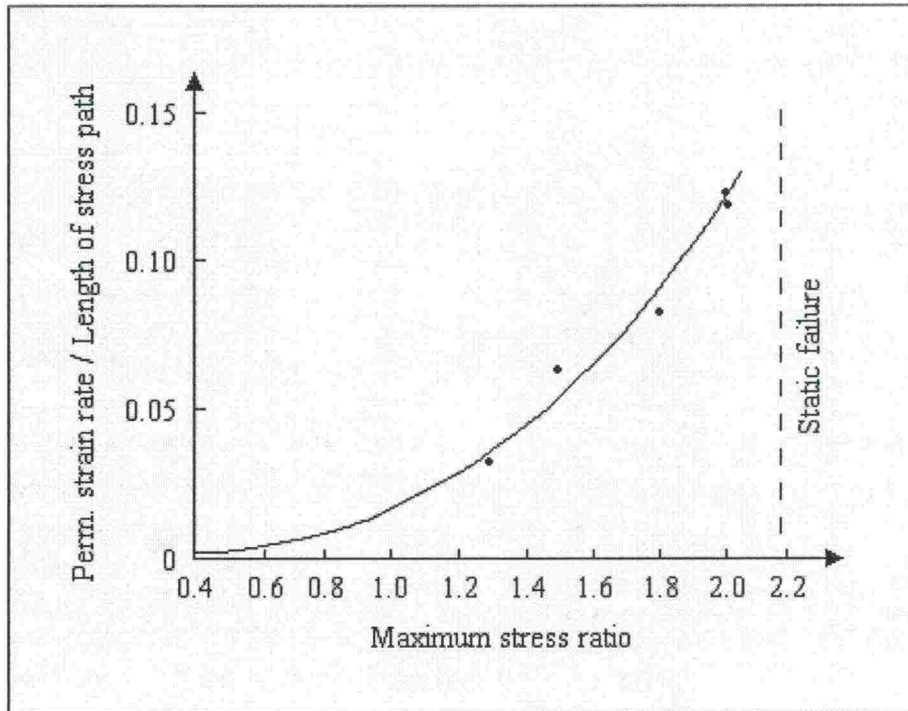


Figure 7.6:2 Relationship between permanent shear strain and the maximum stress ratio of Equation 7.6:3 (Pappin 1979).

Other researchers have reported that the amount of permanent strain is determined by how close the applied stresses are to the static failure stress. Barret and Smith (1976) and Raymond and Williams (1978) made use of the stress ratio  $q_{\max}/q_{\text{failure}}$ , in which  $q_{\max}$  is the maximum deviator stress and  $q_{\text{failure}}$  the deviator stress at failure (continuing along the same stress path), to characterize the results of permanent deformation tests. Thom (1988), on the other hand, suggested that permanent shear strain is better related to the stress ratio  $(q_{\max} - q_{\text{failure}})/q_{\max}$ .

Shaw (1980) attempted to apply the equation suggested by Pappin and to relate permanent shear strain to stress path length and stress ratio, but found it inadequate as the existence of stress paths with identical stress ratio but completely different strain rates was proven possible. He then modified the equation above by replacing the stress ratio variable with an expression which was a function of the minimum distance of the applied stress path from the static failure envelope. Shaw reported that, generally, a good correlation between predicted and experimental shear strains existed for the crushed limestone of nominally 1 mm particle sized used in his study. However, when the permanent strain programme conducted by Pappin on the well-graded material was reanalysed using the modified equation, the results were not satisfactory. Shaw, too, stated that modelling attempts on permanent volumetric strains were unsuccessful.

In a more recent study in France, Paute et al. (1993) defined a practical limit value to the maximum permanent axial strain, the A-value as described before, and suggested that it varies with the maximum shear stress ratio,  $q_{\max}/(p_{\max} + p^*)$ , according to a hyperbolic expression of the form given in the following equation:

$$A = \frac{\frac{q_{\max}}{(p_{\max} + p^*)}}{b \cdot \left[ m - \frac{q_{\max}}{(p_{\max} + p^*)} \right]}, \quad (\text{Eq. 7.6:4})$$

where

$A$	=	practical limit value for maximum permanent axial strain,
$q_{\max}$	=	maximum deviator stress,
$p_{\max}$	=	maximum mean normal stress,
$p^*$	=	stress parameter defined by intersection of the static failure line and the p-axis in p-q space,
$m$	=	slope of the static failure line,
$b$	=	regression parameter.

This hyperbolic relationship indicates that  $A$  increases when the maximum shear stress ratio increases, and that there is a limit value to the maximum shear stress ratio (equal to  $m$ ) for which  $A$  becomes infinite. This approach suggests that the static failure line, defined by parameters  $m$  and  $p^*$ , could be estimated using the results from cyclic triaxial tests, or derived experimentally from static failure tests. On the other hand, if the failure parameters are known, a single triaxial test is needed to define the expression. This approach was later investigated and highly questioned by Lekarp et al. (1996). On the basis of the test results presented, the authors do not support the model in suggesting that the variation in the total permanent strain could be given by comparing the stress state under repeated loading with the static failure condition. Lekarp's study shows clearly that the determination of the failure line according to the Paute model leads either to unreasonable values of failure parameters or a very low correlation between predicted and observed strain values. Dawson and Kolisoja (2004), on the other hand, found a clear indication that the total permanent axial strain is somehow related to the static failure line. This could be due to the fact that the tests were conducted under extreme conditions as far as the stress ratio  $q/q_{\max}$  is concerned.

The stresses induced in an embankment due to traffic are generally well below the static failure stress of the material as defined by the Mohr-Coulomb failure envelope. Wolff and Visser (1994) stated that the gradual increase of permanent deformation in unbound granular materials under repeated loading is caused by the elasto-plastic behaviour of the material and is not the same as the permanent deformation that occurs when a material is subjected to a stress beyond the static Mohr-Coulomb failure envelope. In other words, they did not support relating the development of permanent strain under dynamic loading to the ultimate shear strength of the material.

## 7.7 The Shakedown Theory

A certain theory of plasticity known as the "shakedown theory" (Sharp 1985) has been used for the response of structures subjected to repeated cyclic loading. This theory, which was first introduced by Melan (1936), has been widely applied to structures such as trusses, frames and plates. Some researchers (Sharp 1983, Sharp and Booker 1984, Raad et al. 1989, and Collins et al. 1993) suggested that shakedown principles could



also be employed for pavement design. The use of the shakedown concept is justified by Sharp and Booker with reference to the results of the AASHO Road Test, reported by Kent (1962), where in some cases the distress, as measured by serviceability index, was reported to stabilise after a finite number of load applications. More recently, parallel studies have been made into obtaining upper-bound (Collins and Boulbibalne 1998) and lower-bound (Yu and Hossain 1998) solutions for the shakedown load capacity of simple structures.

Unbound granular layers must be able to resist permanent deformations beyond a certain, tolerable, level. Essentially, only resilient deformations are permitted, while no permanent deformation, or only a small amount of it, can accumulate in the layer.

The performance of unbound granular materials in permanent strain repeated load triaxial tests is highly non-linear with respect to stress. There is a range of permanent strain responses to stress level and load cycles that cannot be described by a single equation. Several researchers (Morgan 1966, Dunlap 1966, Holubec 1969, Barksdale 1972, Chan 1990, Dawson and Wellner 1999, Sharp and Booker 1984, Theyse 2000, Werkmeister et al 2001, Werkmeister 2003, Sharp and Booker 1984) who related the magnitude of the accumulated permanent (plastic) strain to the shear stress level concluded that the resulting permanent strains at low levels of additional stress ratio,  $\Delta\sigma_1/\sigma_3$ , eventually reach an equilibrium state after the process of post-compaction stabilization (i.e. no further increase in permanent strain with an increasing number of loads) for which an asymptotic model is needed (Figure 7.7:1).

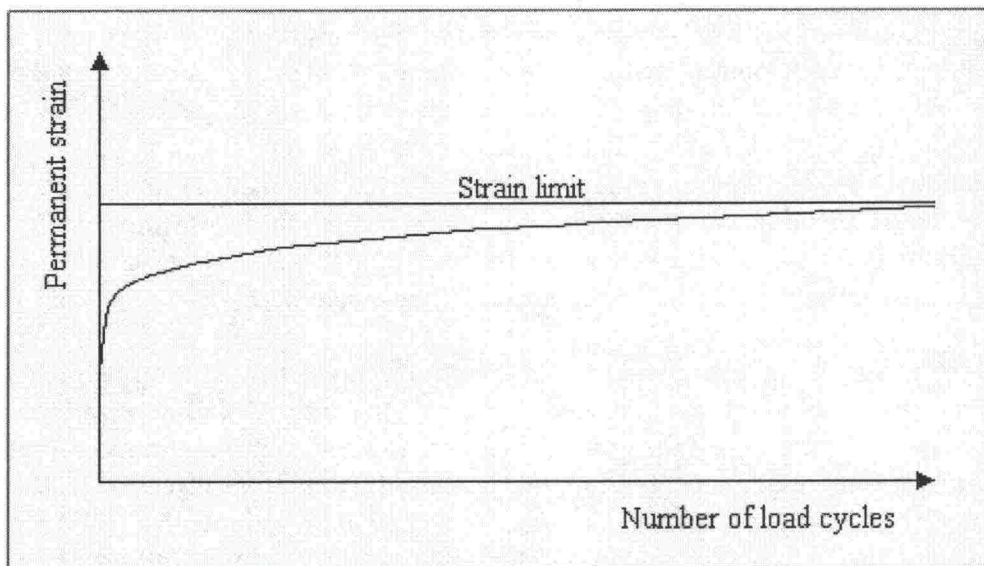


Figure 7.7:1 Permanent deformation behaviour at low stress levels (stable condition) (Werkmeister 2003).

At slightly higher levels of additional stress, however, permanent strains are likely to increase continually, for which a power or logarithmic model is needed. At even higher stresses, strains may increase rapidly, resulting in eventual failure (Figure 7.7:2).

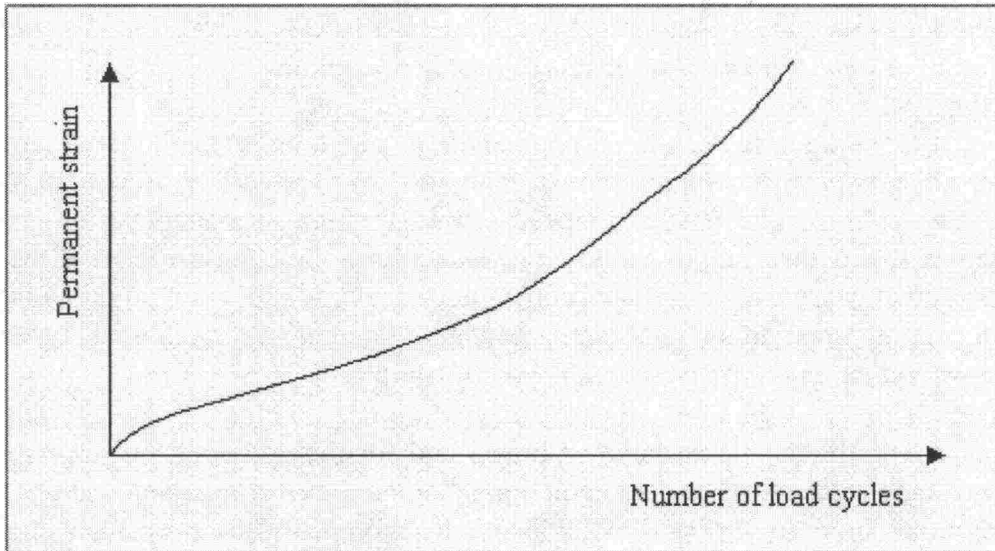


Figure 7.7:2 Permanent deformation behaviour at high stress levels (unstable condition) (Werkmeister 2003).

These ranges of behaviours are illustrated in Figure 7.7:3 and can be described using shakedown concept.

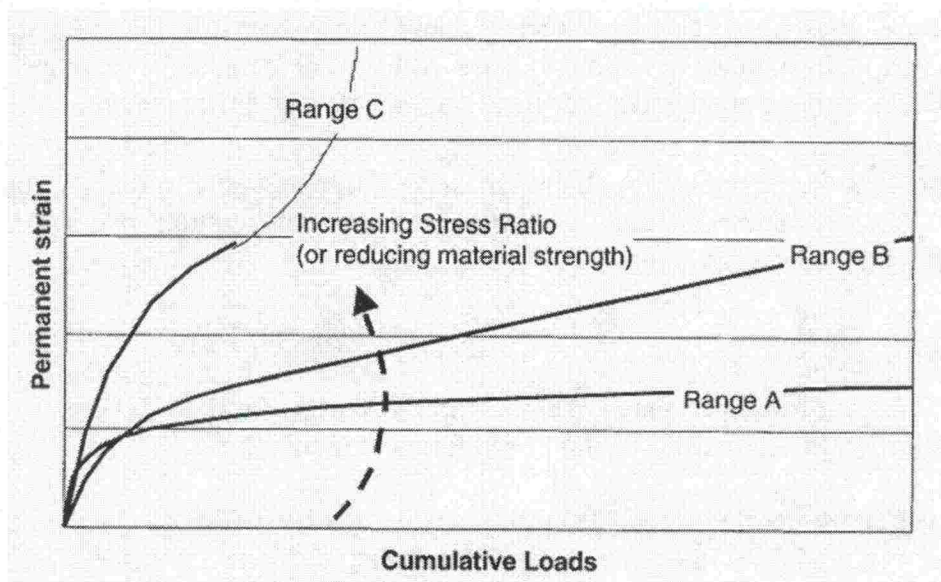


Figure 7.7:3 Shakedown range behaviour for permanent strain versus cumulative loading (Arnold et al. 2004).

The essence of a shakedown analysis is to determine the critical shakedown load. Materials operating above the critical shakedown load are predicted to exhibit an increased accumulation of permanent strains under long term repeated loading conditions that eventually lead to incremental collapse. Those materials operating at load levels below this critical shakedown load may exhibit some distress, but should settle down and reach an equilibrium state in which no further mechanical deterioration occurs (Werkmeister et al. 2001). The shakedown approach can be used to characterize the deformation behaviour of unbound granular materials under



repeated loads, although adaptations have to be made to allow for the particular response of unbound granular materials to repeated loading.

Dawson and Wellner (1999) have applied the shakedown concept to describe the observed behaviour of unbound granular materials in repeated load triaxial permanent strain tests. The results of their repeated load triaxial permanent strain tests are reported as shakedown ranges A, B, or C. This allows the determination of the stress conditions that cause the various shakedown ranges. These stress conditions can then be used to define the stress boundaries between the various behaviour types. The shakedown ranges are (Arnold et al. 2002 and Arnold et al. 2004):

- Range A is the plastic shakedown range, and for this to occur the response must show high strain rates per load cycle for a finite number of load applications during the initial compaction period. After the compaction period the permanent strain rate per load cycle decreases until the response becomes entirely resilient and no further permanent strain occurs. This range occurs at low stress levels.
- Range B is the plastic creep shakedown range, and initially the behaviour is like in Range A during the compaction period. After this time the permanent strain rate (permanent strain per load cycle) either decreases or remains constant. For the duration of the repeated load triaxial test the permanent strain is acceptable, and the response does not become entirely resilient. However, it is possible that if the repeated load triaxial test number of load cycles were increased to perhaps 2 million load cycles, the result could either be Range A or Range C (incremental collapse).
- Range C is the incremental collapse shakedown range where initially a compaction period may be observed, and after this time the permanent strain rate increases with increasing load cycles.

This implies that the maximum load level, which is associated with a resilient response, must be determined and subsequently not exceeded if uncontrolled permanent deformations are to be prevented. This has raised the possibility of the existence of a critical stress level separating the stable and failure conditions.

Maree (1982), as reported by Wolff (1992), studied the behaviour of gravel and crushed stone and reported that, under constant confinement, the specimens stabilized below a certain threshold of repeated deviator stress. Maree called this level of deviator stress the "maximum safe repeated deviator stress" and found a ratio of 0.58 to 0.98 between this stress and the failure strength of the material. He then developed a design procedure based on a failure model where the maximum stress in granular layers is kept below the maximum safe repeated deviator stress. This design procedure was criticized by Wolff (1992), who argued that it is simplistic and does not take into account the non-linear behaviour of unbound materials. Furthermore, the failure model of the procedure is based on the elastic theory of soils, while it is widely accepted that soils and granular materials behave elasto-plastically.

Shakedown limit calculations (critical shakedown load) can be used to predict whether a stable behaviour occurs in unbound granular layers or not (Werkmeister et al. 2001). The shakedown limits are strongly dependent on seasonal effects, especially moisture

content, which is the factor with the largest influence on the mechanical properties of unbound granular materials (Werkmeister et al. 2003).

Analysis of the results from many permanent deformation repeated load triaxial tests revealed an exponential relationship (Equation 7.7:1) between the applied stresses ( $\sigma_{1\max} / \sigma_3$ ) and the boundaries of the various deformation responses, i.e. between ranges A, B, and C, as shown in Figure 7.7:1

$$\sigma_{1\max} = \alpha \cdot \left( \frac{\sigma_{1\max}}{\sigma_3} \right)^\beta, \quad (\text{Eq. 7.7:1})$$

where

$\sigma_{1\max}$	=	peak axial stress	[kPa],
$\sigma_3$	=	cell pressure (minor principal stress)	[kPa],
$\alpha$	=	material parameter	[-],
$\beta$	=	material parameter (Werkmeister 2003)	[-],

With this equation it is possible to deduce the shakedown limit even at small stress ratios (Figure 7.7:4).

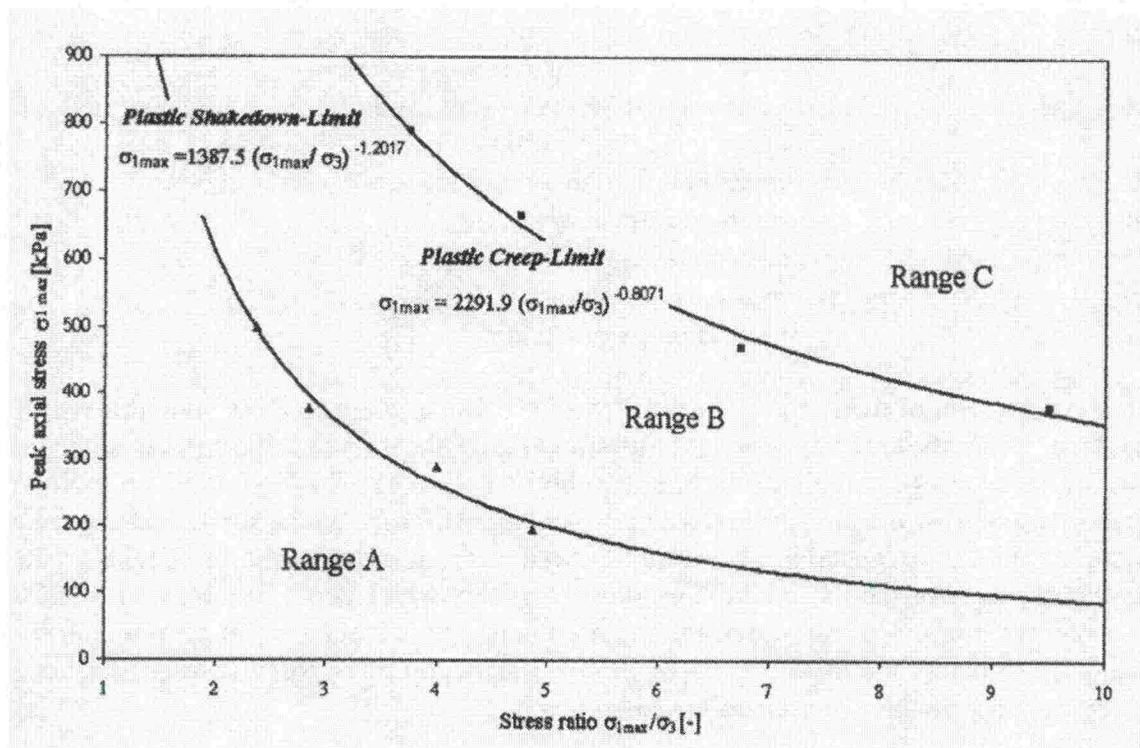


Figure 7.7:4 Stress ratio versus peak axial stress, Granodiorite at 4% water content (Werkmeister 2003 and Werkmeister et al. 2004).

As a practical method of defining the range boundaries (which define the stress conditions at which the type of permanent strain response changes) and, hence, the material parameters for Equation 7.7.1, repeated load triaxial tests are performed on a



series of specimens (or in a multi-stage test on one specimen) at increasing  $\sigma_{1\max} / \sigma_3$  ratios. When the plastic axial strain accumulated from 3,000 to 5,000 load applications is  $0.045 \cdot 10^{-3}$  strain, the range A-B boundary (the "Shakedown Limit") is reached. When this strain equals  $0.4 \cdot 10^{-3}$  strain, the range B-C boundary (the "Plastic Creep Limit") is reached (Werkmeister et al. 2003). As there is an associated change in resilient behaviour for materials operating in the various ranges, it is recommended that the observed response ranges A, B, and C form the basis for modeling permanent and resilient deformation behaviour. Thus, material laws have to be developed for each separate range. Range A is the most important range because stable behaviour will be the predominant requirement for unbound granular layers.

Also Lekarp and Dawson (1998) suggested that the shakedown approach might be employed in explaining the permanent deformation behaviour of unbound granular materials. By carrying out repeated load triaxial tests on different unbound granular materials, they defined a relationship between the accumulated permanent strain after a given number of load cycles, the stress path length, and the maximum shear – normal stress ratio according to the following equation:

$$\frac{\varepsilon_{1,p}(N_{ref})}{(L/p_0)} = a \left( \frac{q}{p} \right)_{\max}^b, \quad (\text{Eq. 7.7:2})$$

where

$\varepsilon_{1,p}(N_{ref})$	=	accumulated permanent axial strain after a given number of cycles $N_{ref}$ , $N_{ref} > 100$ ,
$L$	=	stress path length,
$p_0$	=	reference stress,
$q$	=	deviator stress,
$p$	=	mean normal stress,
$a, b$	=	regression parameters.

The comparison of measured and model-predicted values showed close similarities to the concept of the shakedown theory. At low stress ratios, the growth of permanent strain was shown to level off, resulting in a final equilibrium state. High stress ratios, on the other hand, resulted in a more progressive accumulation of permanent strain and a gradual deterioration of the material. The results showed clear indications that the state of gradual collapse occurs beyond a certain threshold stress ratio, which could be a form of so-called shakedown limit. Lekarp and Dawson, however, pointed out that more research was required to determine this shakedown limit. Analytically, it is possible to estimate peak stresses in a granular layer due to vehicular loading.

For finite element calculations of unbound granular layers, the prerequisite is a stress and load cycles dependent model for the permanent deformation behaviour of unbound granular materials. There are some stress dependent models available (Barksdale 1972, Lashine et al. 1971, Pappin 1979), but there are two models (Theyse-Model and Huurman-Model), which use the shakedown approach, in particular modelling the stable and unstable permanent behaviour, to model the permanent deformation behaviour of unbound granular materials as a function of the number of load cycles.

In his model, Theyse (2000) attempted to accommodate both the mechanism of permanent deformation and the effect of density and water content on the permanent deformation behaviour of unbound granular materials. He used the stress ratio  $R$  (which is a function of the deviator stress and the static failure load) as a stress parameter controlling the permanent deformation of the material. He determined the shear strength parameters  $c$  and  $\phi$  for different combinations of density and saturation and developed stress ratio regression models for both unstable (Equation 7.7:3) and stable (Equations 7.7:4 and 7.7:5) permanent deformation cases:

$$PD = Q \cdot e^{dN} - A \cdot e^{-bN} - Q + A, \quad (\text{Eq. 7.7:3})$$

$$PD = m \cdot N + a(1 - e^{-bN}), \quad (\text{Eq. 7.7:4})$$

$$PD = m \cdot N + \frac{c \cdot N}{\left[1 + \left(\frac{c \cdot N}{a}\right)^b\right]^{1/b}}, \quad (\text{Eq. 7.7:5})$$

where

PD	=	vertical permanent deformation	[mm],
N	=	number of load cycles	[-],
A, Q, a, b, c, d, m	=	stress dependent parameters	[-].

However, Theyse (2000) recognised that Equation 7.7:5 allowed a better control over the initial rate of the permanent deformation of the material under stable conditions than Equation 7.7:4.

Finally Theyse (2000) was able to develop stress ratio – load cycle (S-N) design models for different materials with a defined density and defined degrees of saturation (Figure 7.7:5).



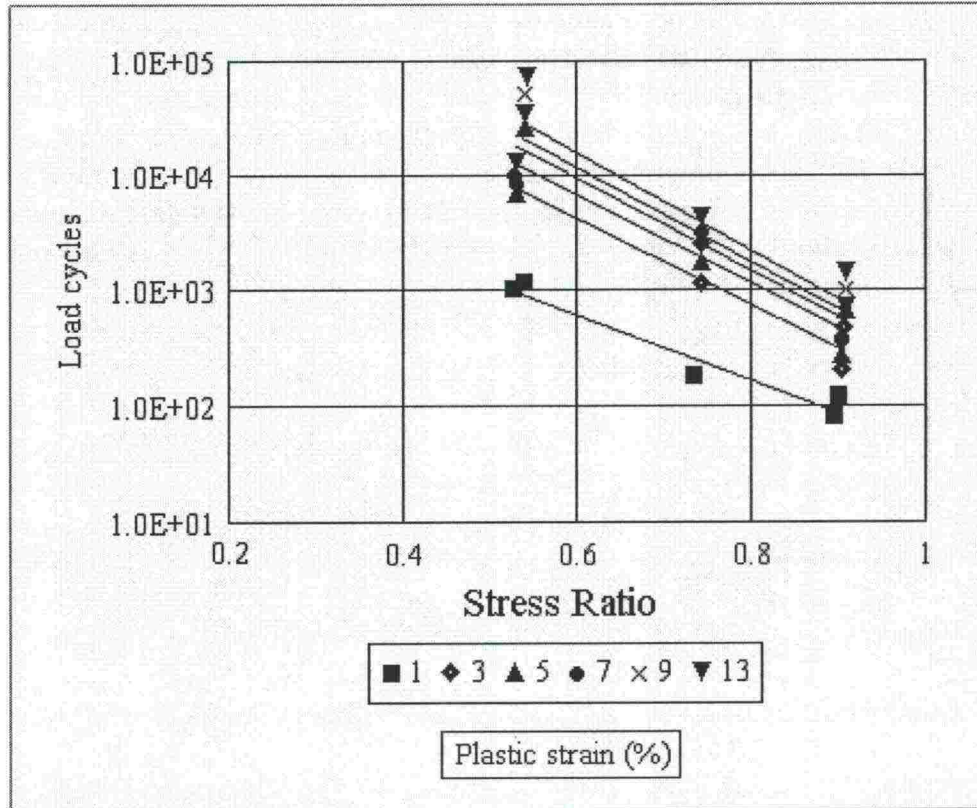


Figure 7.7:5 S-N design models for a clinker ash at 69 % and 75 % saturation for different plastic strains (Theyse 2000).

Similarly to what Sweere (1990) found for his laboratory test results, the log – log approach was also used by Huurman (1997) to describe the permanent strain development (axial and radial) in unbound granular layers under traffic (Equation 7.7:6):

$$\varepsilon_p(N) = A \cdot \left( \frac{N}{1000} \right)^B + C \cdot \left( e^{\frac{D \cdot N}{1000}} - 1 \right), \quad (\text{Eq. 7.7:6})$$

where

- $\varepsilon_p$  = permanent strain,
- $e$  = base of the natural logarithm,
- $N$  = number of load cycles.

Huurman used a repeated load triaxial apparatus to determine the permanent deformation behaviour of different sands.

The first term of the model describes a linear increase of permanent strain with  $N$  on  $\log(\varepsilon_p) - \log(N)$  scales. The parameter  $A$  gives the  $\varepsilon_p$  at 1,000 load cycles, and  $B$  gives the subsequent slope of  $\varepsilon_p$  with a rising number of load cycles. In the case of stable behaviour, the model parameters  $C$  and  $D$  are equal to zero. It is clear that the unstable behaviour at high stress levels cannot be described by the first term alone, because an exponential rather than linear increase of  $\varepsilon_p$  with  $N$  on the same  $\log(\varepsilon_p) - \log(N)$  scales

is observed. For this reason, a second term was added. This term is able to describe, through the values of the parameters C and D, at which value of N and at which value of  $\sigma_1/\sigma_{1,f}$  the incremental collapse will take place. The parameters A, B, C, and D are of course stress dependent. The following equations are used to describe the model parameters as a function of the  $\sigma_1/\sigma_{1,f}$  ratio:

$$A = a_1 \left( \frac{\sigma_1}{\sigma_{1,f}} \right)^{a_2}, \quad (\text{Eq. 7.7:7})$$

$$B = b_1 \left( \frac{\sigma_1}{\sigma_{1,f}} \right)^{b_2}, \quad (\text{Eq. 7.7:8})$$

$$C = c_1 \left( \frac{\sigma_1}{\sigma_{1,f}} \right)^{c_2}, \quad (\text{Eq. 7.7:9})$$

$$D = d_1 \left( \frac{\sigma_1}{\sigma_{1,f}} \right)^{d_2}, \quad (\text{Eq. 7.7:10})$$

where

$a_1, c_1$	=	model parameters	[%],
$a_2, b_1, b_2, c_2, d_1, d_2$	=	model parameters	[-],
N	=	number of load cycles	[-],
$\sigma_1$	=	major principal stress	[kPa],
$\sigma_{1,f}$	=	major principal failure stress according to Equation 7.7:10	[kPa],

$$\sigma_{1,f} = \frac{(1 + \sin \varphi) \cdot \sigma_{3,f} + 2 \cdot c \cdot \cos \varphi}{(1 - \sin \varphi)}, \quad (\text{Eq. 7.7:11})$$

where

$\sigma_{3,f}$	=	minor principal failure stress	[kPa],
$\varphi$	=	angle of internal friction	[°],
c	=	cohesion	[kPa].

The magnitude of the stress to which an unbound granular material is subjected in relation to its maximum failure stress has been found to greatly influence its response to permanent strains (Werkmeister 2003).

From repeated load triaxial test results on several materials, Van Niekerk (2002) found that this model provides a good description of the measured permanent strains.



In conclusion, both Huurman (1997) and Theyse (2000) have attempted to correlate repeated and static failure load test results. This approach has received mixed support because the deformation behaviour of unbound granular materials is regarded as very complex and repeated and static load tests do not necessarily induce the same structural response. For this reason it is convenient to develop an empirical model that is dependent on the number of load cycles and the stresses  $\sigma_1$  and  $\sigma_3$ . However, the Huurman-Model should form the basis for further investigations on the permanent deformation behaviour of unbound granular materials (Werkmeister 2003). This model uses one simple equation with few parameters only to describe the permanent deformation behaviour in the stable range as well as in the unstable range.

## 8 REPEATED LOAD TRIAXIAL APPARATUS

### 8.1 Introduction

In order to study the behaviour of unbound granular materials, it is necessary to create testing conditions as close as possible to those occurring in the field. The stress conditions have been recognised as being one of the most important parameters affecting the behaviour of granular materials. Therefore, most research into this problem uses devices which can simulate the stresses occurring under train traffic loading.

Several simple in-situ testing devices (e.g. static or dynamic loading tests) have been developed over the past years. In the most simplistic approach, the results from static loading tests, such as the California Bearing Ratio Test and the Static Plate Bearing Test, are used to predict the material behaviour under dynamic loading conditions such as those in a railway embankment. Devices such as the Clegg Hammer, the Falling Weight Deflectometer (FWD), and the Dynamic Plate Bearing Test employ dynamic loads which are closer to the actual traffic loading situations. The most realistic approach, however, involves those devices that are able to reproduce the repeated stresses caused by traffic, using an accelerated time scale. Full-scale test facilities, such as the Heavy Vehicle Simulator (HVS) developed at the end of the 1960s in the Republic of South Africa and the Accelerated Loading Facility (ALF) in use in Australia since early 1984, have been effectively used for this purpose.

There are, however, a number of drawbacks involving in situ testing techniques. Due to the inseparable layered nature of a railway embankment and its sub grade, the parameters measured during an in-situ test are often 'construction parameters' rather than 'material parameters' (Lekarp 1997). Furthermore, it is extremely difficult, if not impossible, to isolate and study the influence of different factors affecting material behaviour (Lekarp 1997). Also, the more sophisticated field testing methods are quite expensive in terms of both initial and operating costs. Due to costs, full-scale testing can be carried out only on a very limited number of combinations of materials, climatic conditions, and loading conditions (Werkmeister 2003). In-service embankments experience a much wider spectrum of these variables than the one that it is possible to impose in full-scale testing. For the reasons mentioned, it is normally desirable to study material behaviour by simulating field conditions in a laboratory environment.

By its nature, carefully designed laboratory tests enable the control of many factors which affect the behaviour of the tested material. For unbound granular materials, the repeated load triaxial test is currently the main test used to study the stress-strain behaviour under different conditions of grading, moisture content, and density. The main objective is to determine the deformation behaviour of unbound granular materials under conditions that simulate the physical conditions and stress states of these materials in the real structure under moving loads.

### 8.2 Repeated load triaxial test: advantages and drawbacks

Two types of repeated load tests are usually conducted, being either a resilient or a permanent deformation test (Arnold 2005). Triaxial testing is a research tool with the



aim to simulate as closely as possible the range of conditions that will be experienced in a railway embankment as well as in other earth structures.

The repeated load triaxial apparatus applies repetitive loads on cylindrical test specimens for a range of specified stress conditions. The output is deformation (shortening of the cylindrical specimen) versus number of load cycles (usually 50,000) for a particular set of stress conditions (Arnold 2005). Multi-stage repeated load triaxial tests are used to predict deformation behaviour for a range of stress conditions.

The triaxial test has several advantages. It is relatively inexpensive compared to field tests and it requires less time. In addition, stresses can be applied to a specimen as pulses that simulate those applied to an element in an actual railway embankment. Finally, the combination of the vertical and horizontal stresses can be reproduced during repeated load triaxial tests when both deviator and confining stresses are cycled (Lekarp 1997).

However, the repeated load triaxial test is limited in that only the principal stresses can be directly applied to a test specimen. In addition, because of the axisymmetric arrangement, two of the principal stresses must necessarily be equal. Furthermore, during the passage of a train load the principal stresses axes within the embankment gradually rotate due to shear stress reversal, whereas in a triaxial apparatus they are always the same. In effect, the stresses applied in a triaxial test are equivalent to the in-situ principal stresses directly beneath the centerline of the train wheel load.

The reversal of shear stress requires an apparatus which can apply this mode of stress directly to the boundary of the specimen. One method is the use of a hollow cylinder apparatus, in which a repeated torsion can be applied to a hollow thin-walled cylinder. If the hollow cylinder is at the same time subjected to an axial and a lateral stress over both the inner and outer cylinder faces, then the stress conditions imposed on an element of material will simulate the in situ stress conditions. However, the major limitation of the hollow cylinder test is that it can only accommodate scaled down material samples, and the effects of using scaled down samples are not always known (Lekarp 1997).

### **8.3 Constant (CCP) and variable (VCP) confining pressure tests**

In order to provide a reasonable simulation of traffic-type loading using triaxial equipment, the loading system should be able to cycle both the vertical (deviator) stress and the confining pressure in phase and at levels and frequencies corresponding to the actual field conditions.

There are two test methodologies for conducting repeated load triaxial tests: the constant confining pressure test (CCP) and the variable confining pressure test (VCP). Traffic-type cyclic loading cannot be ideally simulated in the laboratory by CCP type repeated load triaxial tests, which have been commonly used in Europe since the 1970s. In CCP tests, it is only possible to apply one constant stress path at each confining pressure level. Instead, the VCP type repeated load triaxial test offers the capability to apply a wide combination of stress paths by pulsing both the confining pressure and vertical deviator stress. Such stress path loading tests better simulate actual field

conditions, since in an embankment structure the confining stresses acting on unbound granular materials are cyclic in nature. Because of this, the VCP type repeated load triaxial test (specimen size 150 x 300 mm) is a practicable testing method for unbound granular materials and has been increasingly used over the past years.

The repeated load triaxial testing equipment commonly used nowadays is able to cycle the deviator stress at different frequencies. In practice, the triaxial apparatus is an axisymmetric apparatus in which a cylindrical and normally laboratory-compacted specimen is subjected to two independently controlled stress components. The first stress component is an all-around isotropic pressure applied through a confining fluid, confining air, or by internal partial vacuum. The second stress component is a vertically acting deviator, which is stress applied through loading platens at the top and bottom of the specimen.

However, many laboratories still use constant confining pressure (CCP), since this makes testing more cost-effective compared to the application of variable confining pressure (VCP). This is, in fact, a significant technical simplification and its effect on the material response is not properly understood.

Allen (1973) and Allen and Thompson (1974) compared the results obtained from these two types of test and reported generally higher values of resilient modulus computed from CCP test data. The magnitude of the difference was itself non-constant and varied with the stress level. These studies also showed that the CCP tests resulted in larger lateral deformations and higher values of Poisson's ratio. Brown and Hyde (1975), on the other hand, suggested that VCP and CCP tests yield the same values of resilient modulus, provided that the confining pressure in the CCP test is equal to the mean value of the pressure used in the VCP test. As for the value of Poisson's ratio, Brown and Hyde reported considerable differences, as the VCP tests yielded decreasing Poisson's ratio for increasing ratios of deviator stress to confining pressure, while the CCP tests showed the opposite.

#### **8.4 General guidelines on specimen size**

Although the triaxial testing devices presently available are all based on the same principles, the complexities involving their application, especially for testing coarse materials, together with the differences in technical facilities have led to the development of a large variety of equipments and methods for specimen preparation, instrumentation, and loading procedures. Prior to setting up a new testing facility of this kind, it is essential to thoroughly contemplate the type of material to be tested and the level of accuracy desired.

For the study of unbound granular materials using triaxial testing, the specimen must be large enough so as not to have any influence on the behaviour of the material. It is generally accepted that the size of the specimen must be a function of the maximum particle size in the unbound granular material to be tested. Vallergera et al. (1975) suggested that a specimen diameter of 3 – 4 times the maximum particle size is required. In the testing procedure outlined by the Transportation Research Board (TRB) in 1975 a minimum ratio of 4 – 5 times was recommended. The specification released



by the American Association of Highway and Transportation Officials (AASHTO) in 1986 prescribed a minimum specimen diameter 6 times the largest particle size.

Taylor (1971) investigated the effect of specimen height to diameter ratios in repeated load triaxial testing results. He concluded that if the height of the specimen is about twice its diameter, the significance of size effect on the measurements would be negligible. Similar conclusions were drawn by Dehlen (1969) based on a theoretical study on the same issue. The publications from TRB (1975) and AASHTO (1986) also specified a minimum specimen height of twice the diameter.

According to the draft of European Standard prEN 13286-7 (2002), the tested specimen must have a diameter larger than 5 times the maximum particle size of the material and a height twice the diameter ( $\pm 2\%$ ). Most repeated load triaxial testing facilities currently available have specimen diameters of 300 mm, 150 mm, or even less (Lekarp 1999).

### **8.5 General guidelines on instrumentation**

Monitoring the response of the specimen during triaxial testing requires proper instrumentation for measuring both the applied stresses and the induced strains. The tests can be performed more efficiently if the measuring devices are kept outside the pressure vessel. In this way, the deformation transducers would be constantly accessible for possible adjustments. Furthermore, water could be used as the confining medium, being an inexpensive alternative to special hydraulic oils such as silicone oil. However, keeping the devices outside the pressure cell could lead to erroneous measurements due to the impact of factors such as the stiffness of the loading system, the friction between the specimen and the loading platens, and the friction between the loading piston and the cell bush (Lekarp 1999). For reasons such as these, it is generally considered important to place the measuring devices inside the triaxial cell and in direct contact with the specimen. In such cases, however, the devices are directly exposed to the confining medium, and it is essential to make sure that such exposure does not affect the electrical output signals. Silicone oil is normally recommended as the confining medium since it is chemically inert and is an excellent electrical insulator. It is also important for the load cell to be constructed so that its output signal is not influenced by the applied confining pressure (Lekarp 1999).

When measuring the vertical deformation of the specimen, the influence of the deformation due to bedding of the loading platens and the friction between the specimen and these platens could be eliminated if the displacement is measured between two fixed points on the specimen itself. In regard to the monitoring of the lateral deformation, the technique employed should provide a reliable measurement of the average displacement along the circumference of the specimen (Lekarp 1999).

### **8.6 University of Nottingham's repeated load triaxial apparatus**

#### ***8.6.1 Introduction***

A repeated load triaxial apparatus has been developed at the University of Nottingham with some modifications made over the years. The apparatus in question has been used

for projects on the behaviour of unbound granular materials by many researchers, among whom Lekarp in 1997 and Werkmeister in 2003. The principal components of the repeated load triaxial apparatus analysed in this section are illustrated in Figure 8.6.1:1.

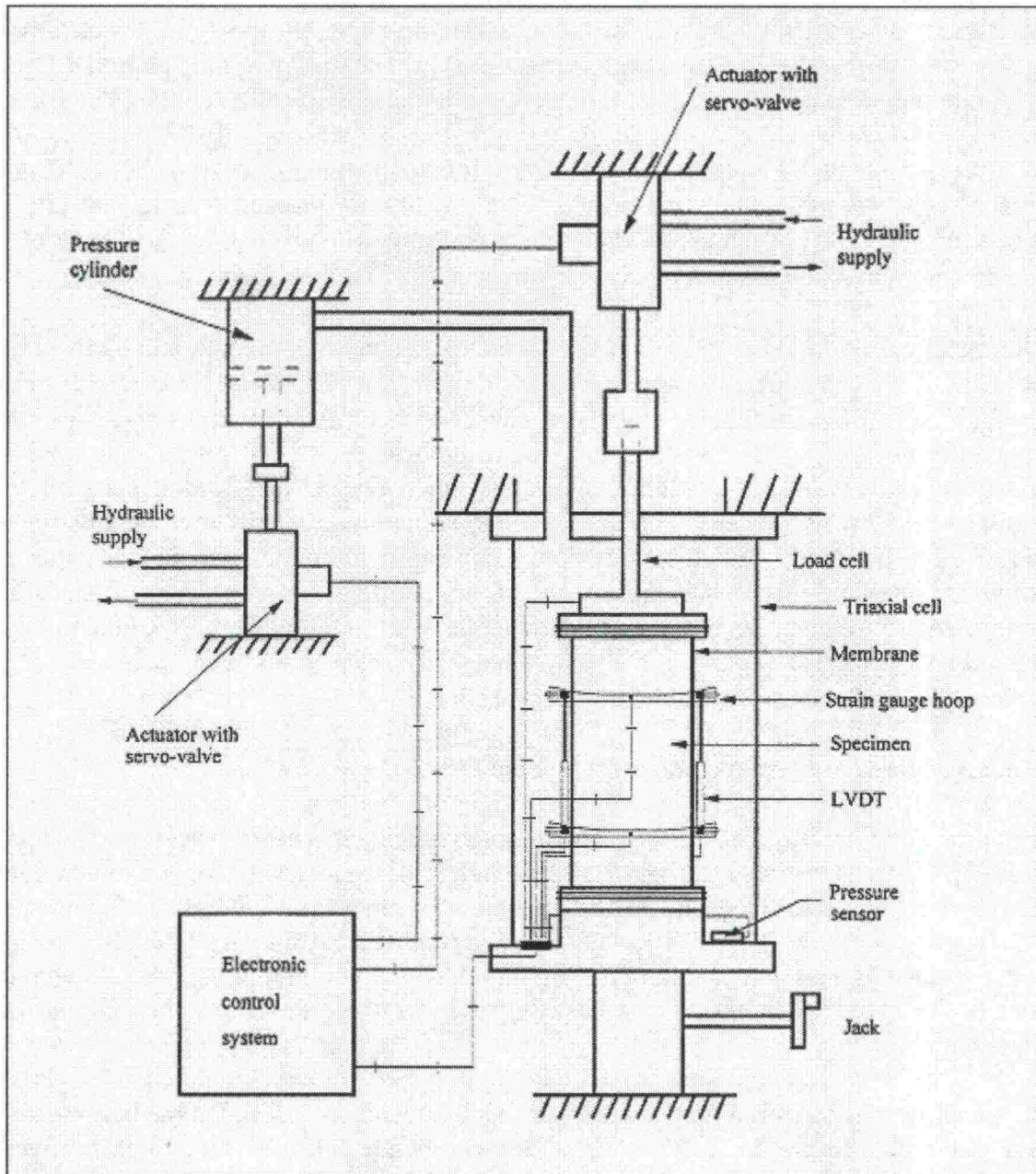


Figure 8.6.1:1 Repeated load triaxial apparatus (Boyce 1976).

The loading system is a hydraulic servo-controlled system, in which both axial load and confining pressure are applied to the specimen in a triaxial cell by hydraulic actuators. The axial load is applied by a 50.8 mm diameter hydraulic actuator, capable of applying a deviator stress of up to 1,200 kPa on samples of 150 mm diameter and 300 mm height, at frequencies up to 16 Hz. The axial load is monitored continuously by a load cell. The electronic control system compares the output from the load cell with a load applied to the servo-valve on the actuator in order to adjust the applied load to that required.



Confining pressure is applied to the specimen through a confining medium in the triaxial cell which is made of an aluminium alloy capable of withstanding an internal pressure of 1,000 kPa. For tests with constant confining pressure, air is used as the pressurising medium. Silicone oil is used as the confining medium for tests with variable confining pressure. This type of oil with relatively low viscosity is chemically inert and is an excellent electrical insulator, having no effect on strain gauge transducers and linear variable differential transformers (LVDT). The oil pressure is controlled by a 25.4 mm diameter hydraulic actuator operating a 127 mm diameter pressure cylinder.

The pressure cylinder is connected by a large flexible tube to a port at the top of the triaxial cell. The equipment can provide cyclic confining pressures up to 400 kPa at frequencies up to 2 Hz. The applied confining pressure is controlled, in a similar way to the deviator stress, by the output of a pressure sensor placed inside the triaxial cell.

During a repeated load triaxial test, the specimen is subjected to both axial and radial stresses. The axial stress is applied through rigid end-platens and the radial stress through a flexible membrane enclosing the specimen. As the test progresses, the specimen undergoes both vertical and radial deformations. The radial deformation, however, is restrained at the ends of the specimen due to the friction between the particles and the end-platens. This causes the specimen to gradually take on a barrelled shape with the largest radial deformation occurring in the middle section. In order to minimise the end-effects (friction between the particles and the end-platens), all deformations are measured within the middle one fourth section of the specimen. A set of two LVDTs at 1/4 and 3/4 of the sample height and two strain gauge hoops are used to measure axial and radial deformations respectively.

#### ***8.6.2 Repeated load triaxial tests by Lekarp (1997)***

The procedure used by Lekarp for preparing each specimen is as follows. Four "location studs" (two on each side at 1/3 and 2/3 of the height) were attached to a latex membrane of 0.3 mm thickness. One end of the membrane was secured around the lower end-platen using a rubber O-ring. Vacuum was then used to hold the membrane against the inner surface of a 2-piece aluminium split mould, which was placed over the lower end-platen. These were then placed on a vibrating table, to which the compaction mould was clamped.

A geotextile filter covered with a thin layer of silicone oil was placed at the bottom. Material was then placed in the mould and compacted in 5 layers (50 mm each, except the first and the last layers which were 75 mm). Each layer was compacted to the desired density using the vibrating table and a KANGO vibrating hammer. Another filter covered with silicone oil was then placed on top and the top end-platen positioned. After placing the mould in the triaxial equipment, a vacuum was applied to the specimen via the end-platens before removing the mould.

In most cases, the first membrane was damaged during the compaction process. In order to seal the specimen, a second membrane (0.3 mm thick) was placed over the first one held at each end to the end-platen by a rubber O-ring. Small holes were made in the outer membrane at the position of each location stud, through which rods were secured to the studs and sealed by O-rings. The LVDTs and the strain gauge hoops were finally

put in place, before the triaxial cell was positioned and tightened. Figure 8.6.2:1 illustrates the compaction arrangement and the attachment of the location studs to the membranes.

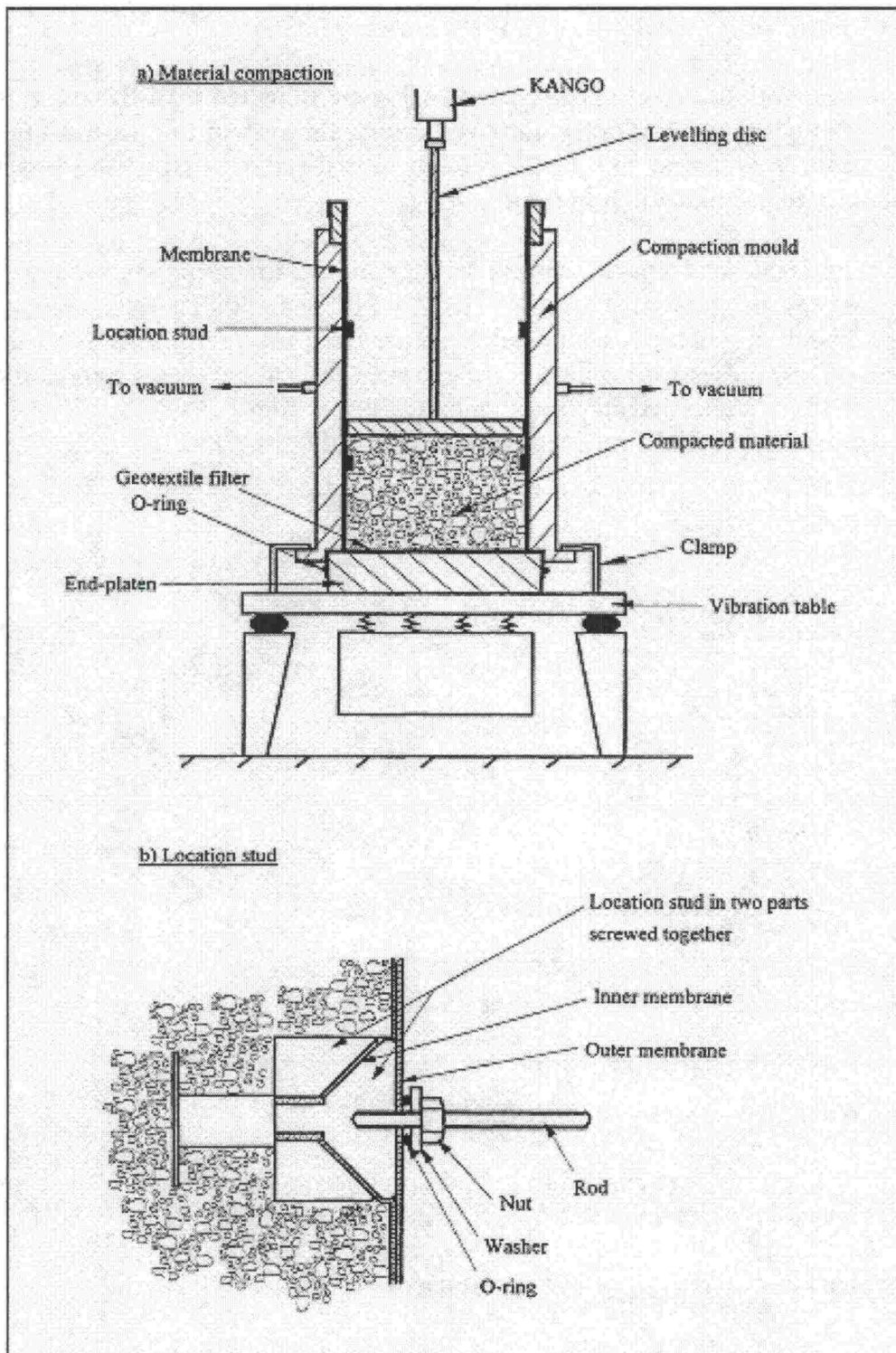


Figure 8.6.2:1 Sample preparation (Boyce 1976).

It should be mentioned that the latex membrane was found inadequate for use as the outer membrane in the case of coarse aggregates. It was noticed that compacting coarse materials sometimes left empty gaps (of different depths) between adjacent large



particles. During the test, the latex membrane was pressed into the gaps and punctured due to over-stretching. A thicker, neoprene membrane was therefore used as the outer membrane and found to be quite satisfactory.

### 8.6.3 Repeated load triaxial tests by Werkmeister (2003)

In her investigations, Werkmeister used CCP-type repeated load triaxial tests and measured only the vertical deformations, because the system to measure the radial deformations was found not to be reliable. Furthermore fixing the hoops was occasionally found to disturb the studs.

For the repeated load triaxial tests the constant confining pressure (applied via pneumatic pressure) was set at levels of 70, 140, 210, and 280 kPa. After the confining pressure had been reached, additional dynamic (frequency = 5 Hz) vertical stress (deviator stress) pulses were applied. The repeated load triaxial tests were carried out with axial stress pulses having stress ratios ( $\sigma_D / \sigma_c$ ) from 0.5 to 1.1. Figure 8.6.3:1 shows the test stress paths for the repeated load triaxial testing.

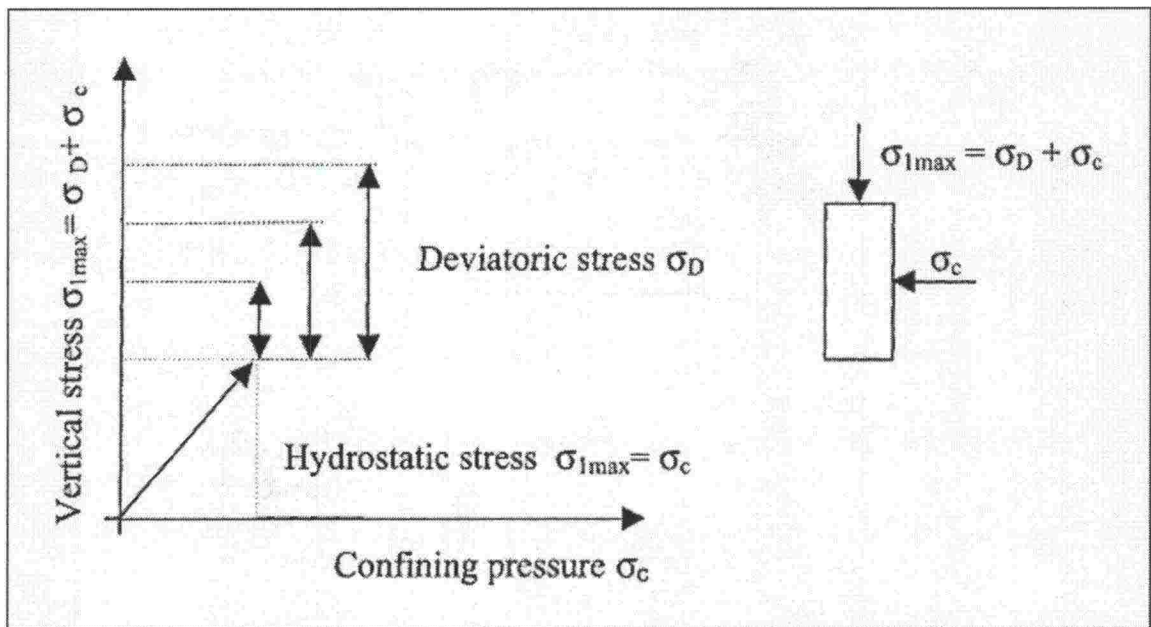


Figure 8.6.3:1 Stress paths (Werkmeister 2003).

Half sine load pulses were applied to the test specimens with each load cycle consisting of a 50 % loading phase followed by a 50 % neutral phase (Figure 8.6.3:2). To define the peaks exactly, 80 reading points were measured (64 reading points during the loading phase and 16 reading points during the neutral phase).

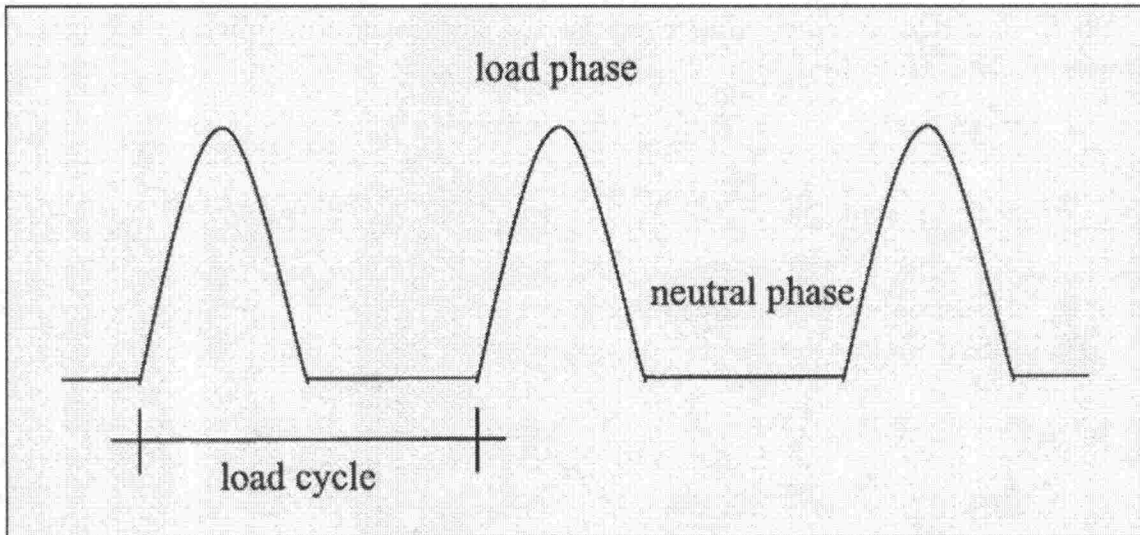


Figure 8.6.3:2 Phases of a load cycle (Werkmeister 2003).

The total strains consisted of three parts: hydrostatic strains (permanent), resilient strains, and permanent strains (Figure 8.6.3:3).

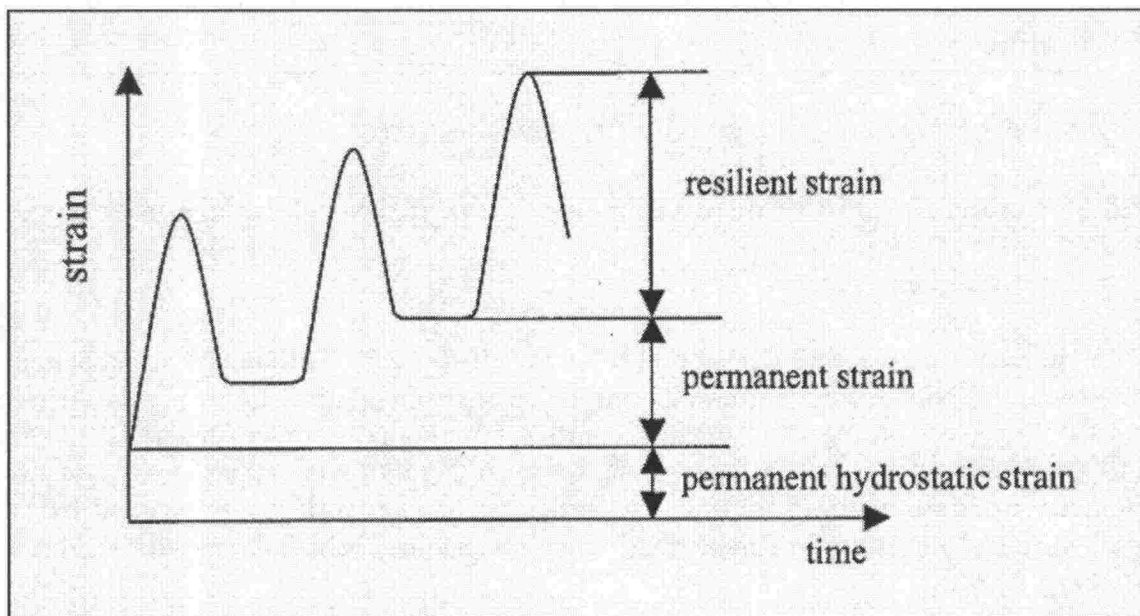


Figure 8.6.3:3 Components of strain (Werkmeister 2003).

The permanent hydrostatic strain part was not considered, because the magnitudes of these strains were too low for an accurate measurement.

A computer with the control software ATS (Automated Testing System) was used to register the test measurements (vertical deformations) and other test parameters such as the number of load cycles, time, confining pressure, and deviatoric stress. However, the analysis of the test results was not carried out by this ATS software, because it takes the peaks of the deformation line as the basis of the test results. Instead, the analysis method adopted by Werkmeister in her investigation was developed by Kiehne (2001). This



method takes the average value over the five maximum and minimum values of deformations (Figure 8.6.3:4).

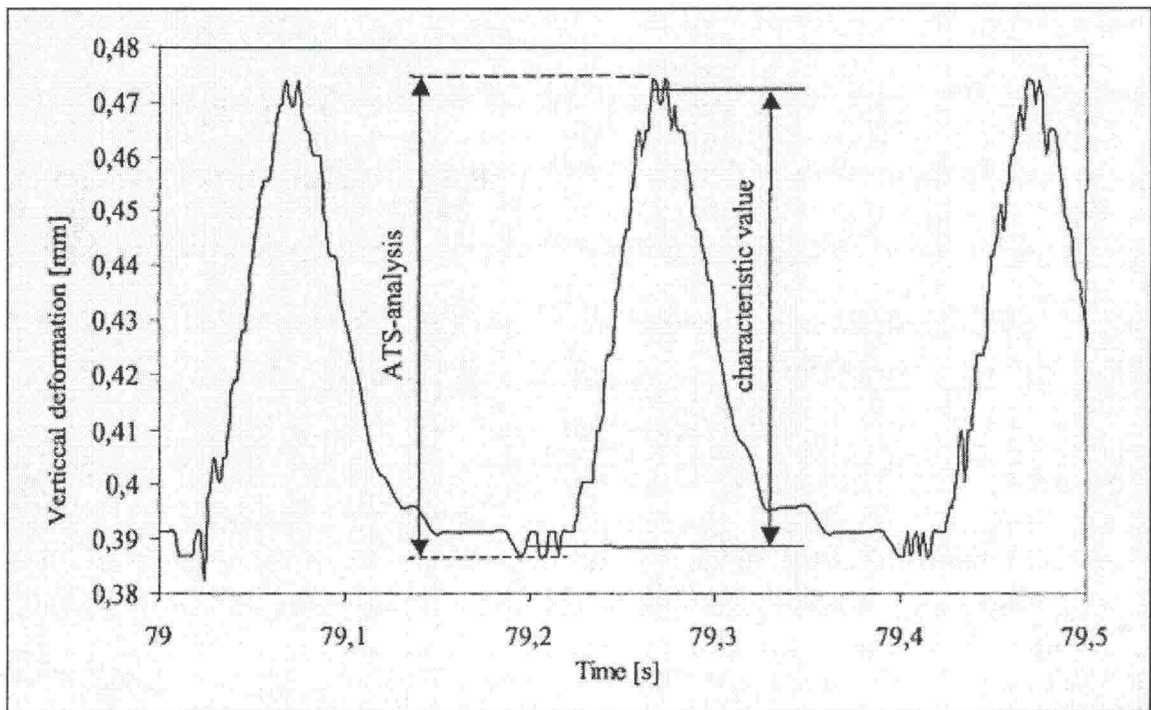


Figure 8.6.3:4 Measured deformations (Werkmeister 2003).

## 8.7 Stockholm Royal Institute of Technology's repeated load triaxial apparatus

### 8.7.1 Introduction

The Highway Engineering Division at the Royal Institute of Technology in Stockholm (Sweden) has conducted research into properties of unbound granular materials. As part of a PhD project on the mechanical properties of unbound granular materials, Lekarp (1999) decided to set up a large-scale repeated load triaxial apparatus for testing granular materials with up to 100 mm particle size. The work was motivated by the growing need for better understanding of the mechanical behaviour of such materials (Lekarp 1999).

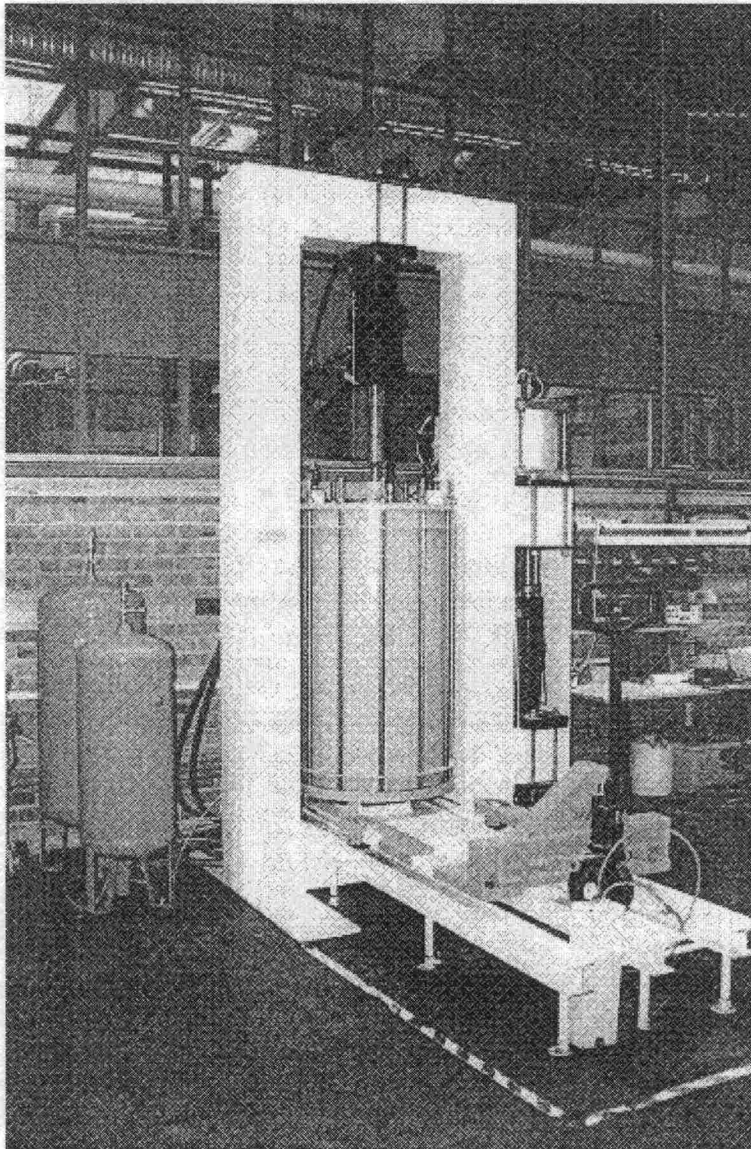
In designing the triaxial apparatus, Lekarp (1999) considered the following basic requirements concerning functional properties:

- Specimen size → A 500 x 1000 mm specimen size was chosen for the new equipment. Such a large specimen size provides proper means for testing granular materials with up to 100 mm particle size.
- Loading System → Based on the importance of proper simulation of loading situations, it was decided that the loading system for the equipment should be able to cycle both deviatoric and confining stresses using appropriate pulse shapes and frequencies. To meet this requirement, a closed servo-controlled hydraulic system was considered to be the best alternative for both cases.

- Instrumentation → The testing equipment is provided with on-sample instrumentation and direct monitoring of the applied stresses by keeping the load cell and the pressure sensor inside the triaxial cell.

### ***8.7.2 Description of the repeated load triaxial apparatus used by Lekarp (1999)***

Figure 8.7.2:1 shows a general view of the triaxial testing equipment. The principal components are schematically illustrated in Figure 8.7.2:2. In this section, the various components of the testing equipment are described in detail.



*Figure 8.7.2:1 General view of the triaxial equipment (Lekarp 1999).*



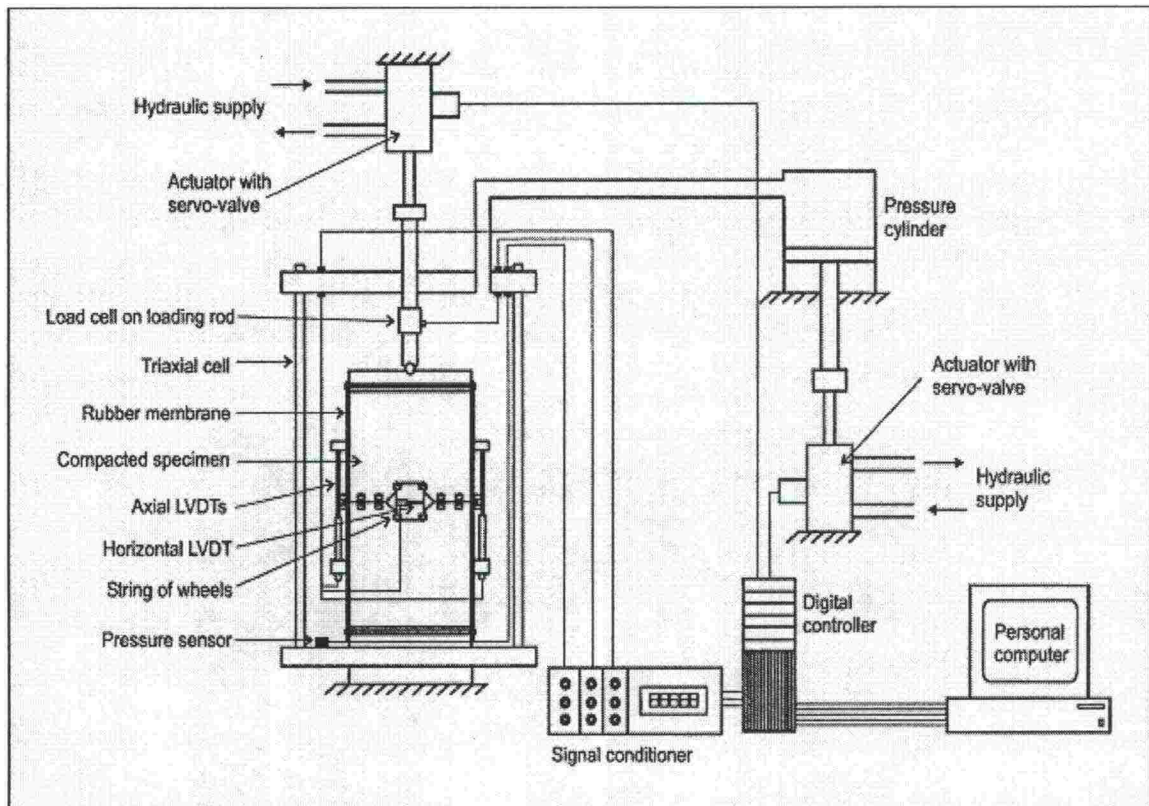


Figure 8.7.2:2 Schematic illustration of the repeated load triaxial apparatus (Lekarp 1999).

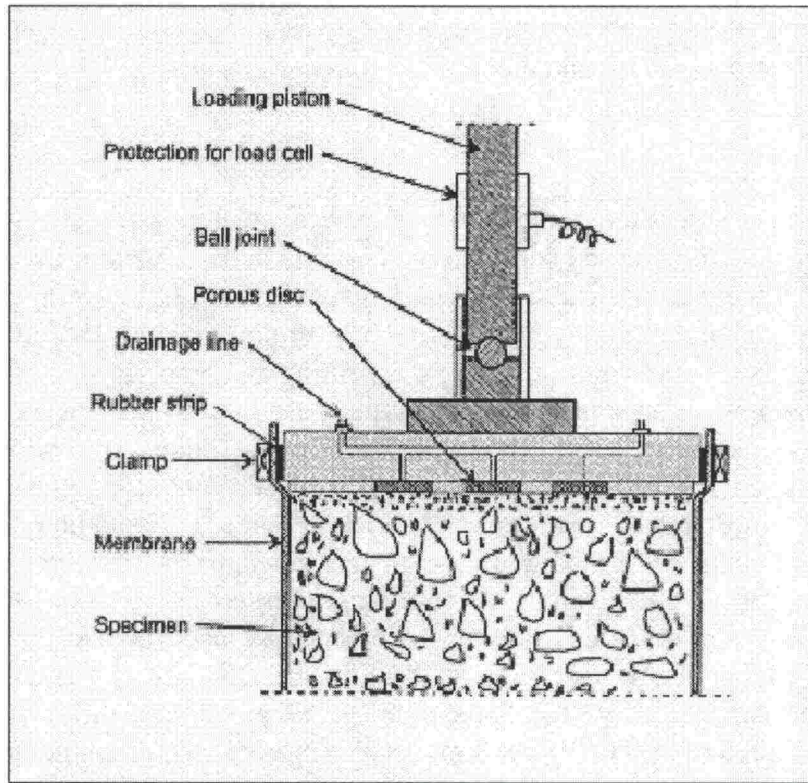
The axial load is applied to the specimen through the loading platens and is continuously monitored by a load cell. The feedback signal from the load cell is compared with the load command signal by the digital control system, after which an error signal is relayed to the servo-valve on the actuator so as to correct the load applied to that required. The confining stress is controlled in a similar manner by the output signal of a pressure sensor placed inside the triaxial cell. For the application of confining stress, silicone oil is used as the confining medium. The compacted specimen, sealed between the top and bottom loading platens by rubber membranes, is instrumented with LVDTs for the measurement of axial and lateral strains. The output signals from the load cell, the pressure transducer, and the LVDTs pass through the conditioning unit and are finally registered at chosen intervals in a data file in the computer. The entire testing process is run with the aid of computer software.

#### Loading frame

The purpose-made loading frame (Figure 8.7.2:1) consists of two joint frames and enables the application of both the deviator stress and confining pressure. The main frame has a capacity of applying a repeated load of 250 kN and is used to generate the axial load. A second frame is welded to the right-hand column of the main frame and is designed for a maximum repeated load of 50 kN required for applying the confining stress. During testing, the triaxial cell rests on a 500 x 500 x 20 mm steel seat welded to the lower beam of the main frame. This seat restricts the movement of the triaxial cell during testing and ensures that the applied load is effectively transmitted to the body of the loading frame.

### Axial loading system

The axial load is applied by a 250 kN servo-hydraulic actuator (MTS 244.31,  $\pm 75$  mm stroke length). The actuator is mounted on the top beam of the main frame (Figure 8.7.2:3) and can apply a repeated deviator stress of up to 1270 kPa to a 500 mm diameter specimen at frequencies of up to 20 Hz.



*Figure 8.7.2:3 Details of the top loading platen and ball joint arrangement (Lekarp 1999).*

A purpose-made 250 kN load cell is used to monitor the axial load applied to the specimen. The load cell consists of strain gauges and is an integral part of the loading piston which is 40 mm in diameter. The load cell is immersed in the confining medium inside the triaxial cell, thereby, eliminating the effects of piston friction from the load measurements. The strain gauges are glued to the surface of the loading piston, responding only to the axial component of the applied load. The output reading of the load cell is therefore unaffected by the confining pressure inside the cell.

The axial load is transmitted to the specimen through the top loading platen described below. The main feature of the connection is a ball joint, illustrated in Figure 8.7.2:3, to ensure the direct transmission of vertical loading and eliminating any shear load transfer.

### Loading platens

The top and bottom loading platens are 40 mm thick and are made of an aluminium alloy. Each platen is fitted with a 20 x 3 mm rubber strip, glued around its outside edge, to ensure proper sealing against the membrane surrounding the specimen. Drainage



through each platen is provided by three drainage connections taken from porous discs around the center part of the platen to brass fittings placed on the outside edge for the bottom platen and on the top surface for the top platen (Figure 8.7.2:3).

Ideally, frictionless contact should be provided between the inside of the loading platens and the specimen. This, however, is not practically feasible (Boyce 1976). When the strain measurements, as in this equipment, are taken at the central part of the specimen, the end frictions do not influence the results. This has been confirmed in the past both experimentally (Boyce 1976) and theoretically (Dehlen 1969).

### Triaxial cell

The triaxial cell is essentially a three-part cylindrical steel pressure chamber consisting of a cell body, a cell base, and top platens. The cell body (750 mm internal diameter, 1330 mm long and 5 mm thick) is capped between the base and top platens using 12 straining rods and bolted nuts. The cell base (840 mm diameter and 50 mm thick) is provided with three outlet connections, two for filling and emptying the cell and one for draining vacuum. The latter is connected by plastic tubing to the drainage lines in the loading platens. The outlets terminate at screwed brass fittings, into which valves are fitted. Each valve is connected to the appropriate pressure line using plastic tubing. The cell top (840 mm diameter and 50 mm thick) is provided at its center with a 40 mm diameter sealed linear ball bushing to ease the movement of the loading piston, a 1-inch diameter port for the application of confining pressure, and three outlets for the electrical terminals. The cell base and top platens are each provided with a mating recess fitted with a flat rubber ring for sealing the cell body at each end. The 12 straining rods are screwed into the threaded holes in the cell base and secured to the cell top platen by bolted nuts. Two 100 x 8 mm steel rings are welded to the outside edge of the cell body to connect the lifting device. These rings add to the structural capacity of the cell, but their contribution has not been considered in the design calculations.

The three sections of the triaxial cell form a sealed pressure chamber designed to withstand an internal pressure of 1000 kPa (approximately 700 kPa safe working pressure). During the testing, the triaxial cell rests on the special seat provided in the main frame. At other times, it can be lifted by a trolley and pulled over the tracks, clear of the loading frame (Figure 8.7.2:1).

### Confining pressure

The confining stress is applied through silicone oil (100 cs viscosity) surrounding the test specimen in the triaxial cell. This oil has a relatively low viscosity and is usually recommended as confining medium since it does not affect the rubber membrane and the electrical devices immersed inside the cell. The applied fluid pressure is controlled by a 50 kN servo-hydraulic actuator (MTS 244.21,  $\pm 75$  mm stroke length) that operates a 320 mm diameter pressure cylinder (MECMAN 167,  $+ 200$  mm stroke length). The large stroke lengths of the actuator and the pressure cylinder ensure that any reduction in the specimen volume during testing can be compensated by extra fluid, assuring satisfactory performances of the equipment. The hydraulic actuator and the pressure cylinder are mounted on the smaller loading frame (Figure 7.7.2:1). The pressurising cylinder is connected to the triaxial cell by a 1-inch bore flexible hydraulic tube.



The confining stress is limited by the area of the piston in the pressure cylinder and the capacity of the hydraulic actuator. With the present arrangement, variable confining pressures of up to 620 kPa can be applied at frequencies of up to about 2 – 3 Hz. The applied pressure is monitored by a pressure transducer placed inside the triaxial cell.

### Specimen preparation

The specimen is manufactured manually by compacting the material into a three-section split mold. First, the bottom loading platen of the specimen is placed in position in the middle of the cell base. A neoprene rubber membrane is then attached to the loading platen by a clamp. The compaction mold is placed over the loading platen, and the membrane is stretched up the inside of the mold and turned over the top. The mold is fixed to the cell base by three straining rods and bolted nuts. A vacuum is applied through the wall of the compaction mold, keeping the membrane against the inside of the mold. The test material is then placed in the mold and compacted in ten layers of equal mass using a vibrating hammer (KANGO 2500) with a base diameter of 300 mm. Since the base diameter is smaller than the diameter of the specimen, shear is induced in the specimen during compaction and higher densities are obtained compared to full-faced compaction. Each layer is compacted to the required height, after which the layer surface, except for the top layer, is roughened by scratching to improve the interlock between the layers. The top 1 cm of the specimen is filled with sand and compacted to ensure a horizontal, smooth surface. After the mold has been filled, the top loading platen is placed in position and the membrane is sealed to the platen with a clamp. The specimen is then lifted over to the trolley in front of the loading frame. Finally, a vacuum is applied to the pore spaces of the specimen via the drainage channels in the loading platens, after which the split mold can be dismantled.

The aggregate normally contains sharp angular particles which could puncture the membrane during the compaction process. A damaged membrane would lead to leakage of confining oil into the specimen, affecting its properties during testing. Therefore, after removing the compaction mold, the first membrane is covered by a second one before the instrumentation is attached. The outer membrane is first stretched, with the help of a vacuum, into a PVC membrane-stretcher with an internal diameter of 520 mm. The membrane-stretcher is then lifted and held over the specimen. When the vacuum is released, the new membrane falls over the first one, and the stretcher can be removed. The clamps used to seal the first membrane are then placed over both membranes. This completes preparation of the specimen, after which the instrumentation can be attached to it.

The neoprene membranes used are specially manufactured for this equipment. Each membrane has been made by rolling a rubber sheet into the shape of a cylinder, with the two ends joined by vulcanising. The membranes are 1200 mm long and 1 mm thick and have an internal diameter of 495 mm.

### Instrumentation for strain measurement

The vertical and lateral strains in the specimen are monitored using direct on-sample instrumentation, as illustrated in Figure 8.7.2:2. To avoid end-effects, the axial strain is



measured for the 600 mm central portion of the specimen's height using three LVDTs placed equally spaced around the circumference of the specimen. Each LVDT measures the relative vertical displacement of the specimen between two "locating anchors" fitted into the body of the specimen after the compaction is completed. It is customary in several other laboratories to use "locating studs" embedded in the sides of the specimen during compaction. Although this arrangement might be suitable for fine aggregates, when the material contains larger particles there is a great risk that the studs are not properly fitted into the material grains, leading to erroneous readings. To avoid this risk, a new method has been successfully implemented for attaching the vertical transducers to the specimen. For this purpose, the compaction mold is provided with three sets of holes, each comprising two 85 mm holes at 600 mm vertical distance. When the specimen is compacted, small holes (8 mm in diameter and 60 mm deep) are drilled into the specimen through the compaction mold and the first membrane. Initial trials have shown that it is necessary to drill the holes before the mold is dismantled, otherwise drilling will disturb the grain arrangement in the area around the holes. After the compaction mold is removed and before placing the second membrane, special aluminium discs are screwed into the holes using standard expandable roller plugs. When the outer membrane is placed in position, threaded rods are screwed into the discs and sealed from the outside using "O"-rings and washers. The LVDTs are then mounted vertically between the rods. The transducer body is attached to the lower rod by the LVDT holder, whereas the core of the transducer is located on an extension rod attached to the upper rod by a bracket. The output voltage of each LVDT indicates the relative movement of the specimen between the two anchors. An enlarged cross-sectional view of the anchoring arrangement used is illustrated in Figure 8.7.2:4.

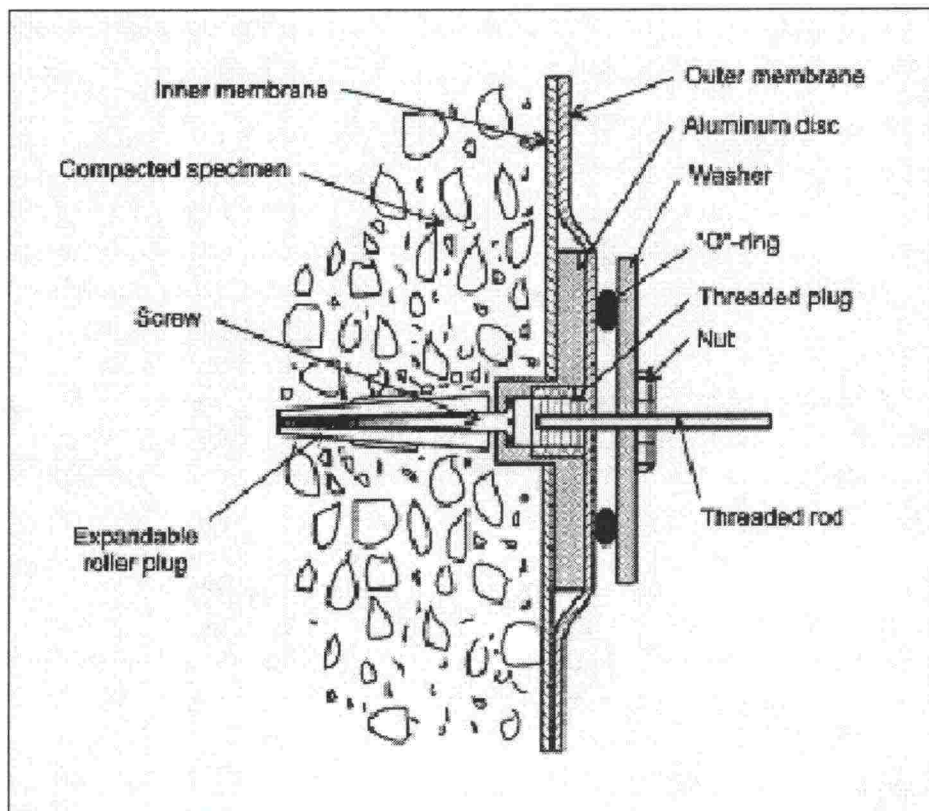


Figure 8.7.2:4 Details of the anchoring arrangement for the axial transducers (Lekarp 1999).

The lateral strain is measured by a "string of wheels" mounted half-way up around the circumference of the specimen, as schematically illustrated in Figure 8.7.2:5.

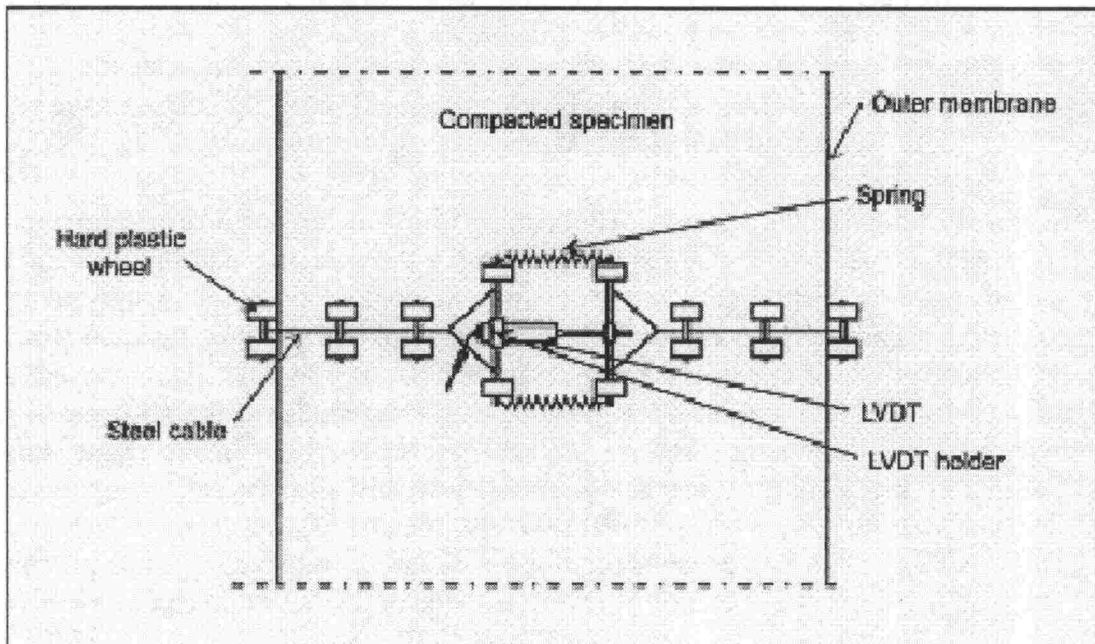


Figure 8.7.2:5 Schematic illustration of the "string of wheels" (Lekarp 1999).

A steel cable of 2 mm diameter is threaded through 15 sets of wheels and attached at each end to an LVDT holder. Each set of wheels comprises a pair of hard plastic wheels fitted on an axle through which the steel cable is threaded. The axles are equally spaced and locked in position by coating the steel cable between adjacent axles with a plastic cover thicker than the hole through the axles. The string of wheels is then wrapped around the specimen and held together by two springs. It is believed that the system adopted here, with several contact points around the circumference of the specimen, gives a very reliable average measurement of the lateral strain. This technique is, therefore, assumed to be superior to other methods used elsewhere, in which the horizontal displacement is measured at two opposite points.

#### Data acquisition and control system

The axial load, confining stress, and displacement transducers all produce electrical output signals that are conditioned and stored by the data acquisition system. As all the measurements are taken inside the triaxial cell, the signals are led through the cell top platen, using sealed outlets. In order to minimise the possibility of electrical disturbance, it was found necessary to provide one outlet to lead the LVDT cables and one for each of the two strain gauge transducers, i.e. the load cell and the pressure sensor. The analogue signals are conditioned and converted to digital electrical signals by the data acquisition and control system (MTS TestStarII ver. 4.0). The resolution of the data acquisition system provides a sensitivity of about 3  $\mu\text{m}$  and 75 N for the LVDTs and the load cell, respectively. The test set is programmed in the computer software provided (MTS TestwareSX ver. 4.0). The experimental measurements can be



displayed on the monitor in graphical form at any time during the test. The results can also be stored in data files for later analysis.

### Equipment factors

There is a possibility that some characteristics of the equipment influence the triaxial testing results. These influences should be appreciated, and the data corrected if necessary. These factors and their possible impacts are discussed below.

- Stiffness of the loading frame → The loading frame undergoes a certain amount of deformation due to the loads applied during testing. If the deformations in the specimen are measured relative to the loading frame, then the results will need to be corrected accordingly. As described previously in this section, the new testing equipment provides direct on-sample instrumentation. The data, therefore, are not affected by the deformation of the loading frame. In addition, the stiffness of the loading frame is important for the amount of energy stored in the frame during loading. If the loading frame is not sufficiently stiff, the energy stored will be released and transferred uncontrollably to the specimen at the start of unloading. This has been taken into consideration in the design of the loading frame provided. The loading frame is sufficiently stiff for the range of loadings that can be applied.
- Friction between the loading piston and the cell bushing → As the loading piston slides through the bushing in the cell top platen, some energy is lost due to friction. When the load cell is placed outside the cell, the axial load measurements will differ from the actual load applied to the specimen. Even though the amount of friction could be reduced greatly by using a linear ball bushing, it will not be eliminated entirely. In the equipment the load cell is placed on the lower portion of the piston inside the triaxial chamber. The effect of friction is, thus, eliminated altogether, and the readings show the actual axial loads applied.
- Friction between the loading platens and the specimen → Large and unquantifiable deformations occur at the interface between the specimen and the loading platens. Consequently, if the axial load displacement is measured over the entire height of the specimen, the strain data will, in fact, be erroneous. To overcome this problem, strain determination in the new equipment is confined to the 600 mm central portion of the specimen.
- Fluid leakage into the specimen → The ingress of the confining silicone oil into the specimen will almost certainly disturb the behaviour of the material. The extent of the impact will depend on the amount of the leaked oil, but is extremely difficult, if not impossible, to quantify. Because of the uncertainties arising from the ingress of oil into the specimen, great care must be taken to avoid this problem. Replacing a specimen spoilt by a leaking membrane is very costly in terms of both time and effort. In the testing equipment, the specimen is covered by two neoprene membranes, sealed at each end to the loading platen by a clamp. Before the outer membrane is placed over the specimen, it is checked for any flaws or leaks. A leak test is performed by sealing each end of the membrane over a purpose-made PVC platen. One of the platens is provided with a sealed outlet, which is connected to a compressed air supply for filling the membrane. As the membrane inflates, it is

examined for leaks through pinholes. At the same time, the surface of the membrane is inspected visually for any flaws, such as trapped air bubbles that could burst during the test. The membrane must be discarded if there are any doubts regarding its sealing capabilities.

- Change in cross-sectional area of the specimen → The actual deviatoric stress applied to the specimen changes during a triaxial test as the cross section of the specimen changes due to radial plastic strain. The cross-sectional area of the specimen,  $A$ , at any moment during triaxial testing can be calculated according to

$$A = A_0 + 2\pi\varepsilon_{3,p}R^2, \quad [\text{Eq. 8.7.2:1}]$$

where

$A_0$	=	original cross-sectional area,
$R$	=	radius of the specimen,
$\varepsilon_{3,p}$	=	radial permanent strain induced.

The amount of plastic strain during a resilient test is practically none, or negligible. This is due to the fact that, prior to resilient testing, the specimen is brought to a stable state by conditioning. However, a significant amount of plastic strain accumulates during conditioning and permanent strain tests. For these cases, the cross-sectional area of the specimen should be corrected accordingly.

- Membrane stiffness → The membranes enclosing a triaxial specimen may have a certain restraining effect on the specimen, thereby reducing the strain caused by the deviator stress. For specimens of high strength materials or with large diameters, the effect of the membrane restraint is considered to be insignificant (Head 1994).

### ***8.7.3 Description of the test series performed by Lekarp (1999)***

Following the construction of the triaxial apparatus, a test program was planned, partly to evaluate the performance of the equipment and partly to characterize the resilient response of several unbound granular materials.

The tests were all conducted in drained condition, thus allowing the pore pressure to dissipate as the tests proceeded. Each test was performed by the application of repeated deviatoric and confining stresses, while the induced axial and radial strains were measured.

## **8.8 Laboratoire des Ponts et Chaussées' repeated load triaxial apparatus**

### ***8.8.1 Introduction***

Gidel et al. (2004) proposed a new approach for studying permanent deformation in unbound granular materials under cyclic loading. The approach is based on results of laboratory tests performed with the repeated load triaxial apparatus. The repeated load triaxial apparatus was developed in order to investigate the mechanical behaviour of



unbound granular materials. The apparatus aims to simulate traffic loads by subjecting a cylindrical specimen to repeated cyclic stresses. Within the network of the Laboratoires des Ponts et Chaussées (LPC), a triaxial apparatus especially designed for the study of unbound granular materials has been developed by the Laboratoire Régional des Ponts et Chaussées in Saint Brieuc (LRPC).

Under cyclic loading, unbound granular materials exhibit elasto-plastic behaviour, characterised by increases in stiffness and permanent deformation with load repetitions. To study this behaviour with a repeated load triaxial apparatus, Gidel et al. (2004) decided to study and model separately:

- the stabilised resilient behaviour obtained after a large number of loading cycles, which can be described by nonlinear elastic models;
- the increase in permanent deformation with the number of cycles.

Much research has been devoted to studying resilient behaviour, and we now possess well established test procedures and appropriate models to describe this behaviour (Balay et al. 1998). Permanent deformation has been less studied, for a number of reasons, which have been discussed earlier in the previous chapters.

This section presents a new procedure for studying the permanent deformation behaviour of unbound granular materials using the repeated load triaxial apparatus developed by Gidel et al. (2004). The technique allows obtaining more information from a single specimen (experimental dispersion is reduced, as is the amount of time and materials required). Examples of loading modes for investigating how stresses increase permanent deformation in order to develop models that relate the increase of permanent deformation with the number of cycles and the cyclic stresses applied are also given. Tests conducted at the Regional Laboratories of Toulouse and Bordeaux (LRPCs) and the LCPC (Nantes Centre) show the usefulness of this procedure when developing models of this type.

### ***8.8.2 Description of the repeated load triaxial test apparatus***

Gidel et al. (2004) used a repeated load triaxial apparatus for specimens with a diameter of 160 mm and a height of 320 mm. These dimensions are suitable for materials with a particle size distribution of 0/20 mm (0/31.5 mm at the very most). The specimens are prepared with a vibrocompression method as described in the French standard NF P 98-230-1. This method consists in compacting the specimen in one layer, under the simultaneous effect of a vertical load and a horizontal vibration. The method is automated, fast (compaction of a specimen takes less than 1 minute) and produces homogeneous specimens (Gidel et al, 2001). The triaxial cell is equipped with sensors to measure applied loads and pressure and axial and radial strains. The apparatus has a pneumatic load application system which allows the confining pressure and the axial stress acting on the specimen to be varied in a cyclic manner. Figure 8.8.2:1 shows the triaxial cell and its instrumentation.

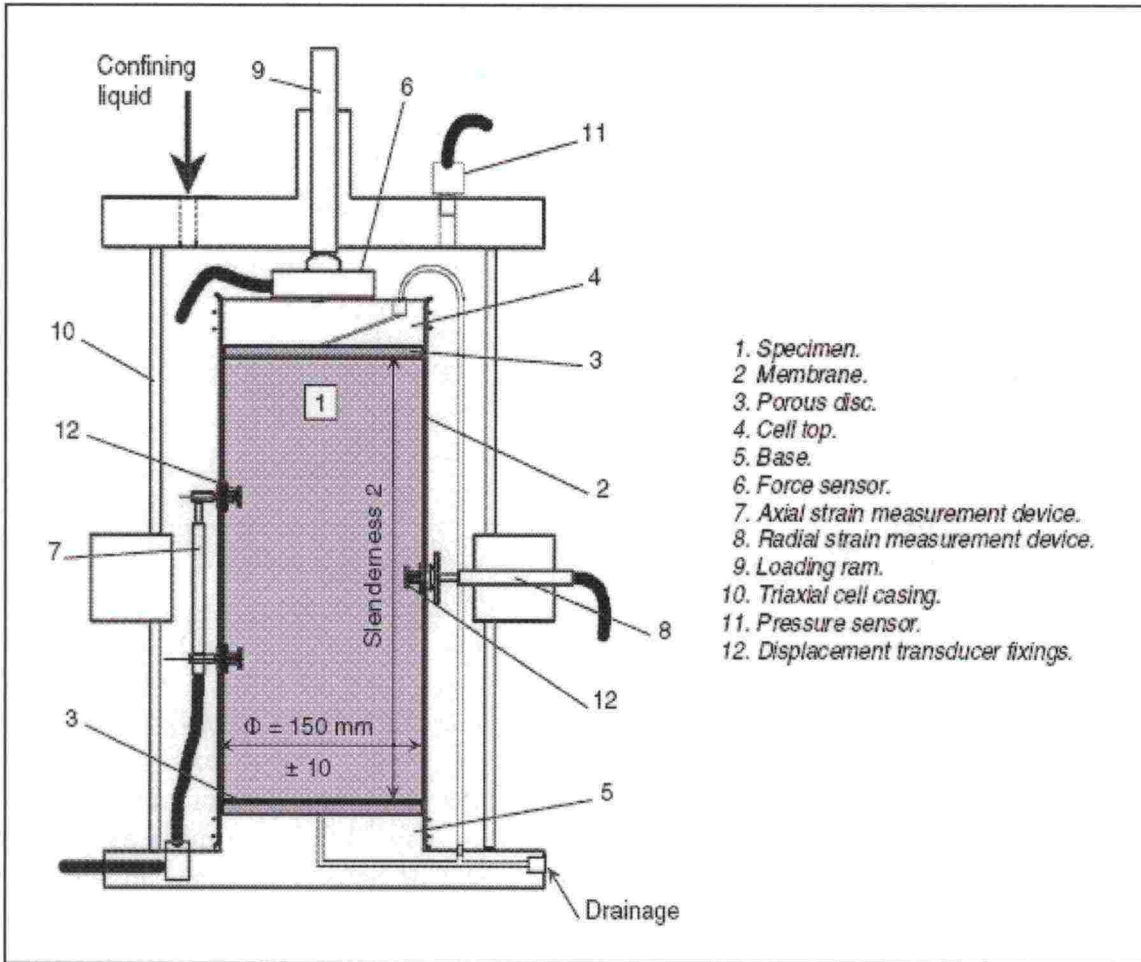


Figure 8.8.2:1 The triaxial cell of the repeated load triaxial apparatus (Gidel et al. 2004).

Figure 8.8.2:2 shows a typical example of the results obtained from this type of test (change in permanent axial deformation  $\epsilon_1^p$  and permanent radial deformation  $\epsilon_3^p$  in relation to the number of cycles). The presented test was performed on a 0/20 mm unbound granular material obtained from micro granite, under average density and water content conditions.



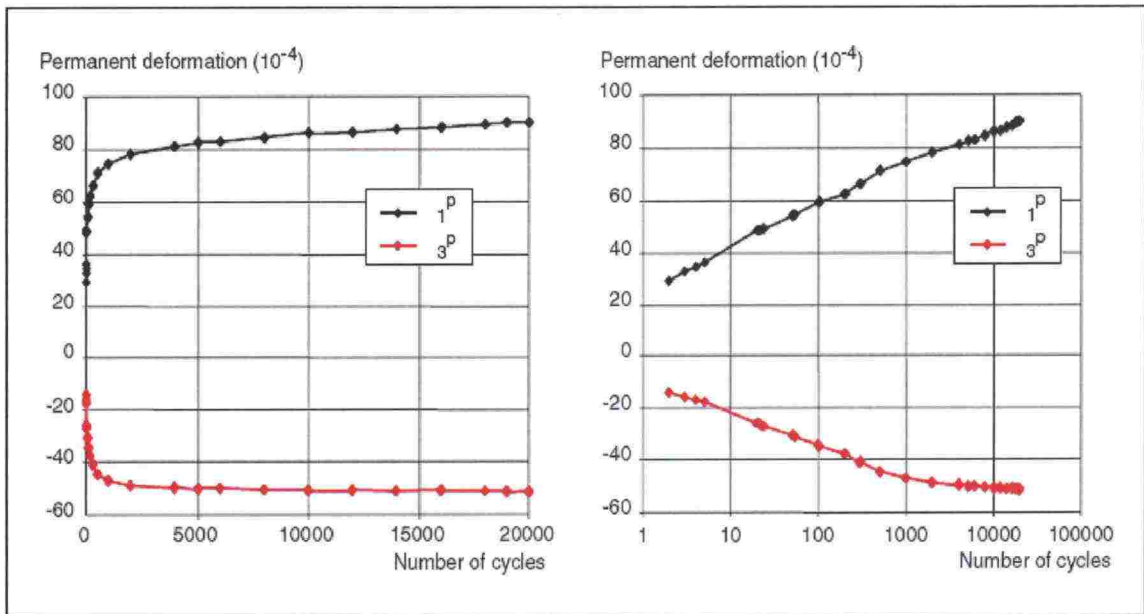


Figure 8.8.2:2 Permanent axial and radial deformation during a triaxial test on a 0/20 unbound granular material obtained from micro granite (Gidel et al. 2004).

When the cyclic stresses are lower than the failure stress of the unbound granular material, permanent deformation is characterized by a rapid increase during the first cycles followed by gradual stabilization. The amount of deformation depends on the characteristics of the material and the applied load.

### 8.8.3 Description of the test procedure

The test procedure usually used to investigate the permanent deformation behaviour of unbound granular materials with a triaxial apparatus consists of applying a single level of stress to each specimen and involves a large number of loading cycles (10<sup>5</sup> and more) with each single level of stress. The disadvantage of this procedure is that investigating the influence of stresses on permanent deformation requires very large numbers of tests. Gidel et al. (2004) therefore tested a different approach which consists of conducting staged loading tests, that is to say subjecting the same specimen successively to several different levels of stress ( $N_1$  loading cycles are applied at the first level of stress, then  $N_2$  cycles at the second level, etc.). This means that the number of tests can be considerably reduced (saving time and materials) but it also reduces experimental dispersion, as a single specimen is used to obtain information under several levels of stress.

Figure 8.8.3:1 illustrates the staged loading procedure. It consists of subjecting the specimen to stress paths with a constant stress ratio  $q/p$  and several increasing amplitudes of cyclic stress application  $\Delta p$  et  $\Delta q$  (four or five successive stages, each with a duration of about 10,000 cycles). The levels of stress chosen are  $\Delta p < 300$  kPa,  $\Delta q < 600$  kPa and  $0 \leq \Delta q/\Delta p \leq 3$ .

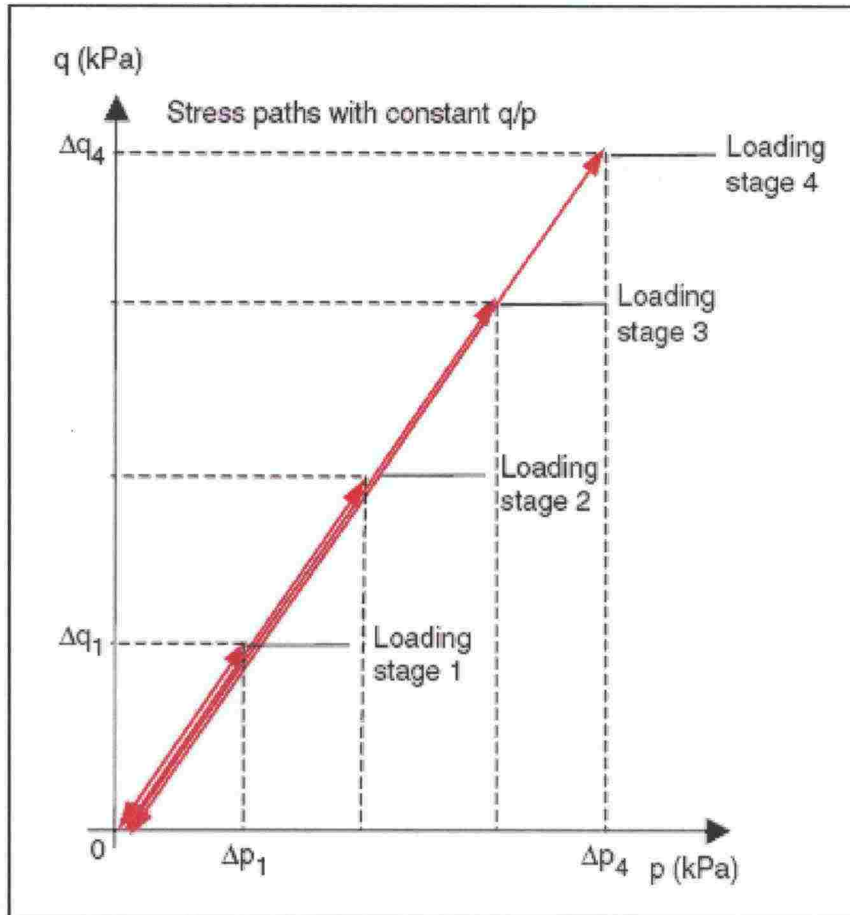


Figure 8.8.3:1 Cyclic loads applied during the staged loading tests (Gidel et al. 2004).

In order to develop and validate this procedure, a first study was conducted by the LRPC of Bordeaux (Gidel 1997) on 0/14 dioritic granular material (Mazière gravel) in order to evaluate the new procedure and compare it with tests involving a single level of stress. All the tests were conducted following stress paths with  $q/p = 2$  on specimens with identical characteristics.

More tests to validate the procedure were carried out later by Gidel et al. (2004). After the tests, they came to the conclusion that the staged loading test procedure seems to provide a good alternative means of studying the behaviour of unbound granular materials in relation to the number of loading cycles and applied stresses. With a minimum number of tests, it provides a means of characterising the increase in permanent deformation that occurs under different levels of stress and reduces the bias caused by experimental dispersion.

### 8.9 Trondheim NTNU/SINTEF's repeated load triaxial apparatus

Recycled concrete aggregate have recently been introduced in Norway as a possible unbound granular material. A number of field projects have revealed good functional properties (proven suitability), despite the fact that the mechanical properties of the materials in many cases do not comply with specifications concerning mechanical strength. Many traditional test methods for mechanical properties are clearly not suitable for this kind of materials. A proper evaluation should therefore be based on



performance-related (functional) tests. In order to make laboratory investigations relevant and comparable to field conditions, the materials should be tested as layers rather than as particles. Also the applied test loadings should be comparable to the real traffic. Aurstad et al. (2005) developed a cyclic triaxial test apparatus (large scale,  $d = 300$  mm,  $h = 600$  mm) suitable for testing these materials.

The testing apparatus, test procedure, sample preparation procedure, etc were developed by NTNU/SINTEF in Trondheim. This equipment allows for testing materials with particle size up to about 60 mm. The apparatus is shown in Figure 8.9:1.

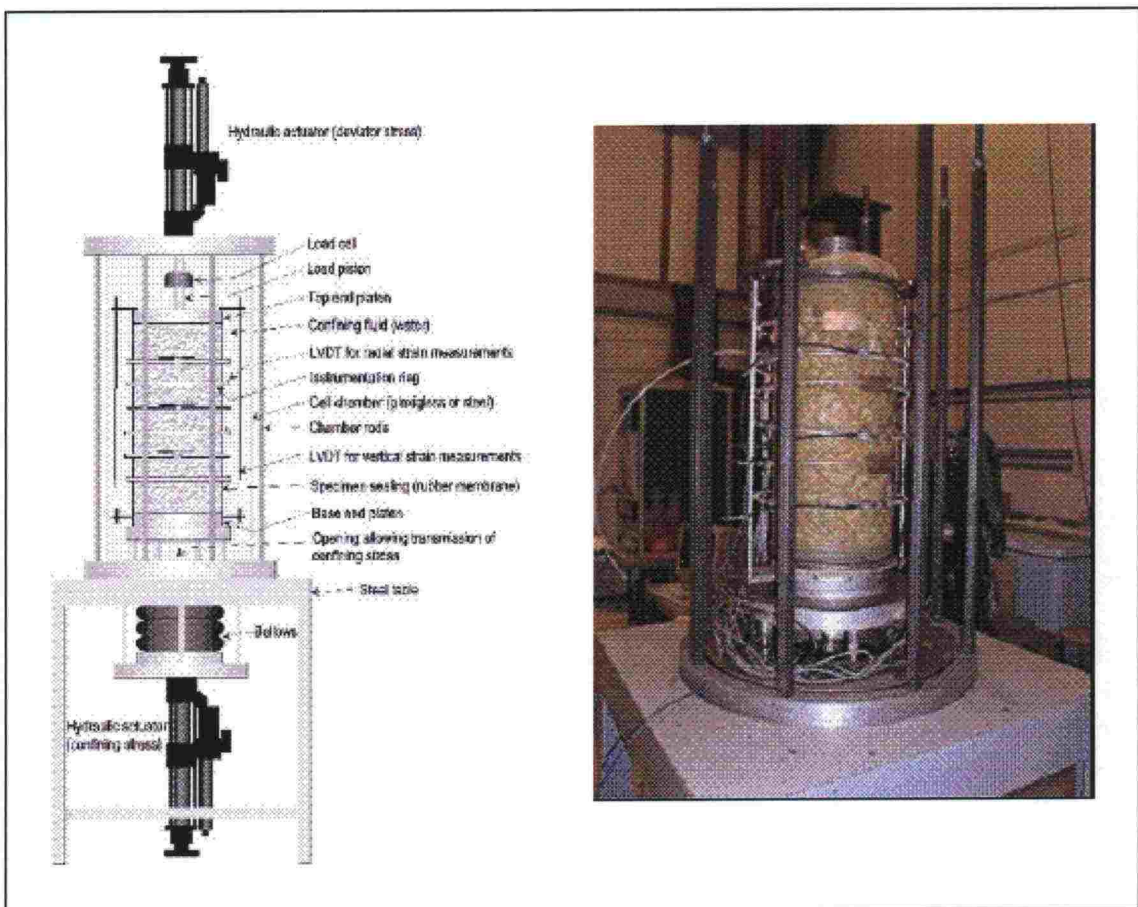


Figure 8.9:1 Large-scale triaxial test apparatus at NTNU/SINTEF with mounted specimen of crushed concrete (Aurstad et al. 2005).

### 8.10 Triaxial tests with measurement of negative pore pressure

Coronado et al. (2005) performed both small and large strains triaxial tests with measurement of negative pore water pressure on two untreated unbound granular materials. The materials have been recomposed with two percentages of fines (10 % and 7 %) and compacted in the laboratory at different water contents to a dry density corresponding to 97 % of the Modified Proctor maximum density. Wetting tests have been carried out to determine suction and water content changes in the material during wetting. One of the goal of Coronados's work is to show how to take moisture changes into account using an effective stress approach.

The materials used by Coronado et al. (2005) are mainly composed of gneiss coming from the "Maraîchères" quarry in France. They are made from the mixture of five different fractions: 0/4 mm sand, 2/6.3 mm gravel, 4/10 mm gravel, 10/14 mm gravel, and 14/20 mm gravel. The reference grain size distributions used contain 7% and 10% of fine elements ( $< 80 \mu\text{m}$ ); they are called MGC and HFC, respectively. Figure 8.10:1 shows the grain size distribution curves and the compaction curves of the mixtures, determined in accordance with the French standards NFP 98-129 and NFP 94-093.

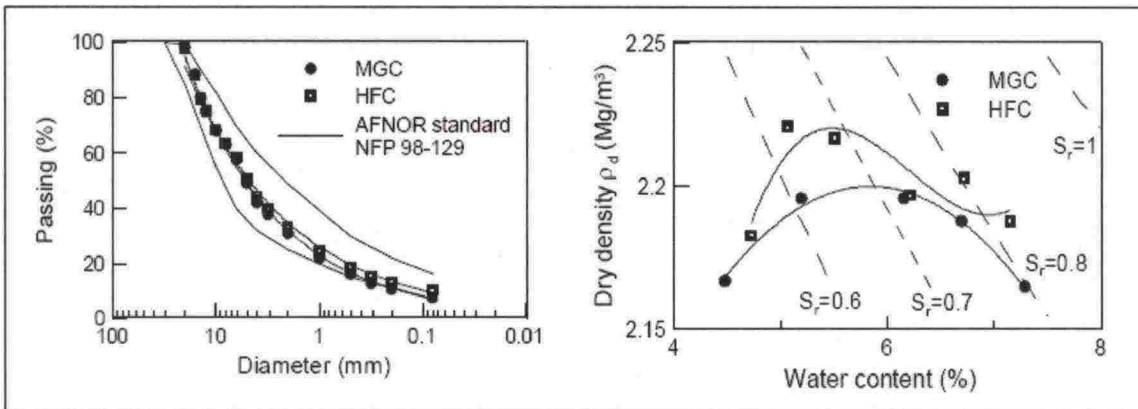


Figure 8.10:1 Grain size distribution and Proctor Modified compaction curves of the two soils (Coronado et al. 2005).

The maxima of the compaction curves correspond to a water content ( $w_{\text{MOP}}$ ) of 5.8% and a dry density of  $2.2 \text{ Mg/m}^3$  for MGC, and a water content of 5.5% and a dry density of  $2.22 \text{ Mg/m}^3$  for HFC. The value of the Los Angeles abrasion coefficient is approximately 20%.

The cyclic tests were carried out in a classic triaxial cell, allowing a direct measurement of the stiffness modulus and Poisson's ratio for homogeneous strains ranging between  $10^{-6}$  and  $10^{-2}$ . To be able to explore the domain of the very small strains with sufficient accuracy, the force and strain measurements are done on the specimen itself. The force transducer is placed inside the cell directly on the head of the specimen, which permits a precise measurement of the force applied to the specimen and eliminates the bearing-piston friction problems. The measure of the axial strains is achieved by means of three LDT (linear displacement transducer) strain sensors placed in the central zone of the specimen, in order to avoid the influence of the constrictions of the bases on the measures. Radial strains are derived from the variations of the perimeter of the specimen measured by a deformable belt placed to mid-height and equipped with a LDT sensor. The LDT sensors are constituted of 4 strain gauges forming a complete Wheatstone bridge fixed on a deformable blade made of beryllium bronze; they were manufactured at the Ecole Centrale Paris on the model of the sensors developed at the University of Tokyo in the team of Professor Tatsuoka (Goto et al. 1991). Supports for the sensors are put in place in the specimen during the compaction. The accuracy of the strain measurements is approximately  $10^{-5}$  with a 21 bits Agilent A/D converter. To prepare the specimens, water is added to the dry mixture in a homogenous way. Dynamic compaction of the specimen, 150 mm in diameter and 300 mm in height, is achieved by hand by means of a Modified Proctor rammer, in 12 layers with 56 strokes of rammer per layer. The initial properties of the specimens are shown in Table 8.10:1. During the manufacture of the specimens, special attention is paid to the setting up of



the six supports of the vertical sensors. Then, the axial and radial strain sensors are put in place, as well as the force transducer.

*Table 8.10:1 Initial conditions of the specimens for the small and large strains tests Coronado et al. (2005).*

		Dry density (Mg/m <sup>3</sup> )	Water content (%)
Test 2	MGC	2.123	2.05
Test 3		2.132	3.86
Test 4		2.167	5.12
Test 5		2.132	2.00
Test 8	HFC	2.150	3.50

To determine the reversible behaviour of the materials, preliminary conditioning of the specimens is carried out in order to simulate the real conditions of laying down of the aggregate: it consists in 20,000 loading-unloading cycles under an isotropic stress of 40 kPa and a deviatoric stress of 280 kPa. After the pre-conditioning, the specimen is submitted to 20 successive paths with increasing levels of stress. All the tests are made under constant confining stress  $\sigma_3$ . Each loading is applied during 100 cycles. The reversible strains of the specimen are measured during the 100<sup>th</sup> cycle.

The measuring device consists of a triaxial cell with a semi-permeable ceramics placed in the base; the porous stone with high air entry pressure (1.5 MPa, from Soil Moisture) does not permit the passage of air in the water circuit. The device can be used either as a densitometer (with  $u_a = 0$  and  $u_w < 0$ ) to measure negative pore water pressures between 0 and 50 kPa, or with an air overpressure at the head of the specimen for higher negative pressures. Pore water pressure measurements are done by means of an absolute pressure sensor with a range of 1000 kPa and a sensitivity of 0.1 kPa/mV. Data logging is achieved by means of a 16 bits GDS data acquisition system.

The size of the specimens is 100 mm in diameter and 200 mm in height. Specimens were dynamically compacted by means of a Modified Proctor rammer in 4 layers, with 56 strokes per layer. A thin layer of kaolinite is placed on the ceramics to ensure a good contact with the specimen and the continuity of the water phase. The measures have been made in the conditions of tests 3 (MGC material,  $w = w_{MOP} - 2\%$ ) and 8 (HFC material,  $w = w_{MOP} - 2\%$ ).

To impose negative pore water pressures ranging between 0 and 30 kPa, tensiometric plates were used. They are made of a low porosity sintered glass filter that plays role of the semi permeable separation, set in a glass funnel. The specimen is placed on the filter to the atmospheric pressure, in contact with a reservoir filled with de-aired water. Imposing a difference of level between the filter and the measurement tube results in controlling the depression of the water placed in the reservoir, and therefore the negative pore water pressure in the specimen.

At the end of the cyclic triaxial tests, the specimen is cut into several pieces, which are placed on the tensiometric plate. The exchanges of water between the specimen and the reservoir are derived from the displacement of the water meniscus in a horizontal measurement tube connected to the reservoir. When the negative pore water pressure in the specimen reaches the imposed value, generally at the end of 5 days, the total volume of the specimen and its water content are derived from immersion in kerdane followed by drying in an oven; the water content, void ratio and degree of saturation of the material are derived from these data.

Figure 8.10:2a shows the non linear increase in pore water pressure  $u_w$  versus the isotropic stress  $\sigma_3$  for the two grain size distributions. The pressure increases up to a value close to 0 under the highest stress (300 kPa) but, even in the initial state, suction values remain unimportant ( $-u_w < 35$  kPa). Under the same isotropic stress, the negative pore water pressure is slightly higher in the case of the material with the highest percentage of fines (HFC), but both curves tend towards the same value when the isotropic stress reaches 300 kPa.

Figure 8.10:2b shows the results of the wetting tests, starting from the initial conditions indicated in Table 8.10:1. The shape of the negative pore water pressure versus water content curves presents the usual aspect. For the material with the highest percentage of fines, there is a slight shift of the wetting curve towards higher water contents (under the same suction) after the air entry point. The air entry point corresponds to the negative pressure for which occurs the fast reduction of the degree of saturation and the water content and can be situated between 1 and 5 kPa for the two materials. One can note the good agreement between the suction values found in the triaxial tests with measurement of negative pore water pressure under  $\sigma_3 = 0$  ("Test 3" and "Test 8" on Figure 8.10:3b) and the wetting tests.

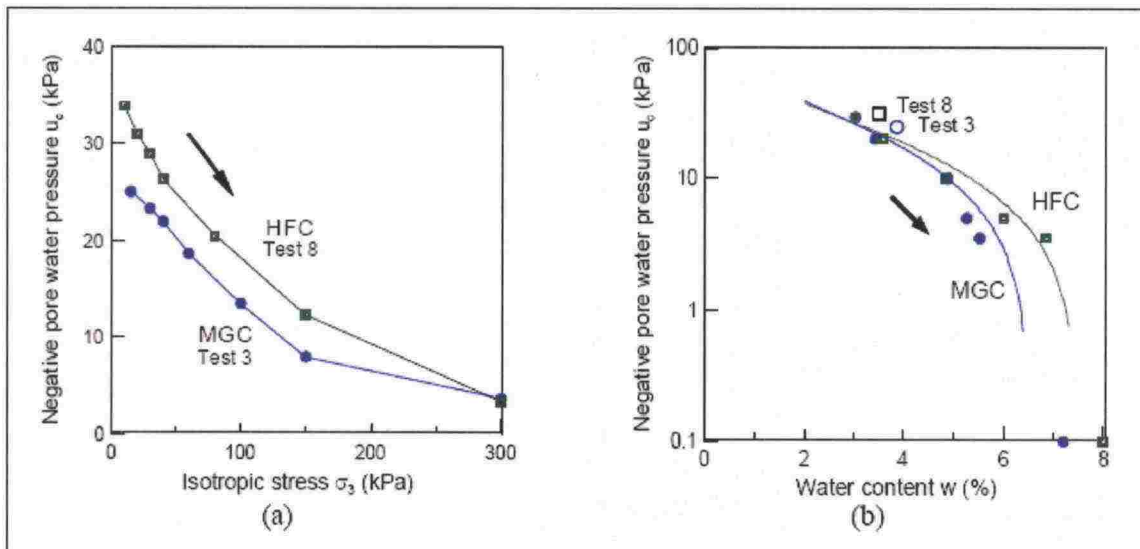


Figure 8.10:2 Changes in the negative pore water pressure according to (a) isotropic stress and (b) water content for the two materials (Coronado et al. 2005).

Figure 8.10:3a shows the variation of the secant modulus of the MGC material as a function of the vertical stress, for different water contents ranging from 2% for tests n° 2 and 5 to 5.1 % for test n° 4. One notes the sensitivity of the material to this parameter:



under the same isotropic stress, the modulus is higher when the water content is smaller because of the increase in the capillary forces in the menisci that form themselves between the grains. As previously noted (Fleureau et al. 2003), the lines are more or less parallel for the different wet soils.

Figure 8.10:3b shows the variation of the secant module with the vertical stress  $\sigma_v$  for the two studied percentages of fines. When the percentage grows from 7% to 10%, the module noticeably increases. However, the influence of the dry density of the material, which is noticeably higher for HFC compared to MGC (2.150 for test 8 against 2.132 for test 3), must be taken into account in the comparison.

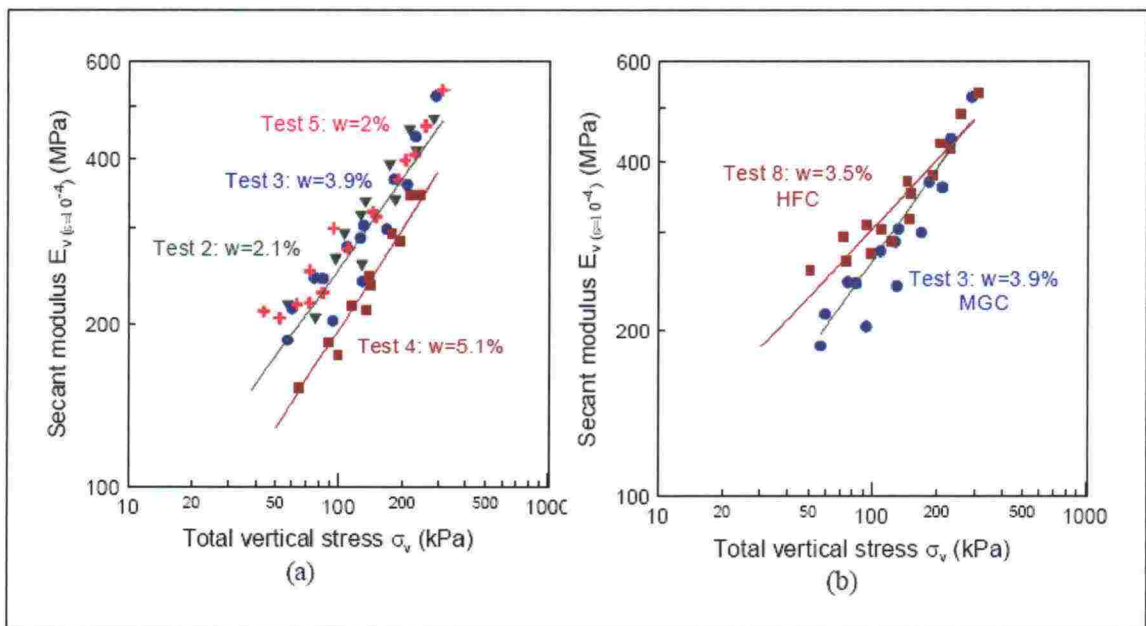


Figure 8.10:3 Influence of (a) the water content and (b) the percentage of fines on the secant modulus (for an axial strain of  $10^{-4}$ ) (Coronado et al. 2005).

Figure 8.10:4 shows the results of the large strains triaxial tests in different corresponding coordinate systems: Stress deviator  $q$  versus axial strain  $\varepsilon_1$  and mean stress  $p$ , pore water pressure  $u_w$  versus axial strain and void ratio  $e$  versus axial strain and mean stress (both in normal and log scales). In the  $[\varepsilon_1, q]$  coordinate system, the curves for the MGC material are classified according to the confining stress. They all present a peak, representative of an overconsolidated behaviour, usual in granular materials under low stresses. There is a small difference in the values of the maximum deviators between the two materials, under  $\sigma_3 = 0.3$  MPa, but the large strains value are similar. In both  $[\varepsilon_1, u_w]$  and  $[\varepsilon_1, e]$  planes, the behaviour of the materials is clearly first contractant, then dilatant for larger strains. The variations of the void ratio are small, probably because the materials are not very far from saturation, as evidenced by the values of the pore water pressure ranging between  $-20$  and  $+5$  kPa.

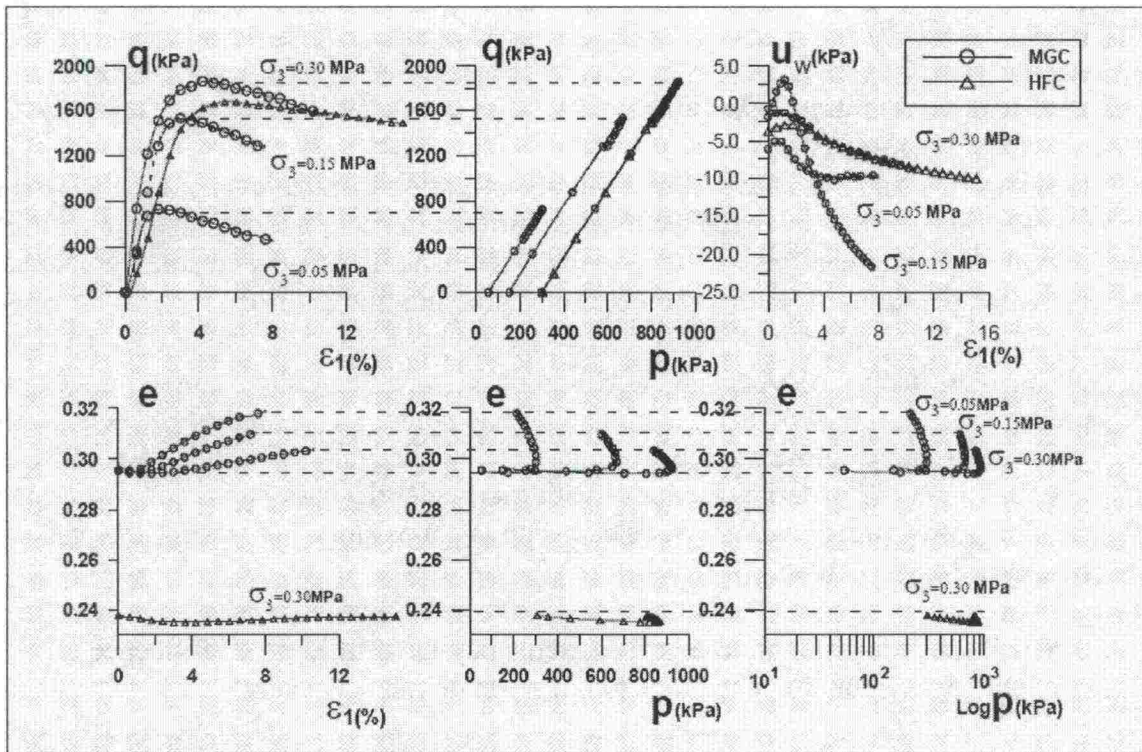


Figure 8.10:4 Results of large strains triaxial tests with measurement of negative pore water pressure for the two materials (Coronado et al. 2005).



## 9 CONCLUSIONS

### 9.1 Resilient deformation behaviour

- Previous investigations have shown without exception that the level of applied stress is one of the factors that have the most significant impact on the resilient properties of unbound granular materials in railway embankments. The resilient modulus increases greatly with increasing confining pressure and sum of principal stresses, but it increases only slightly with increasing deviator stress. However, if excessive plastic deformation is not generated within the embankment, the resilient modulus is practically unaffected by the magnitude of the deviator stress applied. Also resilient Poisson's ratio is influenced by the state of applied stresses. Poisson's ratio of unbound granular materials increases with increasing deviator stress and decreasing confining pressure. However, the relationship is not as simple as that for the resilient modulus.
- Stress history, load duration and load frequency have little or no influence on the resilient behaviour of unbound granular materials typically used in railway embankments.
- The resilient modulus decreases with increasing moisture content, especially at high degrees of saturation. This can be explained with the lubricating effect of water or with the fact that localized pore suctions decrease with higher water content, leading to lower interparticle contact forces. Saturation of unbound granular materials also affects the resilient Poisson's ratio. Poisson's ratio is reduced as the degree of saturation increases.
- If granular materials in a railway embankment become saturated, they develop excess pore water pressure under repeated loading. As pore water pressure develops, the effective stress in the material decreases with a subsequent decrease in both strength and stiffness of the material. It can be argued that it is not the degree of saturation per se that influences the material behaviour, but rather that the pore pressure response controls deformational behaviour.
- The literature available is somewhat ambiguous in regard to the impact of density on resilient response of granular materials under train-type loading. Some investigations have considered the effect of density as relatively insignificant, while some others have evaluated that the effect of density is dependent upon the material. On the other hand, many researchers have come to the logical conclusion that resilient modulus generally increases with increasing density.
- The particle size distribution, or grading, of granular materials seems to have some influence on material stiffness, though it is generally considered to be of minor significance. When moisture is introduced to well-graded materials, the effect of grading can be significantly increased, since these materials can hold water in the pores. They can also achieve higher densities than uniformly graded materials because the smaller grains fill the voids between the larger particles. Therefore grading has an indirect effect on the resilient behaviour of unbound aggregates by controlling the impact of moisture and density of the system.

## 9.2 Permanent deformation behaviour

- Stress level is one of the most important factors affecting the development of permanent deformation in granular materials under repeated, train-type loads. The magnitude of permanent deformations developed strongly depends on the stress level and increases with rising deviator stress and decreasing confining stress. Consequently, permanent deformation behaviour in granular materials is principally governed by a stress ratio consisting of both deviatoric and confining stresses.
- Permanent strain development is clearly dependent on the loading order. The permanent strain resulting from a successive increase in the stress level is considerably smaller than the strain that occurs when the highest stress level is applied immediately. However, from a practical point of view, this phenomenon does not have much importance in a real railway embankment, because train-type loads are not characterised by this “successive increase”.
- The number of load cycles is one of the most important factors to consider in the analysis of the long-term behaviour of granular materials with repeated load triaxial tests because it allows the estimation of how much the railway embankment will deform. Most elasto-plastic material models assume that all the plastic deformations develop under the first loading cycle, and that further loading below this level are purely elastic. This does not fit well with the behaviour of unbound aggregates under train-type loads, which tend to develop some additional permanent deformations for each repeated load step.
- The combination of a high degree of saturation and low permeability due to poor drainage, leads to excessive pore water pressure, low effective stress and, consequently, low stiffness and low deformation resistance. The risk for the development of excess pore water pressure within a railway embankment increases if the material is exposed to sudden changes of the loading conditions. A special risk in the development of the excess pore water pressure has been found to exist when loading applied to the mineral skeleton of the aggregate is repeated several times. These conditions are quite common in a railway embankment under traffic-type loads if the embankment has, for some reason, become saturated.
- Cold weather will cause water in the ground to freeze. As the frozen front moves downwards due to a long, cold period, suctions are established which sucks water towards the freezing front. By this means excess water collects in the ground as ice. When thawing commences in the spring, this water tends to be trapped in the pores of the aggregate and cannot leave as the drainage system remains frozen. The formation of segregation ice should never be allowed in a railway embankment, because the resistance to permanent deformation decreases dramatically when the moisture content of the material increases because of thawing.
- The effect of density is one of the most important factors influencing the long-term behaviour of granular materials and the development of permanent deformations in railway embankments. Resistance to permanent deformation under repetitive loading appears to be highly improved as a result of increased density.



- Permanent deformation resistance in granular materials is reduced as the amount of fines increases. Increasing fines content leads to a significantly higher permanent strain.
- The grain shape apparently affects most significantly the deformation behaviour of coarse-grained aggregates indirectly by means of their compactibility, since the grain shape clearly affects the compactibility of aggregates. Aggregates having very schistose or elongated grain shape are also naturally more sensitive to particle defects. As a result, the amount of fines fraction increases, the proportion of coarse grains decreases and the amount of permanent deformations - due to the rearrangement of the defected particles - increases.

## REFERENCES

- Allaart, A.P. (1992). Design Principles for Flexible Pavements, a Computational Model for Granular Bases. Proefschrift, Technische Universiteit Delft, Delft, The Netherlands.
- Allen, J.J. (1973). The Effect of Non-Constant Lateral Pressure of the Resilient Response of Granular Materials. Ph.D. Thesis, University of Illinois, Urbana, Illinois, USA.
- Allen, J.J. and Thompson, M.R. (1974). Resilient Response of Granular Materials Subjected to Time-Dependent Lateral Stresses. Transportation Research Record, No. 510, pp. 1-13.
- Amber Yau, P.E., Harlod, L., and Von Quintus, P.E. (2004). Predicting Elastic Response Characteristics of Unbound Materials and Soils. Transportation Research Board, 83<sup>th</sup> Annual Meeting, Washington, D.C., USA.
- American Association of Highway and Transportation Officials (AASHTO) (1986). Standard Method of Test for Resilient Modulus of Subgrade Soils. Designation: T272-82.
- Ammi, M., Bideau, D., and Troadec, J.P. (1987). Geometrical Structure of Disordered Packings of Regular Polygons; Comparisons with Disk Packings Structures. Journal of Physics D: Applied Physics, Vol. 20, pp. 422-428.
- Andrei, D., Witczak, M.W., Schwartz, C.W., and Uzan, J. (2004). Harmonized Resilient Modulus Test Method for Unbound Pavement Materials. Transportation Research Board, 83<sup>th</sup> Annual Meeting, Washington, D.C., USA.
- Ansell, P. and Brown, S.F. (1978). Cyclic Simple Shear Apparatus for Dry Granular Material. Geotechnical Testing Journal, Vol. 1, No. 2, pp 82-91.
- Arnold, G. (2005). Performance Tests for Selecting Aggregate for Roads. Nz (Waitangi) 2005 Conference, New Zealand.
- Arnold, G., Dawson, A., Hughes, D., and Robinson, D. (2004). Deformation Behaviour of Granular Pavements. Proceedings, 6<sup>th</sup> International Symposium on Pavements Unbound, Nottingham, England, UK. pp. 169-177.
- Arnold, G., Dawson, A., Hughes, D., Werkmeister, S., and Robinson, D. (2002). Serviceability Design of Granular Pavement Materials. Proceedings, 6<sup>th</sup> International Conference on the Bearing Capacity of Roads and Airfields, Lisbon, Portugal. pp. 957-966.
- Aurstad, J., Aksnes, J., Dahlhaug, J. E., Berntsen, G., and Uthus, N. (2005). Unbound Crushed Concrete in High Volume Roads – A Field and Laboratory Study. Proceedings, 7<sup>th</sup> International Conference on the Bearing Capacity of Roads, Railways and Airfields, Trondheim, Norway.



Balay J., Gomes Correia A., Jouve P., Hornych P., Paute J.-L. (1998). Étude expérimentale et modélisation du comportement mécanique des graves non traitées et des sols supports de chaussées – Dernières avancées. Bulletin des laboratoires des Ponts et Chaussées. pp. 3-18. (In French)

Barrett, P. J. (1980). The Shape of Rock Particles, a Critical Review. *Sedimentology*, 27. pp. 291-303.

Barksdale, R.D. (1972). Laboratory Evaluation of Rutting in Base Course Materials. Proceedings, 3<sup>rd</sup> International Conference on Structural Design of Asphalt Pavements, London, UK. pp 161-174.

Barksdale, R.D. (1991). The aggregate Handbook. National Stone Association, Washington, D.C., USA.

Barksdale, R.D. and Itani, S.Y. (1989). Influence on Aggregate Shape on Base Behaviour. *Transportation Research Record*, No. 1227, pp 173-182.

Barret, J.R. and Smith, D.M. (1976). Stress History Effects in Base Course Materials. Proceedings, 8<sup>th</sup> Australian Road Research Board Conference, Vol. 8, pp 30-39.

Belt, J., Ryynänen, T., and Ehrola, E. (1997). Mechanical Properties of Unbound Base Course. Proceedings, 8<sup>th</sup> International Conference on Asphalt Pavements, Seattle, Washington, USA. Vol. 1, pp. 771-781.

Bishop, A.W. and Blight, G.E. (1963). Some Aspects of Effective Stress in Saturated and Partly Saturated Soils. *Geotechnique*, Vol. 13, pp. 177-197.

Bonaquist, R.F. and Witczak, M.W. (1997). A Comprehensive Constitutive Model for Granular Materials in Flexible Pavement Structures. Proceedings, 8<sup>th</sup> International Conference on Asphalt Pavements, Seattle, Washington, USA. Vol. 1. pp. 783-802.

Boyce, J.R. (1976). The Behaviour of a Granular Material under Repeated Loading. Ph.D. Thesis, Department of Civil Engineering, University of Nottingham, Nottingham, UK.

Boyce, J.R. (1980). A Non-Linear Model for the Elastic Behaviour of Granular Materials Under Repeated Loading. Proceedings, International Symposium on Soils Under Cyclic and Transient Loading, Swansea, UK. pp. 285-294.

Boyce, J.R., Brown, S.F., and Pell, P.S. (1976). The Resilient Behaviour of a Granular Material Under Repeated Loading. Proceedings, 8<sup>th</sup> Australian Road Research Board Conference on Materials Construction and Maintenance, Vol. 8, Part. 3, pp. 1-12.

Brecciaroli, F. and Kolisoja, P. (2004). Ratapenkereen leveys ja luiskakaltevuus, esitutkimus. Ratahallintokeskus, kunnossapitoyksikkö. Helsinki 2004. Ratahallintokeskuksen julkaisu A 9/2004. (In Finnish)

Brown, S.F. (1974). Repeated Load Testing of a Granular Material. Geotechnical Engineering Division, The American Society of Civil Engineers, Vol. 100, No. G77, pp 825-841.

Brown, S.F. (1978). Material Characteristics for Analytical Pavement Design. Developments in Highway Pavement Engineering 1. Applied Science Publishers. pp 41-92.

Brown, S.F. (1985). Structural Role of the Granular Layer. Proceedings, 2<sup>nd</sup> Symposium on Unbound Aggregates in Roads, Nottingham, UK. pp. 1-6.

Brown, S.F. and Chan, F.W.K. (1996). Reduced Rutting in Unbound Granular Pavement Layers Through Improved Grading Design. Proceedings, Institution of Civil Engineers, Vol. 117, pp. 40-49.

Brown, S.F. and Hyde, A.F.L. (1975). Significance of Cyclic Confining Stress in Repeated Load Triaxial Testing of Granular Material. Transportation Research Record, No. 537, pp. 49-58.

Brown, S.F. and Pappin, J.W. (1981). Analyses of Pavements with Granular Bases. Transportation Research Record, No. 810, pp. 17-23.

Brown, S.F. and Pappin, J.W. (1985). Modelling of Granular Materials in Pavements. Transportation Research Record, No. 1022, pp. 45-51.

Brown, S.F. and Pell, P.S. (1967). An Experimental Investigation of the Stresses, Strains and Deflections in a Layered Pavement Structure Subjected to Dynamic Loads. Proceedings, 2<sup>nd</sup> International Conference on Structural Design of Asphalt Pavements, Ann Arbor, USA. pp. 487-504.

Brown, S.F. and Selig, E.T. (1991). The design of Pavement and Rail Track Foundations. Cyclic Loading of Soils: from Theory to Design, O'Reilly and Brown, Editors, Von Nostrand Reinhold, N.Y. pp. 249-305.

CEN 2004 EN 13286. Unbound and Hydraulically Bound Mixtures. Part 7: Cyclic Load Triaxial Tests for Unbound Mixtures. European Standard.

Chan, F.W.K. (1990). Permanent Deformation Resistance of Granular Layers in Pavements. Ph.D. Thesis, Department of Civil Engineering, University of Nottingham, Nottingham, UK.

Chang, C.S., Sivanuja, S.S., and Misra, A. (1989). Initial Moduli of Particulated Mass with Frictional Contacts. International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 13, No. 6, pp. 629-644.

Chazallon C. 2000, An elastoplastic model with kinematic hardening for unbound aggregates in roads, UNBAR 5 Conference, Nottingham, 21-23 June, pp 265 – 270.



Cheung, L.W. (1994). Laboratory Assessment for Pavement Foundation Materials. Ph.D. Thesis, Department of Civil Engineering, University of Nottingham, Nottingham, UK.

Coffman, B.S., Kraft, D.G. and Tamayo, J. (1964). A comparison of Calculated and Measured Deflections for the AASHO (American Association of State Highway Officials) Test Road. Proceedings, Association of Asphalt Paving Technologists, Vol. 33, pp. 54-87.

Collins, I.F., Wang, A.P. and Saunders, L.R. (1993). Shakedown Theory and the Design of Unbound Pavements. Australian Road Research Board, Road and Transport Research, Vol. 2, No. 4, pp.28-37.

Collins, I.F. and Boulbibane, M. (1998). The Application of Shakedown Theory to Pavement Design. Metals and Materials, Vol. 4, No. 4, pp. 28-37.

Coronado, O., Fleureau, J.-M., Correia, A.G., and Caicedo, B. (2005). Influence of Suction on the Properties of Two Granular Road Materials. Proceedings, 7<sup>th</sup> International Conference on the Bearing Capacity of Roads, Railways and Airfields, Trondheim, Norway.

Correia, A.G., Hornych, P., and Akou, Y. (1999). Review of Models and Modelling of Unbound Granular Materials. Workshop on Modelling and Advanced Testing for Unbound Granular Materials, Lisbon, Portugal.

Cumberland, D. J. and Crawford, R. J. (1987). The Packing of Particles. Vol. 6 of the Handbook of Powder Technology Series, Elsevier, Amsterdam, The Netherlands.

Davis, R.O. and Selvadurai, A.P.S. (1996). Elasticity and Geomechanics. Cambridge University Press, Cambridge, UK.

Dawson, A.R. (1990). Introduction to Soils and Granular Materials. Lecture Notes from Residential Course, Bituminous Pavements: Materials, Design and Evaluation. University of Nottingham, Nottingham, UK.

Dawson, A.R., Correia, A.G., Jaure, P., Paute, J., and Galjaard, P.J. (1994). Modelling Resilient and Permanent Deflections in Granular and Soil Pavement Layers. Proceedings, 4<sup>th</sup> International Conference on Bearing Capacity of Roads and Airfields, Vol. 2, Minneapolis, Minnesota, USA. pp. 847-861.

Dawson, A and Kolisoja, K. (2004). Permanent Deformation. Final Report of the Project Roadex II – Northern Periphery.

Dawson, A.R., Thom, N.H., and Paute, J.L. (1996). Mechanical Characteristics of Unbound Granular Materials as a Function of Condition. Flexible Pavements. Edited by A.G. Correia. Proceedings, European Symposium on Flexible Pavements, Euroflex. Laboratório Nacional de Engenharia Civil, Lisbon, Portugal. pp. 35-44.

Dawson, A.R. and Wellner, F. (1999). Plastic Behaviour of Granular Materials. Final Report ARC Project 933. Department of Civil Engineering, University of Nottingham, Reference PRG990, Nottingham, UK.

Dehlen, G.L. (1969). The Effect of Non-Linear Material Response on the Behaviour of Pavements Subjected to Traffic Loads. Ph.D. Thesis, University of California, Berkeley, California, USA.

Desai, C.S., Somasundaram, S., and Frantziskonis, G. (1986). A Hierarchical Approach for Constitutive Modeling of Geologic Materials. *International Journal for Numerical and Analytical Methods in Geomechanics*, John Wiley and Sons, Vol. 10, pp. 225-257.

Dorby, R., Ladd, R.S., Yokel, F.Y., Chung, R.M., and Powell, D. (1982). Prediction of Pore Water Pressure Buildup and Liquefaction of Sand During Earthquakes by Cyclic Strain Method. Building Science Series 138. Washington, D.C.: National Bureau of Standards, U.S. Department of Commerce.

Dorby, R., Ng, T.-T., and Petrakis, E. (1989). Deformation Characteristics of Granular Soils in the Light of Particulate Mechanics. XIV Conferenza di Geotecnica di Torino, Torino, Italy.

Drumm, E.C. and Madgett, M.R. (1997). Subgrade Resilient Modulus Correction for Saturation Effects. *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 123, No. 7.

Duncan, J.M. and Chang, C.Y. (1970). Nonlinear Analysis of Stress and Strain in Soils. *Proceedings, The American Society of Civil Engineers*, Vol. 96, No. SM5, pp. 1629-1653.

Dunlap, W.A. (1963). A Report on Mathematical Model Describing the Deformation Characteristics of Granular Materials. Technical Report No. 1, Project 2-8-62-27, Texas Transportation Institute, Texas, USA.

Dunlap, W.A. (1966). Deformation Characteristics of Granular Materials Subjected to Rapid Repetitive Loading. Ph.D. Thesis, Texas A&M University, College Station, Texas, USA.

Ehrola, E. (1996). Liikenneväylien Rakennesuunnittelun Perusteet. Helsinki: Rakennustieto Oy. (In Finnish)

Ekblad, J. (2004). Influence of Water on Resilient Properties of Coarse Granular Materials. Licentiate Thesis, Kungliga Tekniska Högskolan (KTH), Stockholm, Sweden.

El abd, A., Horny, P., Breyse, D., and Denis, A. (2005). Prediction of Permanent Deformation of Unbound Pavement Layers. *Proceedings, 7<sup>th</sup> International Conference on the Bearing Capacity of Roads, Railways, and Airfields*, Trondheim, Norway.



El abd, A., Hornych, P., Breysse, D., Denis, A., and Chazallon, C. (2004). A Simplified Method of Prediction of Permanent Deformation of Unbound Pavement Layers. Proceedings, 6<sup>th</sup> International Symposium on Pavements Unbound, Nottingham, England, UK. pp. 179-189.

Elhannani, M. (1991). Modélisation et Simulation Numérique des Chaussées Souple. Ph.D. Thesis, University of Nantes, Nantes, France. (In French)

Elliot, R.P. and Lourdesnathan, D. (1989). Improved Characterization Model for Granular Bases. Transport Research Record, No. 1227, pp. 128-133.

Fleureau, J.-M., Hadiwardoyo, S., and Gomes Correia, A (2003). Generalised Effective Stress Analysis of Strength and Small Strains Behaviour of a Silty Sand, from Dry to Saturated State. Soils and Foundations, 43, 4 : 21-33.

Flintsch, G.W., Al-Qadi, I.L., Park, Y., Brandon, T.L., and Appea, A. (2003). Relationship between Backcalculated and Laboratory-Measured Resilient Moduli of Unbound Materials. Transportation Research Board Meeting, Washington, D.C., USA.

Fredlund, D.G., Morgenstern, R.N., and Widger, A. (1978). Shear Strength of Unsaturated Soils. Canadian Geotechnical Journal, Vol. 15, No. 3, pp. 313-321.

Fredlund, D.G. and Rahardjo, H. (1993). Soil Mechanics for Unsaturated Soils. New York: Wiley: 517 p.

Garg, N. and Thompson, M.R. (1997). Triaxial Characterization of Minnesota Road Research Project Granular Materials. Transport Research Record, No. 1577, pp. 27-36.

Gerrard, C.M., Morgan, J.R., and Richards, B.G. (1975). An Approach to the Design of Flexible Pavements for Australian Conditions. Australian Road Research Report 5 (8), Australia.

Gidel G. (1997) Étude des déformations permanentes des graves non traitées sous chargements répétés. École centrale de Paris, Paris, France.

Gidel, G, Hornych, P.; Chauvin, J.J., Breysse, D. and Denis, A. (2001). Nouvelle Approche pour l'Étude de Déformations Permanentes des Graves non Traitées à l'Appareil Triaxial à Chargement Répétés. Bulletin de Liaison des Laboratoires des Ponts et Chaussées, No. 233, pp. 5-21. (In French)

Gleitz, T. 1996. Calculation of Non-Linear Behaviour of Granular Base Materials. M.Sc. Thesis, Faculty of Civil Engineering, Dresden University of Technology, Dresden, Germany. (In German)

Goto, S., Tatsuoka, F., Shibuya, S., Kim Y.-S., and Sato, T. (1991). A simple Gauge for Local Small Strain Measurements in the Laboratory. Soils and Foundations, 31, 1 : 169-180.

Gray, J.E. (1962). Characteristics of Graded Base Course Aggregates Determined by Triaxial Tests. *Engrg. Res. Bull.*, No. 12, National Crushed Stone Association.

Guan, Y., Drumm, E.C. and Jackson, N.M. (1997). Weighting Factor for Seasonal Subgrade Resilient Modulus. Transportation Research Board, 77<sup>th</sup> Annual Meeting, Washington, D.C., USA.

Hansson, J.S. and Lenngren, C.A. (2005). Unbound Material Resilient Behaviour Due to Post Compaction – A Study Comparing FWD and Triaxial Tests. *Proceedings, 7th International Conference on the Bearing Capacity of Roads, Railways and Airfields*, Trondheim, Norway.

Hardin, B.O. and Richart, F.E.Jr. (1963). Elastic Waves Velocities in Granular Soils. *The American Society of Civil Engineers, Journal of the Soil Mechanics and Foundation Engineering Division*, Vol. 89, No. 1, pp. 33-65.

Haynes, J.G. and Yoder, E.J. (1963). Effect of Repeated Loading on Gravel and Crushed Stone Base Course Materials Used in the AASHO (American Association of State Highway Officials) Road Test. *Highway Research Record*, No. 39.

Head, K.H. (1994). *Manual of Soil Laboratory Testing. Volume 2: Permeability, Shear Strength and Compressibility Tests*, 2<sup>nd</sup> Edition, Halsted Press, Wiley and Sons, Inc., New York, New York, USA.

Heydinger, A.G. (2003). Evaluation of Seasonal Effects on Subgrade Soils. Transportation Research Board Meeting, Washington, D.C., USA.

Heydinger, A.G., Xie, Q.L., Randolph, B.W., and Gupta, J.D. (1996). Analysis of Resilient Modulus of Dense and Open-Graded Aggregates. *Transportation Research Record*, No. 1547, pp. 1-6.

Hicks, R.G. (1970). Factors Influencing the Resilient Properties of Granular Materials. Ph.D. Thesis, University of California, Berkeley, California, USA.

Hicks, R.G. and Monismith, C.L. (1971). Factors Influencing the Resilient Response of Granular Materials. *Highway Research Record* 345, pp.15-31.

Hicks, R.G. and Monismith, C.L. (1972). Prediction of the Resilient Response of Pavements Containing Granular Layers Using Non-Linear Elastic Theory. *Proceedings, 3<sup>rd</sup> International Conference on Asphalt Pavements*, Vol. 1, pp. 410-429.

Hoff, I. (1999). Material properties of Unbound Aggregates for Pavement Structures. Ph.D. Thesis, Department of Road and Railway Engineering, Norwegian University of Science and Technology, Trondheim, Norway.

Hoff, I. (2004). GARAP – Evaluation of Different Laboratory Vompaction Methods for Preparation of Cyclic Triaxial Samples. SINTEF Report STF22 A04339.



Hoff, I., Nordal, S., and Nordal, R.S. (1999). Constitutive Model for Unbound Granular Materials Based on Hyperelasticity. Workshop on Modelling and Advanced Testing for Unbound Granular Materials, Lisbon, Portugal.

Hoff, I., Arvidsson, H., Erlingsson, S., Houben, L.J.M., and Kolisoja, P. (2005). Round Robin Investigation on the Cyclic Triaxial Test for Unbound Granular Materials. Proceedings, 7<sup>th</sup> International Conference on the Bearing Capacity of Roads, Railways and Airfields, Trondheim, Norway.

Holubec, I. (1969). Cyclic Creep of Granular Materials. Department of Highways, Ontario, Canada. Report No. RR147.

Hornych, P., Kazai, A., and Piau, J.M. (1998). Study of the Resilient Behaviour of Unbound Granular Materials. Proceedings, 5<sup>th</sup> International Conference on the Bearing Capacity of Roads and Airfields, R.S. Nordal and G Rafsdal, eds., Vol. 3, Trondheim, Norway. pp. 1277-1287.

Huang, Y.H. (1993). Pavement Analysis and Design. Prentice-Hall, Inc. pp. 805.

Hudson, D.R. (1947). Packing of Materials in Bulk. Machinery, Vol. 70, No. 12, pp. 617-622, 681-683.

Huurman, M. (1997). Permanent Deformation in Concrete Block Pavements. Ph.D. Thesis, Department of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands.

Hyde, A.F.L. (1974). Repeated Load Triaxial Testing of Soils. Ph.D. Thesis, Department of Civil Engineering, University of Nottingham, Nottingham, UK.

Janbu, N. (1967). Settlement Calculations Based on the Tangent Modulus Concept. Geotechnical Division, Norwegian Institute of Technology, University of Trondheim, Trondheim, Norway. Bulletin 2.

Jin, M. S., Lee, K.W., and Kovacs, W. D. (1994). Seasonal Variation of Resilient Modulus of Subgrade Soils. Journal of Transportation Engineering, ASCE, Vol. 120, No. 4, pp. 603-616.

Johnson, K.L. (1986). Plastic Flow, Residual Stresses and Shakedown in Rolling Contact. Proceedings, 2<sup>nd</sup> International Conference on Contact Mechanics and Wear of Rail/Wheel Systems, University of Rhode Island, Kingston, Rhode Island, USA.

Johnson, T.C., Berg, R.L., and Dimillo, A. (1986). Frost Action Predictive Techniques: an Overview of Research Results. Transport Research Record, No. 1089, pp. 147-161.

Jorenby, B.N. and Hicks, R.G. (1986) Base Course Contamination Limits. Transportation Research Record, No. 1095. pp 86-101.

- Jouve, P., Martinez, J. Paute, J.L., and Ragneau, E. (1987). Rational Model for the Flexible Pavement Deformations. Proceedings, 6<sup>th</sup> International Conference on Structural Design of Asphalt Pavements, Vol. 1, Ann Arbor, Michigan, USA. pp. 50-64.
- Kamal, M.A., Dawson, A.R., Farouki, O.T., Hughes, D.A.B., and Sha'at, A.A. (1993). Field and Laboratory Evaluation of the Mechanical Behaviour of Unbound Granular Materials in Pavements. Transportation Research Record, No. 1406, pp. 88-97.
- Karasahin, M. (1993). Resilient Behaviour of Granular Materials for Analysis of Highway Pavements. PhD Thesis, Department of Civil Engineering, University of Nottingham, Nottingham, UK.
- Kenis, W.J. (1978). Predictive Design Procedure VESYS User's Manual: an Interim Design Method for Flexible Pavement Using the VESYS Structural Subsystem. Final Report No. FHWA-RD-77-54, Federal Highway Administration, Department of Transportation, Washington, D.C., USA.
- Kenis, W. and Wang, W., (1997). Calibrating Mechanistic Flexible Pavement Rutting Models from Full Scale Accelerated Tests. Proceedings, 8<sup>th</sup> International Conference on Asphalt Pavements, Seattle, Washington, USA. pp. 663-672.
- Kent, M.F. (1962). AASHO (American Association of State Highway Officials) Road Test Vehicle Operating Costs Related to Gross Weight. Highway Research Board, Special Report 73, pp. 149-165.
- Khedr, S. (1985). Deformation Characteristics of Granular Base Course in Flexible Pavement. Transportation Research Record, No. 1043, pp. 131-138.
- Kiehne, A. (2001). Program for Analysis of Triaxial Tests Results. Dresden University of Technology, Dresden, Germany.
- Kim, S.-E., Little, D.N., Masad, E., and Lytton, R.L. (2004). Determination of Anisotropic Moduli Considering Aggregate Particle Shape and Gradation in Unbound Granular Layers. Transportation Research Board, 83<sup>th</sup> Annual Meeting, Washington, D.C., USA.
- Klemt, R. (1990). The Influence of Friction on the Deformation Behaviour of Granular Base Materials. M.Sc. Thesis, Dresden University of Technology, Dresden, Germany.
- Kolisoja, P. (1993). Sitomattomien kerrosten kiviainesten muodonmuutosominaisuudet. Tielaitoksen selvityksiä 38/1993, Helsinki, Finland. (In Finnish)
- Kolisoja, P. (1994). Large Scale Dynamic Triaxial Tests with Coarse Grained Aggregates. Proceedings, 4<sup>th</sup> International Conference on the Bearing Capacity of Roads and Airfields, Minneapolis, Minnesota, USA.
- Kolisoja, P. (1996a) Large Scale Dynamic Triaxial Tests, I. Delprosjektrapport KPG 05.



Kolisoja, P. (1996b) Large Scale Dynamic Triaxial Tests Results of the Supplementary Test Series. Delprosjektrapport KPG 07.

Kolisoja, P. (1997). Resilient Deformation Characteristics of Granular Materials. Ph.D. Thesis, Laboratory of Foundations and Earth Structures, Tampere University of Technology, Tampere, Finland.

Kolisoja, P. (1998). Large Scale Dynamic Triaxial Tests III. Institute of Foundations and Earth Structures, Tampere University of Technology, Tampere, Finland.

Kolisoja, P., Vuorimies, N., and Saarenketo, T. (2004). Assessment of the Effect of Seasonal Variations on the Unbound Granular Materials of Low Volume Roads by Laboratory Testing. 6<sup>th</sup> International Symposium on Pavements Unbound, Nottingham, England, UK.

Kuhn, L.T., McMeeking, R.M. and Lange, F.F. (1991). A Model for Power Consolidation. Journal of the American Ceramic Society, Vol. 74, No. 3, pp. 682-685.

Ladd, R.S. (1978). Preparing Test Specimens Using Undencompaction. Geotechnical Testing Journal, Vol. 1, No. 1, pp. 16-23.

Lade, P.V. and Nelson, R.D. (1987). Modelling the Elastic Behaviour of Granular Materials. International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 2, pp.521-542.

Lambe, T.W. and Whitman, R.V. (1979). Soil Mechanics, SI Version. Wiley, New York. 553 p.

Lashine, A.K., Brown, S.F., and Pell, P.S. (1971). Dynamic Properties of Soils. Report No. 2 Submitted to Koninklijke/Shell Laboratorium, Department of Civil Engineering, University of Nottingham, Nottingham, UK.

Lekarp, F. (1997). Permanent Deformation Behaviour of Unbound Granular Materials. Licentiate Thesis, Kungl Tekniska Högskolan, Stockholm, Sweden.

Lekarp, F. (1999). Resilient and Permanent Deformation Behaviour of Unbound Granular Aggregates Under Repeated Loading. Ph.D. Thesis, Kungl Tekniska Högskolan, Stockholm, Sweden.

Lekarp, F., Richardson, I.R., and Dawson, A. (1996). Influences on Permanent Deformation Behaviour of Unbound Granular Materials. Transportation Research Record, No. 1547, pp 68-75.

Lekarp, F. and Dawson, A.R. (1998). Some Influences on the Permanent Deformation Behaviour of Unbound Granular Materials. US Transportation Research Board, Transportation Research Board RE 960372.

Lentz, R.W. and Baladi, G.Y. (1980). Simplified Procedure to Characterize Permanent Strain in Sand Subjected to Cyclic Loading. Proceedings, International Symposium on Soils Under Cyclic and Transient Loading, Swansea, UK. pp 89-95.

Lentz, R.W. and Baladi, G.Y. (1981). Constitutive Equation for Permanent Strain of Sand Subjected to Cyclic Loading. Transportation Research Record, No. 810, pp- 50-54.

Liu, M. (1993). Numerical Prediction of Pavement Distress with Geotechnical Constitutive Laws. Ph.D. Thesis, Texas A&M University, College-Station, Texas, USA.

Lytton, R.L. (1995). Foundations and Pavements on Unsaturated Soils. 1<sup>st</sup> International Conference on Unsaturated Soils, Paris, France.

Lytton, R.L., Uzan, J., Fernando, E., Roque, R., Hiltunen, D., and Stoffels, S.M. (1993). Development and Validation of Performance prediction Models and Specifications for Asphalt Binders and Paving Mixers. Strategic Highway Research Program. SHRPA A-357, National Research Council, Washington, DC, USA.

Maree, J.H. (1982). Aspects of the Design and the Behaviour of Road Pavements with Granular Material Base Layers. Ph.D. Thesis, Department of Civil Engineering, University of Pretoria, Pretoria, South Africa. (In Afrikaans)

Maree, J.H., Freeme, C.R., Van Zyl, N.J., and Savage, P.F. (1982). The permanent Deformation of Pavements with Untreated Crushed Stone Bases as Measured in Heavy Vehicle Simulator Tests. Proceedings, 11<sup>th</sup> Australian Road Research Board Conference, Part 2, pp. 16-28.

Marek, C.R. (1977). Compaction of Graded Aggregate Bases and Sub-Bases. Proceedings, The American Society of Civil Engineers, Vol. 103, pp. 103-113.

Masad, E. (2000). Unified Imaging Approach for Measuring Aggregate Angularity and Texture. Computer-Aided Civil and Infrastructure Engineering, 15. pp. 273-280

Masad, E. (2001). Correlation of Fine Aggregate Imaging Shape Indices with Asphalt Mixture Performance. Transportation Research Record No. 1757, Transportation Research Board, National Research Council, Washington, DC, USA. Pp.148-156.

May, R.W. and Witczak, M.W. (1981). Effective Granular Modulus to Model Pavement Responses. Transport Research Record, No. 810, pp. 1-9.

Mayhew, H.C. (1983). Resilient Properties of Unbound Road Base under Repeated Triaxial Loading. Laboratory Report 1088, Transport and Road Research Laboratory, Crowthorne, UK.

Melan, E. (1936). Theorie Statisch Unbestimmter Systeme aus Ideal-Plastischen Baustoff. Sitzungsberichte der Akademie der Wissenschaften in Wien, Vol. Iia, pp 145-195.



Mitry, F.G. (1964). Determination of the Modulus of Resilient Deformation of Untreated Base Course Materials. Ph.D. Thesis, University of California, Berkeley, California, USA.

Monismith, C.L., Seed, H.B., Mitry, F.G., and Chan, C.K. (1967). Prediction of Pavement Deflections from Laboratory Tests. Proceedings, 2<sup>nd</sup> International Conference on Structural Design of Asphalt Pavements, Ann Arbor, USA. pp. 109-140.

Moore, W.M., Britton, S.C., and Scrivner, F.H. (1970). A Laboratory Study of the Relation of Stress to Strain for a Crushed Limestone Base Material. Research Report 99-5F, Study 2-8-65-99, Texas Transportation Institute, Texas A and M University, Texas, USA.

Morgan, J.R. (1966). The Response of Granular Material to Repeated Loading. Proceedings, 3<sup>rd</sup> Australian Road Board Conference, Sydney, Australia. pp. 1178-1192.

Mroz, Z., Norris, V.A., and Zienkiewicz, O.C. (1978). An Anisotropic Hardening Model for Soils and its Application to Cyclic Loading. International Journal for Numerical and Analytical Methods in Geomechanics, John Wiley and Sons, Vol. 2, pp. 203-221.

Naji, K., Zaman, M., Nevels, J., and Mann, J. (2003). Effect of Soil Suction on Resilient Modulus of Subgrade Soil Using the Filter Paper Technique. Transportation Research Board 82<sup>nd</sup> Annual Meeting, Washington, D.C., USA.

Nataatmadja, A. (1992). Resilient Modulus of Granular Materials Under Repeated Loading. Proceedings, 7<sup>th</sup> International Conference on Asphalt Pavements, Vol. 1, pp. 172-185.

Nataatmadja, A. and Parkin A.K. (1989). Characterization of Granular Material for Pavement Analysis. Canadian Geotechnical Journal, No. 26, pp. 725-730.

NCHRP Project 1-37A <http://www4.trb.org> [internet document] [referred 17.11.2005]. Available at <http://www4.trb.org/trb/crp.nsf/All+Projects/NCHRP+1-37A>.

Numrich, R. (2003). Non-Linear Resilient Deformation Behaviour of Unbound Granular Materials. Ph.D. Thesis, Dresden University of Technology, Dresden, Germany. (in German)

Nurmikolu, A. (2005). Degradation and Frost Susceptibility of Crushed Rock Aggregates Used in Structural Layers of Railway Track. Ph.D. Thesis, Laboratory of Foundations and Earth Structures, Tampere University of Technology, Tampere, Finland.

Ooi, P.S.K., Archilla, A.R., and Kealohi, G.S. (2004). Resilient Modulus Models for Compacted Cohesive Soils. Transportation Research Board, 83<sup>th</sup> Annual Meeting, Washington, D.C., USA.

Pappin, J.W., (1979). Characteristics of a Granular Material for Pavement Analysis. Ph.D. Thesis, Department of Civil Engineering, University of Nottingham, Nottingham, UK.

Pappin, J.W. and Brown, S.F. (1980). Resilient Stress-Strain Behaviour of a Crushed Rock. Proceedings, International Symposium on Soils under Cyclic and Transient Loading, Swansea, Wales, UK. Vol. 1, pp. 169-177.

Park, S.-W. (2000). Evaluation of Accelerated Rut Development in Unbound Pavement Foundations and Load Limits on Load-Zoned Pavements. Ph.D Thesis, Department of Civil Engineering (Transportation Engineering), Texas A&M University, College Station, Texas, USA.

Park, S.-W. and Fernando, E. (1998). Sensitivity Analysis of Stress-Dependency and Plastic Behaviour for Load Zoning. Proceedings, 5<sup>th</sup> International Conference on the Bearing Capacity of Roads, Railways, and Airfields, Trondheim, Norway. pp 627-636.

Park, H.-M., Kim, Y.R., and Park, S.-W. (2003). Condition Evaluation of the Pavement Foundations Using Multi-Load Level FWD Deflections. Journal of Korean Geotechnical Society, Seoul, Korea. Vol. 19, No. 6, pp. 261-271.

Park, S.-W. and Lytton, R.L. (2002). Prediction of Flexible Pavement Response Using Non-Linear Stress-Dependent Material Models. Proceedings, 6<sup>th</sup> International Conference on the Bearing Capacity of Roads and Airfields, Lisbon, Portugal. Vol. 1, pp. 231-240.

Park, S.-W. and Park, H.-M. (2005). Prediction of Deformation Behaviours on Stress-Dependent Unbound Pavement Foundations. Proceedings, 7<sup>th</sup> International Conference on the Bearing Capacity of Roads, Railways, and Airfields, Trondheim, Norway.

Paute, J.L., Jouve, P., Martinez, J. and Ragneau, E. (1988). Modèle de Calcul pour le Dimensionnement des Chaussées Souples. Bulletin de Liaison de Laboratoires des Ponts et Chaussées, No. 156, pp 21-36.

Paute, J.L., Hornych, P., and Benaben, J.P. (1993). Repeated Load Triaxial Testing of Granular Materials in the French Network of Laboratoires des Ponts et Chaussées. Flexible Pavements. Edited by A.G. Correia. Proceedings, European Symposium on Flexible Pavements, Euroflex. pp. 53-64.

Paute, J.L., Hornych, P., and Benaben, J.P. (1994). Comportement Mécanique des Graves Non-Traitées au Triaxial à Chargement Répétés. Bulletin de Liaison de Laboratoires Central des Ponts et Chaussées, Paris, France.

Pezo, R.F. (1993). A General Method of Reporting Resilient Modulus Tests of Soils – A Pavement Engineers's Point of View. 72<sup>nd</sup> Annual Meeting of the Transportation Research Board, Washington, D.C. USA.



Plaistow, N.C. (1994). Non-Linear Behaviour of Some Pavement Unbound Aggregates. M.Sc. Thesis, Department of Civil Engineering, University of Nottingham, Nottingham, UK.

Powell, M.J. (1980). Computer-Simulated Random Packing of Spheres. *Powder Technology*, Vol. 25, No. 1, pp. 45-52.

prEN 13286-7 (2002). Unbound and Hydraulically Bound Mixtures for Roads – Test Methods, Part 7 : Cyclic Load Triaxial Tests for Unbound Mixtures. Draft of European Standard Submitted to CEN Members, Brussels, Belgium.

Raad, L., Weichert, D. and Haidar, A. (1989). Shakedown and Fatigue of Pavements with Granular Bases. Transportation Research Board, Transportation Research Record, No. 1227, pp. 159-172.

Raad, L., Minassian, G., and Gartin, S. (1992). Characterization of Saturated Granular Bases under repeated Loads. Transportation Research Record, No. 1369. pp 73-82.

Rada, G. and Witczak, M.W. (1981). Comprehensive Evaluation of Laboratory Resilient Moduli Results for Granular Materials. Transportation Research Record, No. 810, pp. 23-33.

Raymond, G.P. and Williams, D.R. (1978). Repeated Load Triaxial tests on a Dolomite Ballast. Proceedings, The American Society of Civil Engineers, Vol. 104, No. GT7, pp. 1013-1029.

Robinson, R.G. (1974). Measurement of the Elastic Properties of Granular Materials Using a Resonance Method. Transportation Research Record, Supplementary Report No. 111UC.

Saarenketo, T. and Scullion, T. (1996). Laboratory and GPR Tests to Evaluate Electrical and Mechanical Properties of Texas and Finnish Base Course Aggregates. Proceedings, 6<sup>th</sup> International Conference on Ground Penetrating Radar, GPR '96, Sendai, Japan. pp. 477-482.

Seed, H.B., Mitry, F.G., Monismith, C.L., and Chan, C.K. (1965). Prediction of Pavement Deflection from Laboratory Repeated Load Tests. Report No. TE-65-6, Soil Mechanics and Bituminous Materials Research Laboratory, University of California, Berkeley, California, USA.

Seed, H.B., Mitry, F.G., Monismith, C.L., and Chan, C.K. (1967). Prediction of Flexible Pavement Deflections from Laboratory Repeated Load tests. National Cooperative Highway Research Program Report No. 35.

Seed, H.B. and Idriss, I.M. (1970). Soil Moduli and Damping Factors for Dynamic Response Analyses. Report No. EERC 70-10, Earthquake Engineering Research Center, University of California, Berkeley, California, USA.

Sharp, R.W. (1983). Shakedown analysis and the Design of Pavements. Ph.D. Thesis, University of Sydney, Sydney, Australia.

Sharp, R.W. (1985). Pavement Design Based on Shakedown Analysis. Transportation Research Record, No. 1022, pp. 99-107.

Sharp, R.W. and Booker J.R. (1984). Shakedown of Pavements under Moving Surface Loads. The American Society of Civil Engineers, Journal of Transportation Engineering, Vol. 110, No. 1, pp. 1-14.

Shaw, P.S. (1980). Stress-Strain Relationships for Granular Materials Under Repeated Loading. Ph.D. Thesis, Department of Civil Engineering, University of Nottingham, Nottingham, UK.

Slyngstad, T. (1985). Overbyggings- og Undergrunnsmaterialers Egenskaper. Friksjonsjordarter. VTI Maddelande 512. Linköping: Statens Väg- och Trafikinstitut. pp 30-44. (In Norwegian).

Smith, W.O., Foote, P.D., and Busang, P.F. (1929). Packing of Homogeneous Spheres. Physical Review, Vol. 34, No. 2, pp. 1271-1274.

Smith, W.S. and Nair, K. (1973). Development of Procedures for Characterization of Untreated Granular Base Course Materials and Asphalt Treated Base Course Materials. Federal Highway Administration, Report No. FHWA-RD-74-61, Washington, D.C., USA.

Standish, N. and Borger, D.E. (1979). The Porosity of Particulate Mixtures. Powder Technology, Vol. 22, No. 1, pp. 121-125.

Stock, A.S. and Brown, S.F. (1980). Non-Linear Characterization of Granular Materials for Asphalt Pavement Design. Transportation Research Record, No. 755, pp. 14-30.

Strategic Highway Research Program (SHRP) (1989). Resilient Modulus of Unbound Granular Base/Subbase Materials and Subgrade Soils. Strategic Highway Research Program Protocol P-46, UG07, SS07, Washington, D.C., USA.

Suzuki, M. and Oshima, T. (1985a). Comparisons Between the Computer-Simulated Results and the Model for Estimating the Co-Ordination Number in a Three-Component Random Mixture of Spheres. Powder Technology, Vol. 43, No. 1, pp.19-25.

Suzuki, M. and Oshima, T. (1985b). Co-Ordination Number of a Multi-Component Randomly Packed Bed of Spheres with Size Distribution. Powder Technology, Vol. 44, No. 3. pp. 213-218.

Sweere, G.T.H. (1990). Unbound Granular Bases for Roads. Ph.D. Thesis, University of Delft, Delft, The Netherlands.



Tam, W.A. and Brown, S.F. (1988). Use of the Falling Weight Deflectometer for In Situ Evaluation of Granular Materials in Pavements. Proceedings, 14<sup>th</sup> Australian Road Research Board Conference, Vol. 14, Part 5, pp. 155-163.

Taylor, K.L. (1971). Finite Element Analysis of Layered Road Pavements. Ph.D. Thesis, University of Nottingham, Nottingham, UK.

Terzaghi, K. (1936). Shear Resistance of Saturated Soils. Proceedings, 1<sup>st</sup> International Conference on Soil Mechanics and Foundation Engineering, Cambridge, Massachusetts, USA. Vol. 1, pp. 54-56.

Theyse H.L. (2000). The Development of Mechanistic-Empirical Permanent Deformation Design Models for Unbound Pavement Materials from Laboratory and Accelerated Pavement Test Data. UNBAR 5. pp. 285-293.

Thom, N.H. (1988). Design of Road Foundations. Ph.D. Thesis, Department of Civil Engineering, University of Nottingham, Nottingham, UK.

Thom, N.H. and Brown, S. F. (1987). Effect of Moisture on the Structural Performance of a Crushed-Limestone Road Base. Transportation Research Record, No. 1121, pp. 50-56.

Thom, N.H. and Brown, S. F. (1988). The effect of Grading and Density on the Mechanical Properties of a Crushed Dolomitic Limestone. Proceedings, 14<sup>th</sup> Australian Road Research Board Conference, Materials and Testing, pp. 94-100.

Thom, N.H. and Brown, S. F. (1989). The Mechanical Properties of Unbound Aggregates from Various Sources. UNBAR 3, Nottingham, UK. pp 130-142.

Thom, N.H. and Dawson, A.R. (1989). The Permanent Deformation of a Granular Material Modelled Using Hollow Cylinder Apparatus. Paper Submitted to Transportation Research Board Meeting in Washington, D.C., USA.

Tian, P., Zaman, M.M., and Laguros, J.G. (1998). Variation of Resilient Modulus of Aggregate Base and Its Influence on Pavement Performance. Journal of Testing and Evaluation, ASTM, Vol. 26, No. 4, pp. 329-355.

Ting, J.M., Corkum, B.T., Kauffman, C.R., and Creco, C. (1989). Discrete Numerical Model for Soil Mechanics. ASCE, Journal of Geotechnical Engineering, Vol. 115, No. 3, pp. 379-398.

Ting, J.M. and Khwaja, M. (1993). An Ellipse-Based Discrete Element Model for Granular Materials. International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 17, No. 9, pp. 603-623.

Transportation Research Board (TRB) (1975). Test Procedures for Characterizing Dynamic Stress-Strain Properties of Pavement Materials. Special Report 162, Washington, D.C., USA.

Trollope, D.H., Lee, J.K. and Morris, J. (1962). Stress and Deformation in Two-Layer Pavement Structures Under Slow Repeated Loading. Proceedings, Australian Road Research Board, Vol. 1, Part 2, pp. 693-718.

Uthus, L, Hoff, I., and Horvli, I. (2005). A Study on the Influence of Water and Fines on the Deformation Properties of Unbound Aggregates. Proceedings, 7<sup>th</sup> International Conference on the Bearing Capacity of Roads, Railways and Airfields, Trondheim, Norway.

Uzan, J. (1985). Characterization of Granular Materials. Transportation Research Record, No. 1022, pp. 52-59.

Uzan, J. (1992). Resilient Characterization of Pavement Materials. International Journal for Numerical and Analytical Methods in Geomechanics, Chichester, New York, USA. No. 16(6), pp. 435-459.

Uzan, J., Witczak, M.W., Scullion, T., and Lytton, R.L. (1992). Development and Validation of Realistic Pavement Response Models. Proceedings, 7<sup>th</sup> International Conference on Asphalt Pavements, Seattle, Washington, USA.

Vallerga, B.A., Seed, H.B., Monismith, C.L., and Cooper, R.S. (1975) Effect of Shape, Size and Surface Roughness of Aggregate Particles on the Strength of Granular Materials. Road and Paving Material, ASTM, Special Technical Publication, No. 212, pp. 63-76.

Van Niekerk, A.A. (2002). Mechanical Behaviour and Performance of Granular Bases and Sub-Bases in Pavements. Ph.D. Thesis, Delft University of Technology, Delft, The Netherlands.

Van Niekerk, A.A., Houben, L.J.M., and Molenaar, A.A.A. (1998). Estimation of Mechanical Behaviour of Unbound Road Building Materials from Physical Material Properties. Proceedings, 5<sup>th</sup> International Conference on the Bearing Capacity of Roads and Airfields, Trondheim, Norway, 6-8 July 1998. Vol. 3, pp.1221-1233.

Veverka, V. (1979). Raming Van de Spoordiepte Bij Wagen met een Bitumineuze Verharding. De Wegentechniek, Vol. XXIV, No. 3, pp. 25-45.

Vik, G. (1996). Sykliske Vakuum-Treksialforsok på Pukk og Grus (NGI). (In Norwegian)

Vuong, B. (1992). Influence of Density and Moisture Content on Dynamic Stress Strain Behaviour of a Low Plasticity Crushed Rock. Australian Road Research Board, Road and Transportation Research, Vol. 1, No. 2, pp 88-100.

Wadsworth, J. (1960). Experimental Examination of Local Processes in Packed Beds of Homogeneous Spheres. National Research Council of Canada, Mechanical Engineering Report MT-41. NRC 5895.



Wellner, F. (1996). Influence of the Stress Dependent Strain Behaviour of Unbound Road Bases on the Stress of Superposined Top Layers. In *Flexible Pavements, [Proceedings of the Euroflex Symposium, 1993, Lisbon, Portugal]*, edited by A. Gomes Correia. Rotterdam: Balkema. pp. 311-318.

Werkmeister, S. (2003). Permanent Deformation Behaviour of Unbound Granular Materials in Pavement Constructions. Ph.D. Thesis, Dresden University of Technology, Dresden, Germany.

Werkmeister, S., Dawson, A., and Wellner, F. (2001). Permanent Deformation Behaviour of Granular Materials and the Shakedown Concept. Transportation Research Board, 80<sup>th</sup> Annual Meeting, Washington, D.C., USA.

Werkmeister, S., Numrich, R., Dawson, A., and Wellner, F. (2003). Design of Granular Pavement Layers Considering Climatic Conditions. Transportation Research Board 82<sup>nd</sup> Annual Meeting, Washington, D.C., USA.

Werkmeister, S., Numrich, R. and Wellner, F. (2000). Resilient and Permanent Deformation Behaviour of Unbound Granular Materials. In *Unbound Aggregates in Road Construction. Proceedings, 5<sup>th</sup> International Symposium on Unbound Aggregates in Roads (UNBAR 5)*, Nottingham, UK. pp. 171-180.

Werkmeister, S., Numrich, R., and Wellner, F. (2002). Modelling of Granular Layers in Pavement Constructions. Proceedings, 9<sup>th</sup> International Society of Asphalt Pavement Conference, Copenhagen, Denmark.

Werkmeister, Wellner, F., Oeser, M., and Moeller, B. (2004). Design Criteria of Granular Pavement Layers. Proceedings, 6<sup>th</sup> International Symposium on Pavements Unbound, Nottingham, England, UK. pp. 209-218.

Witczak, M.W., and Uzan, J. (1988). The Universal Airport Pavement Design System, Report I of IV: Granular Material Characterization. University of Maryland, Maryland, USA.

Wolff, H. (1992). The Elasto-Plastic Behaviour of Granular Pavement Layers in South Africa. Ph.D. Thesis, Department of Civil Engineering, University of Pretoria, Pretoria, South Africa.

Wolff, H. and Visser, A.T. (1994). Incorporating Elasto-Plasticity in Granular Layer Pavement Design. Proceedings, Instn. Civil Engineers Transp., No. 105. pp. 259-272

Wu, S., Gray, D.H. and Richart, F.E.Jr. (1984). Capillary effects on Dynamic Modulus of Sands and Silts. The American Society of Civil Engineers, Journal of Geotechnical Engineering, Vol. 110, No. 9, pp. 1188-1203.

Youd, T.L. (1972). Compaction of Sands by Repeated Shear Staining. Proceedings, The American Society of Civil Engineers, Journal of Soil Mechanics and Foundation Division, Vol. 98, SM7, pp. 709-725.

Yu, H.S. and Hossain, M.Z. (1998). Lower Bound Shakedown Analysis of Layered Pavements Using Discontinuous Stress Fields. *Computer Methods in Applied Mechanics and Engineering*, Vol. 167, pp. 209-222.

Yu, A.B. and Standish, N. (1988). An Analitical-Parametric Theory of the Random Packing of Particles. *Powder Technology*, Vol. 55, pp. 171-186.

Yu, A.B. and Standish, N. (1993). Characterisation of Non-Spherical Particles from Their Packing Behaviour. *Powder Technology*, Vol. 74, pp. 205-213.

Zheng, J., Johnson, P.F., and Reed, J.S. (1990). Improved Equation of the Continuous Particle Size Distribution for Dense Packing. *Journal of the American Ceramic Society*, Vol. 73, No. 5, pp. 1392-1398.

ZTVT-StB 95 (1995). Zusätzliche Technische Vertragsbedingungen und Richtlinien für Tragschichten im Straßenbau (Specification for Unbound Granular Materials Used in Pavement Constructions). Bundesministerium für Verkehr, Bonn, Germany. (In German)



**Lyhennelmä kirjallisuusselvityksestä****RATAPENKEREIDEN MATERIAALIEN MUODONMUUTOS-  
KÄYTTÄYTYMINEN TOISTUVIEN KUORMIEN ALAISUUDESSA**

1 JOHDANTO JA TAUSTAA.....	2
2 TAVOITTEET.....	3
3 TUTKIMUKSEN SISÄLTÖ JA KIRJALLISUUSTUTKIMUKSEN RAKENNE.....	3
4 JOHTOPÄÄTÖKSET .....	7
4.1 Kirjallisuusselvityksen tulokset koko tutkimushankkeen kannalta .....	7
4.2 Palautuva muodonmuutoskäyttäytyminen.....	8
4.3 Pysyvä muodonmuutoskäyttäytyminen.....	9

## 1 JOHDANTO JA TAUSTAA

Tämä kirjallisuusselvitys aiheesta ”toistokuormitetut ratapengermaamateriaalit” on osa tutkimusprojektia, joka käsittelee ratapenkereitten leveyttä ja luiskakaltevuutta. Selvityksen on tehnyt diplomi-insinööri Fabrizio Brecciaroli Maa- ja pohjarakenteiden laitoksella professori Pauli Kolisojan ohjauksessa. Selvitys on jatkoa Brecciarolin aiempaan tutkimukseen, jonka tulokset on raportoitu Ratahallintokeskuksen julkaisuissa A 8/2004 ”Stabiliteetiltaan kriittiset ratapenkereet, esitutkimus” ja A 9/2004 ”Ratapenkereitten leveys ja luiskakaltevuus, esitutkimus”. Tutkimusprojekti jatkuu laboratoriokokeilla, maastomittauksilla ja ratarakenteiden mallintamisella, jotka täydentävät tutkimusta. Kun projekti päättyy, tulokset raportoidaan väitöskirjan muodossa.

Ilmastollisten olosuhteiden ankaruudesta johtuen kaikilla korkealuokkaisemmilla radoilla Suomessa käytettävät radan rakennekerrospaksuudet ovat huomattavan suuret moniin muihin, leudommilla alueilla sijaitseviin maihin verrattuna. Toisaalta ratapenkereet on kustannusten säästämiseksi jouduttu jättämään suhteellisen kapeiksi ja jyrkkäluiskaisiksi. Tästä johtuen eri yhteyksissä on esitetty arveluja, että ratapenkereissä tapahtuu toistuvan junakuormituksen alaisena vähittäistä ratapenkereen muodon latistumiseen johtavaa deformatumista.

Ratapenkereiden optimaalisella leveydellä ja luiskakaltevuudella on suuri taloudellinen merkitys, koska penkereiden leventäminen ja/tai pengerluiskien loiventaminen edellyttäisi huomattavan suuria taloudellisia investointeja. Toisaalta ratapenkereen latistuminen lisää tapahtuessaan oleellisesti radan kunnossapitotarvetta ja välillisenä vaikutuksena raiteen toistuva tukeminen taas lisää merkittävästi muun muassa raide-sepelin kulumista. Erityisen suuri merkitys ratapenkereen leveyteen ja muotoon liittyvillä valinnoilla on rataosilla, joiden sallittuja akselikuormia tai junanopeuksia ollaan nostamassa, koska junakuorman kasvun voidaan joka tapauksessa otaksua vaikuttavan ratapenkereen deformatumista kiihdyttävästi.

Tutkimuksen esiselvitysvaiheessa tehtyjen mallinnuslaskelmien perusteella ratapenkereitten staattisten murtokuormitusten arvot ovat pääsääntöisesti varsin suuria rataverkolla sallittaviin junakuormiin verrattuna (Ratahallintokeskuksen julkaisu A 9/2004 ”Ratapenkereitten leveys ja luiskakaltevuus, esitutkimus”). Toisaalta myös siirtymät ovat tällöin niin suuria, että vaikka todelliset siirtymät olisivat vain pienen osan näistä arvoista, siirtymät eivät käytännössä olisi enää kimmoisia eli penger luhistuisi vaiheittain varsin nopeasti. Näin ollen on ilmeistä, että ratapenkereitten leveyksiä ja luiskakaltevuuksia ei ole mahdollista arvioida pelkästään stattiseksi idealisoidun kuormituksen perusteella, vaan junakuormituksen lukuisia kertoja toistuva luonne on otettava tehtävissä tarkasteluissa huomioon.



## 2 TAVOITTEET

Kirjallisuusselvityksen päätavoitteet ovat:

1. Tehdä katselmus ajan tasalla olevaan tietämykseen karkearakeisten materiaalien muodonmuutoskäyttäytymisestä ratapenkereille tyypillisissä toistuvan kuormituksen olosuhteissa.
2. Selvittää kirjallisuuteen perustuen, miten ja missä määrin eri tekijät kuten jännitys- ja muodonmuutostasotaso, kuormituksen kesto, kuormitustaajuus ja kuormitus-historia sekä materiaalin tyyppi, vesipitoisuus, tiheys, rakeisuusjakautuma, hieno-rakeisten lajitteiden määrä, maksimiraekoko ja rakeiden muoto vaikuttavat karkea-rakeisten materiaalien sekä palautuvaan että pysyvään muodonmuutos-käyttäytymiseen ratapenkereissä toistuvan kuormituksen alaisuudessa.
3. Tehdä yhteenveto tarjolla olevista malleista, joita käytetään mallintamaan karkearakeisten materiaalien sekä palautuvaa että pysyvää muodonmuutos-käyttäytymistä ratapenkereille tyypillisen toistuvan kuormituksen alaisuudessa.
4. Tehdä yhteenveto erilaisista maailmalla yleisimmin käytössä olevista toisto-kuormituslaitteista, joilla tutkitaan karkearakeisten materiaalien käyttäytymistä syklisiä liikennekuormitusta simuloivissa olosuhteissa.

## 3 TUTKIMUKSEN SISÄLTÖ JA KIRJALLISUUSTUTKIMUKSEN RAKENNE

**Kappaleessa 2 tehdään yhteenveto maarakenteisiin kuuluvien insinööri-rakenteiden materiaalien mekaanisesta peruskäyttäytymisestä ja annetaan yleiset suuntaviivat karkearakeisten materiaalien muodonmuutosominaisuuksista. Lopuksi analysoidaan veden ja karkearakeisten materiaalien vuorovaikutuksia.**

Ratapenkereet muodostuvat karkearakeisista maamateriaaleista. Karkearakeinen materiaali on usein epähomogeeninen ja anisotrooppinen konglomeraatio, joka koostuu suuresta määrästä toistensa kanssa kosketuksessa olevia yksittäisiä makroskooppisia rakeita. Yksi seuraus rakeisesta luonteesta on, että käsiteltäessä karkearakeisilla materiaaleilla ei ole luontaista lujuutta kontinuumina eivätkä ne kestä käytännössä lainkaan vetojännitystä. Toisaalta ne voivat kestää suurehkoja puristusjännityksiä ja kohtullisia leikkausjännityksiä loputtomasti. Kun tällaiset materiaalit rakennetaan kerroksittain ja tiivistetään hyvin, ne kykenevät kantamaan liikennekuormia ja jakamaan kuormat alla oleviin kerroksiin tai pohjamaahan. Karkearakeisten materiaalien muodonmuutosvastus riippuu kuitenkin aina vaikuttavista jännityksistä.

Karkearakeisten materiaalien käyttäytyminen puristusjännityksen alaisuudessa on hyvin monimutkaista, koska pienilläkin jännitystasoilla esiintyy sekä palautuvia että pysyviä muodonmuutoksia. Palautuvat muodonmuutokset vaikuttavat ratapenkereen kykyyn kantaa ja jakaa kuormia, kun taas pysyvät muodonmuutokset vaikuttavat rata-penkereen pitkäaikaiseen muodonmuutoskäyttäytymiseen. Toisin sanoen rata-penkereissä esiintyy palautuvia muodonmuutoksia, jotka palautuvat kunkin



kuormituskerran jälkeen, ja pysyviä muodonmuutoksia, jotka kertyvät kunkin kuormituskerran jälkeen.

**Kappale 3 käsittelee karkearakeisten materiaalien mallinnusta yleisestä näkökulmasta. Mallit jaetaan kontinuumimekaniikan malleihin ja partikkelimekaniikan malleihin.**

Karkearakeisten materiaalien tämänhetkistä osaamisen huipputasoa edustava mallintaminen vaatii konstitutiivisten mallien käyttöä kaikille materiaaleille rata-penkereessä ja pohjamaassa. Mallit jaetaan kontinuumimekaniikan malleihin ja partikkelimekaniikan malleihin. Kontinuumimekaniikan menettelytapa ei ota huomioon materiaalin rakeista luonnetta. Sen sijaan ympäröivästä maamassasta eristetyn karkearakeisen materiaalin kuvitteellisen elementin oletetaan olevan homogeeninen koostumukseltaan. Jännityksien kuvitteellisessa elementissä oletetaan olevan jatkuvasti jakautuneita, kun taas rakeiden välissä oleviin kosketuspisteisiin vaikuttavia voimia ei oteta huomioon. Toisaalta partikkelimekaniikan mallinnuksessa vuorovaikutuksia partikkelien välillä tutkitaan eksplisiittisesti. Menettelytavan perusidea on se, että kun partikkelien määrä on riittävän suuri, partikkeliryhmän käyttäytyminen voidaan laajentaa kuvaamaan kyseisten materiaalien makroskooppista käyttäytymistä.

**Kappaleissa 4 ja 6 analysoidaan eri tekijöiden vaikutusta karkearakeisten materiaalien palautuvaan ja pysyvään muodonmuutuskäyttäytymiseen.**

Rakenteiden mitoitustarkoituksiin on tärkeää huomioida, miten rakenteessa mukana olevat materiaalit reagoivat ja palautuva muodonmuutuskäyttäytyminen muuttuu, kun eri vaikuttavat tekijät muuttuvat. Kirjallisuudesta löytyneiden tutkimusten mukaan näyttää siltä, että karkearakeisten materiaalien palautuva muodonmuutuskäyttäytyminen toistuvan liikennekuorman alaisuudessa riippuu vaihtelevin tärkeysastein monesta tekijästä kuten jännitystasosta, kuormituksen kestosta, kuormitustaajuudesta ja kuormitushistoriasta sekä materiaalin tyypistä, vesipitoisuudesta, tiheydestä, rakeisuudesta, hienorakeisten lajitteiden määrästä, maksimiraekoosta ja rakeiden muodosta.

Aiemmat tutkimukset ovat selvästi osoittaneet, että palautuviin muodonmuutoksiin vaikuttavat erityisesti jännitystaso ja materiaalin vesipitoisuus. Joidenkin muiden tekijöiden osalta johtopäätökset niiden vaikutuksesta karkearakeisen materiaalin palautuviin muodonmuutoksiin ovat epäyhtenäisempiä ja jossakin määrin jopa keskenään ristiriitaisia. Tällöin on kuitenkin syytä muistaa se, että tuloksia on eri yhteyksissä saatu varsin paljonkin toisistaan poikkeavilla materiaaleilla ja myös erilaisilla koejärjestelyillä.

Suurin osa tutkimuksista, jotka on tehty karkearakeisten materiaalien mekaanisista ominaisuuksista, käsittelee näiden materiaalien palautuvaa muodonmuutuskäyttäytymistä. Toisaalta pysyvästä muodonmuutuskäyttäytymisestä tehty tutkimus on suhteellisen vähäistä. Tämä johtuu todennäköisesti siitä, että pitkäaikaisen käyttäytymisen monitorointi on hyvin aikaa vievä ja hankala prosessi, kun tarvitaan erittäin suuri määrä kuormituskertoja ( $10^5$ – $10^6$  sykliä). Jokainen näyte voidaan lisäksi altistaa periaatteessa vain yhdellä jännitystasolla tehtävälle kuormitukselle, koska pysyvä muodonmuutuskäyttäytyminen riippuu huomattavasti näytteen jännityshistoriasta.



Ratapenger altistuu suurelle määrälle kuormituskertoja käyttöikänsä aikana. Vaikka pysyvä muodonmuutos on tavallisesti hyvin pieni osa yksittäisen kuormituskerran aiheuttamasta kokonaismuodonmuutoksesta, näiden pienten pysyvien muodonmuutosten asteittainen kerääntyminen voi johtaa lopulliseen murtumiseen. Ratapenkereen murtuminen sekä liian suuret pysyvät muodonmuutokset täytyy tietysti estää. Tämän tavoitteen saavuttamiseksi karkearakeisten materiaalien pysyvän muodonmuutoskäyttäytymisen tunteminen on hyvin tärkeää.

Palautuvien muodonmuutosten tavoin myös pysyvien muodonmuutosten kehittymiseen karkearakeisissa materiaaleissa vaikuttavat monet tekijät kuten jännitystaso, jännityshistoria, kuormituskertojen määrä, pääjännitysten kiertyminen, vesipitoisuus, tiheys, rakeisuus, materiaalin tyyppi sekä materiaalin partikkeleiden fysikaaliset ominaisuudet.

**Kappaleissa 5 ja 7 annetaan yleiskatsaus malleista, joita on tähän mennessä käytetty mallintamaan karkearakeisten materiaalien palautuvaa ja pysyvää muodonmuutoskäyttäytymistä.**

Mallintaminen on välttämätöntä karkearakeisten materiaalien mekaanisen toiminnan kuvaamisessa. Monet tutkijat ovat esittäneet erilaisia menetelmiä ennustamaan karkearakeisten materiaalien palautuvia ja pysyviä muodonmuutoksia. Suuri määrä olemassa olevia malleja on toisaalta sinällään todiste tutkimusalueen monimutkaisuudesta ja hankaluudesta. Monet tutkijat ovat esittäneet mallinnusmenetelmiä, jotka sopivat heidän omiin tutkimustuloksiinsa. Enemmän työtä tarvittaisiin kuitenkin kehittämään yleisempiä malleja, joilla on tukeva teoreettinen perusta ja laaja käytettävyys.

Vuodesta 1960 lähtien on tehty paljon tutkimusta karakterisoimaan karkearakeisten materiaalien palautuvaa muodonmuutoskäyttäytymistä. On yleisesti tunnettua, että nämä materiaalit käyttäytyvät monimutkaisesti, epälineaarisesti, ajasta riippuvasti ja elasto-plastistisesti toistuvan liikennekuorman alaisuudessa. Jotta tätä epälineaarisuutta voitaisiin käsitellä ja siirtyä pois perinteisestä kimmoteoriasta, karkearakeisten materiaalien palautuvan muodonmuutoskäyttäytymisen matemaattisessa mallintamisessa käytetään yleensä kahta menettelytapaa. Ensimmäisessä menettelytavassa jännitys-muodonmuutos-suhde annetaan jännityksestä riippuvana resilient-moduulina ja vakiona tai jännityksestä riippuvana Poissonin lukuna (esim. K- $\theta$  malli ja Uzanin malli). Toisessa menettelytavassa jännitys-muodonmuutos-suhde kuvataan jakamalla jännitykset ja muodonmuutokset tilavuudenmuutos- ja leikkausmuodonmuutos-komponentteihin. Materiaalin palautuvat muodonmuutokset määritellään tällöin käyttäen tilavuusmoduulia ja leikkausmoduulia resilient-moduulin ja Poissonin luvun sijaan. Tällaiset mallit ovat yleensä luonteeltaan monimutkaisempia ja parametriarvojen fysikaalinen merkitys on usein vaikeampi päätellä kerätyistä mittaustuloksista.

Karkearakeisten materiaalien pitkäaikaisen käyttäytymisen mallintamisessa on olennaista ottaa huomioon asteittainen pysyvien muodonmuutosten kerääntyminen, kuormituskertojen määrä ja jännitysolosuhteiden tärkeä rooli. Siksi yksi tärkeimmistä tavoitteista karkearakeisten materiaalien pitkäaikaikäkäyttäytymiseen liittyen olisi kehittää konstitutiivinen malli, joka mahdollistaa pysyvien muodonmuutosten ennustamisen millä tahansa kuormituskertojen määrällä annetulla jännitystasolla.

Vuosien varrella useat tutkijat ovat yrittäneet kehittää menetelmiä karkearakeisten materiaalien pysyvien muodonmuutosten ennustamiseen ja materiaalien pysyvää muodonmuutuskäyttäytymistä on mallinnettu useilla eri tavoilla. Jotkut malleista ovat logaritmisia kuormituskertojen määrän suhteen, kun taas toiset ovat hyperbolisia lähestyen asyymptootisesti muodonmuutoksen raja-arvoa kuormituskertojen määrän kasvaessa.

Huolimatta vuosien varrella tapahtuneesta edistyksestä karkearakeisten materiaalien muodonmuutuskäyttäytymisen ja erityisesti palautuvan muodonmuutuskäyttäytymisen ymmärtämisessä, kirjallisuustutkimuksen perusteella on todettavissa, että edelleen on olemassa tarvetta tutkimukselle, jonka avulla kehitettäisiin yleisempiä ja teoreettisesti parempia malleja ja menetelmiä karkearakeisten materiaalien sekä palautuvan että pysyvän muodonmuutuskäyttäytymisen ennustamiseen.

**Kappale 8 käsittelee erilaisia maailmalla yleisimmin käytettyjä kolmiaksiaalisia toistokuormituslaitteita, joita käytetään tutkimaan karkearakeisten materiaalien käyttäytymistä syklisen liikennekuorman alaisena.**

On huomattavaa, että karkearakeisten materiaalien käyttäytymistä tutkitaan yleensä laboratoriossa käyttämällä kolmiaksiaalisia toistokuormituskokeita. Vaikkakin nykyisin tarjolla olevat kolmiaksiaalikoelaitteet perustuvat samaan peruseriaatteeseen, koejärjestelyjen laatu ja rajoitteet vaihtelevat laboratorioden välillä. Kuinka tällaiset erot vaikuttavat koetuloksiin, pitäisi myös selvittää tarkemmin. Tavallinen kolmiaksiaalikoelaitte käyttää toistuvaa kuormaa sylinterinmuotoiselle näytteelle erilaisissa jännitysolosuhteissa. Keskeisin mitattava suure on näytteen aksiaalinen muodonmuutos (sylinterinmuotoisen näytteen lyhenemä) kuormituskertojen määrän funktiona (tavallisesti suuruusluokkaa 50000) tietyissä jännitysolosuhteissa. Moniportaisia kolmiaksiaalikokeita käytetään muodonmuutuskäyttäytymisen mittaamiseen useissa erilaisissa jännitysolosuhteissa. Jotta voitaisiin toteuttaa mahdollisimman realistinen simulaatio liikenteentyypiselle kuormitukselle käyttäen kolmiaksiaalikoelaitetta, kuormitussysteemin tulisi pystyä tuottamaan sekä pystysuuntainen pääjännitysero että sellipaine sellaisilla tasoilla ja taajuuksilla, jotka vastaavat todellisia kenttäolosuhteita.



## 4 JOHTOPÄÄTÖKSET

### 4.1 Kirjallisuusselvityksen tulokset koko tutkimushankkeen kannalta

Tämä kirjallisuusselvitys aiheesta ”toistokuormitetut ratapengermaamateriaalit” on osa ratapenkereiden leveyttä ja luiskakaltevuutta käsittelevää tutkimusprojektia ja syventää esitutkimusvaiheen yhteydessä tehtyä kirjallisuusselvitystä erityisesti toistokuormitettujen karkearakeisten maamateriaalien mekaaniseen käyttäytymiseen vaikuttavien tekijöiden ja mekaanisen käyttäytymisen mallintamisen osalta.

Tutkimusprojektin muut osat ovat:

- **Ratarakenteiden mekaanisen toiminnan mallintaminen:** kartoitetaan, testataan ja tarvittavassa määrin edelleen kehitetään toistokuormitettujen ratarakenteiden mekaanisen toiminnan kuvaamiseen soveltuvat mallinnustyökalut ja -menetelmät.
- **Laboratoriotutkimukset:** ratapengermaamateriaalien toistokuormituskokeet TTY:n Maa- ja pohjarakenteiden laitoksen suurimittakaavaista syklistä kolmiaksisiaalikoelaitteistoa käyttäen.
- **Kenttämittaukset:** laboratoriomittaustuloksiin perustuvat ratarakenteiden laskennalliset **mallinnustarkastelut** verifioidaan todellisista ratarakenteista tehtävin mittauksin. Mittauskohde sijaitsee Kokemäen–Rauman välisellä rataosalla. Koe-kohteella varioitiin hallitusti sekä ratapenkereen leveyttä että luiskakaltevuutta ja mitataan rakenteiden todellista käyttäytymistä sekä lyhyt- että pitkäaikaisen juna-kuormituksen alaisena.

Kirjallisuusselvityksen sekä tutkimusprojektin muiden osien tuloksista laaditaan väitöskirja. Väitöskirjatutkimuksen lopputavoitteena on muodostaa suuntaviivat erilaisille pohjamaaolosuhteille sijoittuvien ja eri tavoin kuormitettujen ratarakenteiden optimointiin rakenteen elinkaaren aikaisten kokonaisvaikutusten kannalta. Toisin sanoen tavoitteena on määritellä erilaisissa kuormitusolosuhteissa vaadittavat ratapenkereen minimileveydet sekä optimaaliset luiskakaltevuudet. Toisaalta pyrkimyksenä on luonnollisesti ratojen rakentamiseen ja ylläpitoon tarvittavien kiviainesten mahdollisimman säästeliäs käyttö. Yhtäältä kuormituksiin nähden liian heikosti rakennettujen ratapenkereiden ylläpidosta aiheutuvat kohtuuttoman suuret kunnossapitokustannukset ja muut haitat tulee kuitenkin myös voida välttää.

Kirjallisuusselvityksen päätulokset on lueteltu seuraavissa kappaleissa. Kappale 4.2 käsittelee palautuvaan muodonmuutoskäyttäytymiseen liittyviä tuloksia, kun taas kappale 4.3 käsittelee pysyvään muodonmuutoskäyttäytymiseen liittyviä tuloksia.

#### 4.2 Palautuva muodonmuutoskäyttäytyminen

- Aiemmat tutkimukset ovat kiistatta osoittaneet, että käytetty jännitystaso vaikuttaa eniten karkearakeisten materiaalien palautuvaan muodonmuutoskäyttäytymiseen ratapenkereessä. Resilient-moduuli kasvaa selkeästi pääjännitysten summan kasvaessa, mutta ratapengerrakenteille tyypillisellä jännitysalueella se kasvaa vain vähän pääjännityseron kasvaessa. Resilient-moduuliin ei käytännössä vaikuta määrityksessä käytetyn pääjännityseron taso olettaen, että ylenmääräistä plastista muodonmuutosta ei synny. Käytetty jännitystaso vaikuttaa myös Poissonin lukuun. Karkearakeisten materiaalien Poissonin luku kasvaa pääjännityseron kasvaessa ja hydrostaattisen jännityksen pienentyessä. Kuitenkaan suhde ei ole yhtä yksinkertainen kuin resilient-moduulin tapauksessa.
- Jännityshistoria, kuormituksen kesto ja kuormitustaajuus vaikuttavat vain vähän tai eivät lainkaan ratapenkereissä tyypillisesti käytettyjen karkearakeisten materiaalien palautuvaan muodonmuutoskäyttäytymiseen
- Resilient-moduuli pienenee vesipitoisuuden kasvaessa erityisesti korkeilla kyllästysasteilla. Tämä voidaan selittää veden voitelevalla efektillä tai sillä, että paikalliset huokosveden alipaineet pienenevät korkeammilla vesipitoisuuksilla johtaen pienempiin partikkelien välisiin kosketusvoimiin. Karkearakeisten materiaalien kyllästäminen vaikuttaa myös Poissonin lukuun. Poissonin luku pienenee, kun kyllästysaste kasvaa.
- Jos karkearakeiset materiaalit ratapenkereessä kyllästyvät täysin vedellä, niihin voi kehittyä huokosveden ylipainetta toistuvan kuorman alaisuudessa. Kun huokosvedenpainetta kehittyy, tehokas jännitys materiaalissa alenee aiheuttaen materiaalin lujuuden ja jäykkyyden pienenemistä. Voidaankin väittää, että itse kyllästysaste ei sinällään juurikaan vaikuta materiaalin käyttäytymiseen, vaan muodonmuutoskäyttäytymiseen vaikuttavat ensi sijaisesti huokosvedenpaineessa tapahtuvat muutokset.
- Kirjallisuudessa esitetyt lähteet ovat epäyhtenäisiä mitä tulee tiheyden vaikutukseen karkearakeisten materiaalien palautuvaan muodonmuutoskäyttäytymiseen toistuvan liikennekuorman alaisuudessa. Jotkut tutkimukset ovat päätyneet johtopäätöksiin, että tiheys on suhteellisen merkityksetön, kun taas toiset ovat päätelleet, että tiheyden vaikutus riippuu materiaalista. Useimmat tutkijat ovat kuitenkin tulleet siihen loogiselta tuntuvaan johtopäätökseen, että resilient-moduuli yleensä kasvaa tiheyden kasvaessa.
- Karkearakeisten materiaalien rakeisuus näyttää vaikuttavan jossakin määrin materiaalin jäykkyyteen, vaikkakin sen merkitystä pidetään yleisesti pienenä. Kun vettä on saatavilla suhteistuneisiin materiaaleihin, rakeisuuden merkitys kasvaa, koska nämä materiaalit voivat pidättää vettä huokosissa. Ne voivat myös saavuttaa korkeampia tiheyksiä kuin tasarakeiset materiaalit, koska pienimmät rakeet täyttävät huokokset isompien rakeiden välillä. Siksi rakeisuudella on epäsuora vaikutus karkearakeisten materiaalien palautuvaan muodonmuutoskäyttäytymiseen kontrolloimalla systeemin vesipitoisuuden ja tiheyden vaikutusta.



### 4.3 Pysyvä muodonmuutoskäyttäytyminen

- Jännitystaso on yksi tärkeimmistä tekijöistä, jotka vaikuttavat pysyvien muodonmuutosten kehittymiseen karkearakeisissa materiaaleissa toistuvien junaliikennekuormien alaisuudessa. Syntyneiden pysyvien muodonmuutosten taso riippuu suuresti jännitystasosta ja kasvaa pääjännityseron kohotessa ja hydrostaattisen jännityksen pienentyessä. Pysyvä muodonmuutoskäyttäytyminen karkearakeisissa materiaaleissa riippuu pääasiassa jännityssuhteesta, joka koostuu sekä pääjännityserosta että hydrostaattisesta jännityksestä.
- Pysyvien muodonmuutosten kehittyminen riippuu selvästi kuormitusjärjestyksestä. Jännitystason kasvusta johtuva pysyvä muodonmuutos on selvästi pienempää kuin muodonmuutos, joka syntyy, kun suurin jännitystaso kohdistuu kuormitettavaan materiaaliin välittömästi. Todellisissa ratapenkereissä tällä ilmiöllä ei kuitenkaan ole juurikaan käytännöllistä merkitystä, koska todellisissa junien akselikuormissa ei tämäntyyppistä ”kausittaista” vaihtelua esiinny.
- Kuormituskertojen määrä on yksi tärkeimmistä tekijöistä analysoitaessa karkearakeisten materiaalien pitkäaikaista muodonmuutoskäyttäytymistä kolmiaksisiaalisilla toistokuormituskokeilla, koska tämä mahdollistaa ratapenkereessä tapahtuvien muutosten arvioimisen. Useimmat elasto-plastiset mallit olettavat, että kaikki pysyvät muodonmuutokset kehittyvät ensimmäisen kuormituskerran aikana ja että lisäkuormitukset, jotka ovat tämän tason alapuolella, ovat ainoastaan elastisia. Tämä ei vastaa karkearakeisten materiaalien todellista muodonmuutoskäyttäytymistä toistuvien junaliikennekuormien alaisuudessa. Nämä synnyttävät yleensä lisää pysyviä muodonmuutoksia jokaisen kuormituskerran aikana.
- Kuormitettavan materiaalin korkean kyllästysasteen ja huonosta vedenpoistosta johtuvan matalan vedenläpäisevyyden yhdistelmä johtaa huokosveden ylipaineeseen, matalaan tehokkaaseen jännitykseen ja siten matalaan jäykkyyteen ja alhaiseen muodonmuutosten vastustuskykyyn. Huokosveden ylipaineen kehittymisen riski kasvaa, mitä äkillisemmille kuormitusolosuhteiden muutoksille materiaali altistetaan. Erityinen riski huokosveden ylipaineen kehittymiselle esiintyy silloin, kun materiaalin raerunkoa kuormitetaan useita kertoja. Tällaiset olosuhteet ovat varsin ominaiset myös junaliikenteen kuormittamalle ratapenkereelle, jos se jostain syystä on päätyntä veden kyllästämään tilaan.
- Kylmä sää saa maassa olevan veden jäätymään. Jäätymisrintaman kulkiessa alaspäin pitkän kylmän jakson aikana syntyy imu, joka imee vettä kohti jäätymisrintamaa. Tämä tarkoittaa, että routivaan maahan kerääntyy liikaa vettä jäänä. Kun sulaminen alkaa keväällä, vesi ei pääse poistumaan, koska alapuolinen rakenne pysyy vielä jäätyneenä. Routimista ei pitäisikään koskaan sallia ratapenkereessä, koska sen vastus pysyvää muodonmuutosta vastaan pienenee suuresti, kun pengermateriaalin vesipitoisuus kasvaa sulamisen vaikutuksesta.
- Tiheys on yksi tärkeimmistä karkearakeisten materiaalien pitkäaikaiseen muodonmuutoskäyttäytymiseen vaikuttavista tekijöistä ja näin ollen keskeinen tekijä myös pysyvien muodonmuutosten kehittymiseen ratapenkereissä. Pysyvien muodon-

muutosten vastustuskyky toistuvien kuormien alaisuudessa näyttää paranevan huomattavasti, kun tiheys eli materiaalin tiiviys paranee.

- Pysyvien muodonmuutosten vastustuskyky karkearakeisissa materiaaleissa alenee, kun hienoainespitoisuus kasvaa. Kasvava hienoainepitoisuus johtaa huomattavasti suurempaan alttiuteen pysyville muodonmuutoksille.
- Rakeiden muoto vaikuttaa nähtävästi eniten karkearakeisten materiaalien muodonmuutuskäyttäytymiseen epäsuorasti tiivistämisen kautta, koska rakeiden muoto vaikuttaa selvästi maa-aineksien tiivistämiseen. Maa-ainekset, joilla on hyvin liuskeinen tai pitkänomainen rakeiden muoto ovat luonnollisesti herkempiä hienontumiselle. Tämän seurauksena hienoainespitoisuus kasvaa ja pysyvät muodonmuutokset lisääntyvät samalla kun hienontuvat rakeet järjestäytyvät uudelleen.



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